Starter - Past Paper 2016

MARKS

7. During a viral epidemic a doctor examines 150 people suffering from symptoms commonly associated with the virus. Of the 150 people examined, 90 are male of whom 40 actually have the virus. 10 of the examined females have the virus, the rest do not.

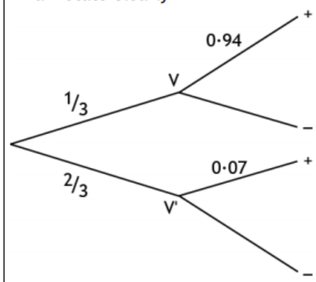
(a) Calculate the probability that an individual selected at random from this group is infected with the virus. **1**

(b) If 3 different people are selected at random without replacement from this group, what is the probability that all 3 have the disease? **2**

Of the people in this group with the virus 94% react positively to a clinical test to confirm the viral infection, as do 7% of the people without the virus.

(c) (i) Calculate the probability that a person selected at random reacts positively. **4**

May 30-09:55

Question			Generic Scheme	Illustrative Scheme	Max Mark
7	(a)		•¹ correct probability	•¹ $\frac{1}{3}$	1
Notes:					
Commonly Observed Responses:					
	(b)		•² correct probabilities	•² $\frac{50}{150}\times\frac{49}{149}\times\frac{48}{148}$	2
			•³ calculate probability	•³ $0\cdot0356$	
Notes:					
Commonly Observed Responses:					
	(c)	(i)	•⁴ appropriate strategy	•⁴ eg create tree diagram	4
			•⁵ continue strategy	•⁵ annotate clearly 	
			•⁶ complete strategy	•⁶ $\frac{1}{3}0\cdot94+\frac{2}{3}0\cdot07=$	
			•⁷ calculate probability	•⁷ $=0\cdot36$	

Random Variables

Today we are learning...

How we represent random variables as a probability distribution.

I will know if I have been successful if...

I understand what a random variable is and give its range space.

I understand what a probability distribution is.

I can calculate the mean and variance of a discrete random variable.

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Random Variables

Definition:

A random variable is a **function** which associates a unique real value with each outcome in the **sample space** of a random experiment.

(Textbook)

or...

A random variable is a **variable** whose possible values are outcomes of a random phenomenon or experiment.

(Wikipedia)

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Random Variables

Consider an experiment where two dice are rolled.

The sample space is given in the textbook. (Page 33)

Suppose the random variable is the number of sixes showing on the dice.

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X = The number of 6's

$X =$

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Random Variables

Consider an experiment where two dice are rolled.

The sample space is given in the textbook. (Page 33)

Suppose the random variable is the number of sixes showing on the dice.

The **range space** of X is therefore $\{x_1, x_2, x_3\} = \{0, 1, 2\}$.

In this case X is called a **discrete** random variable.

With each value of x_i we may associate a probability of its occurrence. $P(X = x) = p(x)$

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Probability Distributions

With each value of x_i we may associate a probability of its occurrence. $P(X = x) = p(x)$

x			
p(X = x)			

Notice that if we add all of the probabilities up we get 1.

	1	2	3	4	5	6
1	0	0	0	0	0	1
2	0	0	0	0	0	1
3	0	0	0	0	0	1
4	0	0	0	0	0	1
5	0	0	0	0	0	1
6	1	1	1	1	1	2

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Probability Distribution

Sometimes probability distributions may look a little different.

The discrete random variable W has the distribution function given by...

$$p(w) = \begin{cases} 0.6 - 0.2w & \text{for } w = 1, 2, 3 \\ 0.4 & \text{for } w = 4 \\ 0 & \text{otherwise} \end{cases}$$

What would this look like if it were in table form?

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$$p(w) = \begin{cases} 0.6 - 0.2w & \text{for } w = 1, 2, 3 \\ 0.4 & \text{for } w = 4 \\ 0 & \text{otherwise} \end{cases}$$

w				
P(W = w)				

Do the probabilities add up to 1?

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Your task...

An experiment consists of rolling two dice.

Suppose the random variable Y is the **sum** of the two dice.

Using a table draw the probability distribution of Y.

Use the sample space shown on page 33 to help you.

Challenge: Can you draw this in the alternative form using a bracket?

	1	2	3	4	5	6
1	2					
2						
3						
4						
5						
6						

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y	2	3	4	5	6	7	8	9	10	11	12
P(Y=y)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

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Cumulative Distribution Function

A cumulative distribution function of a discrete random variable gives us information about $P(X \leq x)$.

Previously a probability distribution only gave us information about $P(X = x)$.

Going back to our random variable X, the number of sixes showing on the dice we came up with this probability distribution.....

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Probability Distribution

x	0	1	2
$P(X = x)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Cumulative Distribution Function

x	0	1	2
$P(X \leq x)$			

$P(X \leq 0)$ $P(X \leq 1)$ $P(X \leq 2)$

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Your Task

We previously defined Y to be the sum of the two dice.

Draw a table to represent the cumulative distribution function for Y.

Use your cumulative distribution function to find....

a) The probability that the sum of two dice when rolled is less than or equal to 9.

b) The probability that the sum of two dice when rolled is strictly greater than 8.

y	2	3	4	5	6	7	8	9	10	11	12
$P(Y \leqslant y)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{15}{36}$	$\frac{21}{36}$	$\frac{26}{36}$	$\frac{30}{36}$	$\frac{33}{36}$	$\frac{35}{36}$	1

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EX2.1A - Page 34

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