

### Starter

1. A biased, five-sided spinner is numbered with scores 2, 4, 6, 8 and 10.

Let  $S$  be the score on a single spin, where  $P(S = s) = \frac{s}{k}$ , for some constant  $k$ .

(a) Determine the value of  $k$  and hence tabulate the probability distribution of  $S$ . 2

Question		Generic scheme	Illustrative scheme	Max mark												
1.	(a)	<ul style="list-style-type: none"> <li>•<sup>1</sup> calculate <math>k</math></li> <li>•<sup>2</sup> tabulate probability distribution</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> 30</li> <li>•<sup>2</sup> <table border="1" style="margin-left: 20px; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px;"><math>s</math></td> <td style="padding: 2px;">2</td> <td style="padding: 2px;">4</td> <td style="padding: 2px;">6</td> <td style="padding: 2px;">8</td> <td style="padding: 2px;">10</td> </tr> <tr> <td style="padding: 2px;"><math>P(S = s)</math></td> <td style="padding: 2px;"><math>\frac{1}{15}</math></td> <td style="padding: 2px;"><math>\frac{2}{15}</math></td> <td style="padding: 2px;"><math>\frac{1}{5}</math></td> <td style="padding: 2px;"><math>\frac{4}{15}</math></td> <td style="padding: 2px;"><math>\frac{1}{3}</math></td> </tr> </table> </li> </ul>	$s$	2	4	6	8	10	$P(S = s)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$	2
$s$	2	4	6	8	10											
$P(S = s)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$											

Aug 8-5:18 PM

### Mean & Variance

**Today we are learning...**

How to calculate the mean and variance of a random variable.

**I will know if I have been successful if...**

I can tabulate the probability distribution of a random variable.

I can find the  $E(X)$  and  $E(X^2)$

I can find the  $Var(X)$

Aug 8-5:18 PM

### Mean & Variance

The mean (denoted  $\mu$  or  $E(X)$ ) of the discrete random variable  $X$  is the average value that would be recorded in if an experiment was carried out.

$$\mu = E(X) = \sum_{\text{all } x} x p(x)$$

Calculate  $E(Y)$  for the sum of two dice.

What would we expect the sum of two dice to be?

Aug 8-5:18 PM

### Mean & Variance

The variance,  $V(X)$ , of a discrete random variable is a measure of how spread out the recorded values of  $X$  would be if the experiment were carried out a large number of times.

$$V(X) = E(X^2) - ((E(X))^2)$$

where  $E(X^2) = \sum_{\text{all } x} x^2 p(x)$

Calculate the  $V(Y)$  for the sum of the two dice.

Aug 8-5:22 PM

Page 34 - Exercise 2.1A

Page 36 - Exercise 2.1B

Aug 8-5:28 PM

1. A biased, five-sided spinner is numbered with scores 2, 4, 6, 8 and 10.

Let  $S$  be the score on a single spin, where  $P(S = s) = \frac{s}{k}$ , for some constant  $k$ .

- (a) Determine the value of  $k$  and hence tabulate the probability distribution of  $S$ . 2
- (b) Calculate  $E(S)$  and  $V(S)$ . 2

Jun 18-10:01

Question		Generic scheme	Illustrative scheme	Max mark												
1.	(a)	<ul style="list-style-type: none"> <li>•<sup>1</sup> calculate <math>k</math></li> <li>•<sup>2</sup> tabulate probability distribution</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> 30</li> <li>•<sup>2</sup> <table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td><math>s</math></td> <td>2</td> <td>4</td> <td>6</td> <td>8</td> <td>10</td> </tr> <tr> <td><math>P(S = s)</math></td> <td><math>\frac{1}{15}</math></td> <td><math>\frac{2}{15}</math></td> <td><math>\frac{1}{5}</math></td> <td><math>\frac{4}{15}</math></td> <td><math>\frac{1}{3}</math></td> </tr> </table> </li> </ul>	$s$	2	4	6	8	10	$P(S = s)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$	2
$s$	2	4	6	8	10											
$P(S = s)$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$											
	(b)	<ul style="list-style-type: none"> <li>•<sup>3</sup> calculate <math>E(S)</math></li> <li>•<sup>4</sup> calculate <math>V(S)</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>3</sup> <math>\frac{22}{3}</math></li> <li>•<sup>4</sup> <math>\frac{56}{9}</math></li> </ul>	2												

Jun 18-10:02

12. In a 'low stakes' area of a Las Vegas casino, a player pays 1 dollar to play a game where three unbiased regular octahedral dice with faces marked 1 to 8 are thrown.

- If all 3 dice show a 1 the player receives 100 dollars
- If 2 dice show a 1 the player receives 10 dollars
- If only 1 dice shows a 1 the player receives 1 dollar
- Otherwise the player receives nothing

The random variable  $X$  represents the player's profit for one game.

(a) Tabulate the probability distribution of  $X$ , with probabilities correct to 4 decimal places, and show that  $E(X) = -0.1029$  and  $SD(X) = 4.8562$ .

5

Jun 18-10:05

Question		Generic scheme	Illustrative scheme	Max mark										
12.	(a)	<ul style="list-style-type: none"> <li>•<sup>1</sup> correct values of <math>X</math></li> <li>•<sup>2&amp;3</sup> correct probabilities</li> <li>•<sup>4</sup> calculate <math>E(X)</math></li> <li>•<sup>5</sup> calculate <math>SD(X)</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1-3</sup></li> </ul> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td><math>X</math></td> <td>99</td> <td>9</td> <td>0</td> <td>-1</td> </tr> <tr> <td><math>P(X)</math></td> <td>0.0020</td> <td>0.0410</td> <td>0.2871</td> <td>0.6699</td> </tr> </table> <ul style="list-style-type: none"> <li>•<sup>4</sup> <math>E(X) = -0.1029</math> dollars</li> <li>•<sup>5</sup> <math>SD(X) = 4.8562</math> dollars</li> </ul>	$X$	99	9	0	-1	$P(X)$	0.0020	0.0410	0.2871	0.6699	5
$X$	99	9	0	-1										
$P(X)$	0.0020	0.0410	0.2871	0.6699										
<p><b>Notes:</b> Evidence of working required for •<sup>4</sup> and •<sup>5</sup></p>														

Jun 18-10:06