

National Qualifications

X803/77/11

Statistics Paper 1

Marking Instructions

Please note that these marking instructions have not been standardised based on candidate responses. You may therefore need to agree within your centre how to consistently mark an item if a candidate response is not covered by the marking instructions.



General marking principles for Advanced Higher Statistics

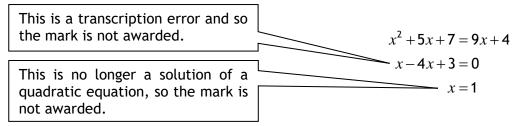
Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

The marking instructions for each question are generally in two sections:

- generic scheme this indicates why each mark is awarded
- illustrative scheme this covers methods which are commonly seen throughout the marking

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

- (a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.
- (b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.
- (c) One mark is available for each •. There are no half marks.
- (d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.
- (e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.
- (f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.
- (g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.
- (h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example



The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended	$x^{2} + 5x + 7 = 9x + 4$ x - 4x + 3 = 0 (x - 3)(x - 1) = 0
quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	<i>x</i> = 1 or 3

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

Horizontal: ${}^{6}x = 2$ and x = -4 ${}^{6}y = 5$ y = -7Horizontal: ${}^{6}x = 2$ and x = -4 ${}^{6}y = 5$ and y = -7Vertical: ${}^{6}x = 2$ and y = 5 ${}^{6}x = -4$ and y = -7

You must choose whichever method benefits the candidate, not a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$	$\frac{43}{1}$ must be simplified to 43
$\frac{15}{0.3}$ must be simplified to 50	$\frac{\frac{4}{5}}{3}$ must be simplified to $\frac{4}{15}$
$\sqrt{64}$ must be simplified to 8*	

*The square root of perfect squares up to and including 144 must be known.

- (k) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:
 - working subsequent to a correct answer
 - correct working in the wrong part of a question
 - legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
 - omission of units
 - bad form (bad form only becomes bad form if subsequent working is correct), for example

$$(x^{3}+2x^{2}+3x+2)(2x+1) \text{ written as}$$
$$(x^{3}+2x^{2}+3x+2) \times 2x+1$$
$$= 2x^{4}+5x^{3}+8x^{2}+7x+2$$

gains full credit

- repeated error within a question, but not between questions or papers
- (I) In any 'Show that...' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.
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- (n) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(o) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Detailed Marking Instructions for each question

Q	uesti	on	Generic scheme	Illustrative scheme	Max mark
1.	(a)		• ¹ describe improvement	•1 add title for each side, and add a key eg $0 5=0.5$	2
			• ² describe improvement	• ² reverse the order of the digits (leaves) on the left hand side	
	(b)		• ³ calculate upper fence	• ³ upper fence = 6·5 + 1·5 (6·5 - 3·7) = 10·7	2
			• ⁴ state number of outliers	• ⁴ 5 outliers	
	(c)	(i)	• ⁵ state hypothesis test	• ⁵ <i>z</i> -test for a difference in population proportions	1
		(ii)	• ⁶ state hypotheses	• ⁶ $H_0: p_{cat} = p_{dog}$ $H_1: p_{cat} \neq p_{dog}$	1
	(d)		• ⁷ state assumption	• ⁷ populations of times to draw the doodles of both cats and dogs have the same shape	2
			• ⁸ state assumption	• ⁸ and variability	
	(e)		• ⁹ correct parameters	• ⁹ $\begin{cases} E(W) = 16153 \cdot 5 \\ V(W) = 390376 \cdot 25 \end{cases}$	3
			 •¹⁰ standardise W •¹¹ with correct continuity correction 	$\bullet^{10\pounds11} \begin{cases} \frac{12048 \cdot 5 - 16153 \cdot 5}{\sqrt{390376 \cdot 25}} \\ (= -6 \cdot 57009) \end{cases}$	
	(f)		• ¹² state hypotheses	 •¹² H₀: median time to draw a cat = median time to draw a dog H₁: median time to draw a cat ≠ median time to draw a dog 	3
			• ¹³ state significance level	• ¹³ 1% significance level	
			• ¹⁴ state conclusion	 ¹⁴ we reject H₀ and we have evidence to suggest that the median time to draw a cat is different to the median time to draw a dog. 	

Q	Question Generic		Generic scheme	Illustrative scheme	Max mark
1.	(g)		• ¹⁵ state assumption	• ¹⁵ assume that the standard deviations of the population doodle times for cats and dogs are equal	2
			• ¹⁶ correct information	• ¹⁶ the sample standard deviations of 2·307 and 2·655	

Q	uestion	Generic scheme		Max mark	
2.	(a)	 •¹ appropriate comment •² appropriate comment 	 there is a positive relationship but it appears to be nor linear 		2
	(b)	 ³ appropriate comment ⁴ appropriate comment 	• ³	both models may be suitable both have small p-values (to reject H ₀)	2
	(C)	 ⁵ correct value ⁶ appropriate explanation 	•5 •6	$R^2 = 0.805$ Model A explains 80% of the variation in square root of cost dependent upon the length of the yacht	2
	(d)	 ^{•7} appropriate comment ^{•8} appropriate comment ^{•9} appropriate comment 	• ⁷ • ⁸ • ⁹	both plots appear to have random scatter centred on zero both plots appear to have constant variance both models appear valid	3
	(e)	 •¹⁰ calculate sqrt(cost) •¹¹ calculate estimate •¹² calculate interval 	•10 •11 •12	estimated sqrt(cost) = $\begin{cases} -204 \cdot 693 + 55 \cdot 437 \times 15 \\ = 626 \cdot 8627 \end{cases}$ estimated cost = $626 \cdot 8627^2 = £392956$ 95% Confidence interval $(592 \cdot 0915^2, 661 \cdot 634^2)$ = $(£350572, £437760)$	3
	(f) (i)	• ¹³ appropriate reason	•13	both models are set up regressing transformed cost on length.	1
	(ii)	• ¹⁴ appropriate suggestion		He needs to design a model that has regressed length on (transformed) cost	1

[END OF MARKING INSTRUCTIONS]



National Qualifications

X803/77/12

Statistics Paper 2

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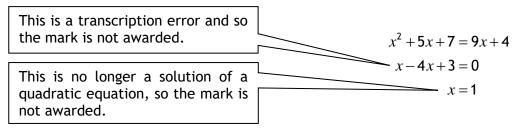
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Marking Instructions for each question

Section 1

Q	uesti	on	Generic scheme	Illustrative scheme						Max mark		
1.	(a)		 ¹ calculate k ² tabulate probability 	• ¹ 30						2		
			distribution		S	2	4	6	8	10		
					P(S=s)	1 15	2 15	1 5	$\frac{4}{15}$	$\frac{1}{3}$		
	(b)		• ³ calculate $E(S)$	•3	<u>22</u> 3							2
			• ⁴ calculate V(S)	•4	<u>56</u> 9							

Q	uestio	on	Generic scheme	Illustrative scheme	Max mark
2.	(a)		•1 correct probability	• ¹ 0·36	2
			• ² correct probability	• ² 0.94	
	(b)	(i)	• ³ appropriate strategy	• ³ $\frac{30}{50} \times \ldots + \frac{10}{50} \times \ldots + \frac{10}{50} \times \ldots$	2
			• ⁴ calculate probability	$\bullet^{4} \left(\frac{30}{50} \times 0.8 + \frac{10}{50} \times 0.5 + \frac{10}{50} \times 0.3 \right)$	
				= 0.64	
		(ii)	• ⁵ appropriate denominator	$\bullet^5 \frac{\cdots}{1 - 0.64}$	3
			• ⁶ appropriate strategy	$\bullet^6 \frac{\frac{10}{50} \times 0.5}{1 - 0.64}$	
			• ⁷ calculate probability	• ⁷ $\frac{5}{18}$	

Q	Question		Generic scheme		Illustrative scheme	Max mark
3.			• ¹ appropriate assumption	•1	the distribution of the mite count is symmetrical	8
			• ² appropriate hypothesis	 H₀: The median mite count is 7 H₁: The median mite count is greater than 7 		
			• ³ deal with zero difference	•3	discard the zero difference	
			• ⁴ process ranking	• ⁴	ranks processed (see below)	
			• ⁵ calculate statistics	•5	$n = 13, W_{-} = 14, W_{+} = 77$	
			• ⁶ correct critical value	• ⁶	5% cv is 21	
			\bullet^7 deal with H_0	•7	$14 < 21~$ so we reject H_0 at the 5% level of significance	
			• ⁸ appropriate conclusion	• ⁸	and conclude there is evidence of a risk to colony health	

Q	uesti	on	Generic scheme	Illustrative scheme	Max mark
4.	(a)	(i)	•1 correct strategy	• ¹ 1−P(<i>W</i> ≤10)	2
			• ² calculate probability	• ² 0.0137	
		(ii)	• ³ correct distribution	• ³ $F + M \sim \text{Po}(3.5)$	2
			• ⁴ calculate probability	• ⁴ $P(F + M = 2) = 0.1849$	
	(b)		• ⁵ correct strategy	• ⁵ <i>T</i> ~ Po(357)	5
			• ⁶ correct approximation	•6 $T \approx N(357,357)$	
			• ⁷ correct strategy	• ⁷⁶⁸ P(T < 340) = P(Z < $\frac{339 \cdot 5 - 357}{\sqrt{357}}$)	
			• ⁸ continuity correction	(√357)	
			• ⁹ calculate probability	• ⁹ 0·1762	

Q	uestio	n	Generic scheme		Max mark	
5.			• ¹ calculate s^2	● ¹	<i>s</i> ² = 81	5
			• ² calculate t	• ²	<i>t</i> = 1·959	
			• ³ correct cv	• ³	$t_{24,0.975} = 2.064$	
			\bullet^4 deal with $H_{\!_0}$	• ⁴	1.959 < 2.064 so we do not reject H_0 at the 5% level of significance	
			● ⁵ appropriate conclusion	● ⁵	and conclude there is no difference in warm up time between brands	

C	uestion	Generic scheme	Illustrative scheme	Max mark
6.	(a)	•1 state sampling method	•1 stratified random sampling	1
	(b)	• ² state sampling method	• ² convenience sampling	2
		• ³ appropriate reason	• ³ it is possible that the parents who subscribe to the newsletter are particularly involved with their child's education and the selected students are therefore not representative of the population	
	(C)	• ⁴ correct strategy	•4 409±1.96 $\frac{130}{\sqrt{25}}$	2
		● ⁵ calculate interval	• ⁵ (358·0, 460·0)	
	(d)	• ⁶ start explanation	• ⁶ 458 minutes is not captured by the 90% interval but it is by the 95% interval	2
		• ⁷ continue explanation	• ⁷ they have presented the evidence that would lead to a reward being given	

Q	uestion	Generic scheme	Illustrative scheme	Max mark
7.	(a)	• ¹ correct probability	$\bullet^1 \frac{0 \cdot 38}{5} = 0 \cdot 0760 \qquad \qquad$	
	(b)	• ² correct mean and variance	• ² $E(X) = 80.5$ 3 $V(X) = \frac{25}{12} = 2.0833$	
		• ³ approximate distribution of \overline{X}	• ³ $\overline{X} \approx N\left(80 \cdot 5, \frac{1}{36}\right)$	
		• ⁴ appropriate justification	• ⁴ since $n \ge 20$ the CLT can be used	
	(C)	• ⁵ appropriate strategy	• ⁵ $P(Z \le 1.98) - P(Z \le -0.30)$ 2	
		• ⁶ calculate probability	• ⁶ 0 · 5940	

Q	uestion	Generic scheme	Illustrative scheme Max mark
8.		• ¹ appropriate hypotheses	• $H_0: \mu = 15$ $H_1: \mu > 15$ 8
		• ² correct distribution	• ² $\overline{X} \sim N\left(15, \frac{4}{50}\right)$
		• ³ calculate z	• ³ $z = \frac{16 \cdot 1 - 15}{2 / \sqrt{50}} = 3 \cdot 89$
		• ⁴ correct critical value	• ⁴ 5% cv is 1·64
		\bullet^5 deal with H_0	 •⁵ 3·89>1·64 so we reject H₀ at the 5% level of significance
		• ⁶ appropriate conclusion	• ⁶ conclude that there is cause for concern
		• ⁷ appropriate explanation	• ⁷ the task may be too complex for the 15-minute time limit

Q	uestion	Generic scheme	Illustrative scheme	Max mark
9.	(a)	 •¹ correct expectation •² correct variance 	• ¹ $E(C) = 2.50 - 1.00 = 1.50$ • ² $V(C) = 4^2 + 5^2 = 41$	2
	(b)	• ³ correct description	• ³ how much more profit policy A generates than policy B, for each £10 premium	1
	(c)	 ⁴ appropriate strategy ⁵ appropriate application of variance law 	• ⁴ total, $T = A_1 + + A_{33} + B_1 + B_{26}$ • ⁵ $V(T) = V(A_1) + + V(A_{33}) + V(B_1) + + V(B_{26}) = 33V(A) + 26V(B)$	3
		 6 calculate standard deviation 	$= 33 \times (A) + 26 \times (B)$ $\bullet^{6} = 33 \times 4^{2} + 26 \times 5^{2}$ = 1178 $\Rightarrow SD(T) = (£)34 \cdot 32$	

Q	Question		Generic scheme		Illustrative scheme	Max mark
10.	(a)		• ¹ correct strategy	• ¹	$\hat{p} \pm z \sqrt{\frac{\hat{p}\hat{q}}{n}}$	3
			• ² appropriate substitution	•2	$0\cdot 26\pm 1\cdot 96\sqrt{\frac{0\cdot 26\times 0\cdot 74}{50}}$	
			• ³ calculate interval	• ³	(0.1384,0.3816)	
	(b)		• ⁴ correct strategy	•4	$\hat{p}\pm$ 0 \cdot 02	3
			• ⁵ appropriate substitution	• ⁵	$0 \cdot 02 = 1 \cdot 96 \sqrt{\frac{0 \cdot 26 \times 0 \cdot 74}{n}}$	
			• ⁶ calculate sample size	• ⁶	1848 shops	

Q	uestion	Generic scheme	Illustrative scheme	Max mark
11.	(a)	• ¹ appropriate comment	 ¹ distribution appears to be normally distributed 	1
	(b)	• ² appropriate assumption	• ² scores for workers in group B are independent of those in group C	7
		• ³ correct hypotheses	• ³ $ \begin{array}{l} H_0: \mu_B = \mu_C \\ H_1: \mu_B \neq \mu_C \end{array} $	
		• ⁴ correct test statistic	• ⁴ $Z = \frac{\overline{X}_B - \overline{X}_C - (\mu_B - \mu_C)}{\sqrt{\frac{\sigma_B^2}{n_B} + \frac{\sigma_C^2}{n_C}}}$	
		● ⁵ correct z-value	• ⁵ $z = \frac{55 \cdot 4 \cdot 51 \cdot 8}{\sqrt{\frac{10 \cdot 08^2}{70} + \frac{10 \cdot 49^2}{60}}} = 1.99$	
		• ⁶ calculate p-value	•6 $\begin{cases} p - \text{value} = 2P(Z > 1.99) \\ = 2(1 - 0.9767) \\ = 0.0466 \end{cases}$	
		$ullet^7$ deal with H_0	$\bullet^7 0\cdot 0466 < 0\cdot 05$, so we can reject H_0at the 5% level of significance	
		• ⁸ appropriate conclusion	 and conclude that there is evidence that the mean score for group B is different to that for group C 	
	(c)	• ⁹ correct strategy	• 9 $z = \frac{56 \cdot 0 - \overline{x}_B}{\sqrt{\frac{10 \cdot 71^2}{120} + \frac{10 \cdot 08^2}{70}}}$	3
		• ¹⁰ correct critical value	• ¹⁰ $z = 1.28$	
		• ¹¹ calculate maximum score	• ¹¹ $\overline{x}_B = 54 \cdot 01$	

Section 2 - Part A

Q	Question		Generic scheme	Illustrative scheme	Max mark
12.	(a)		 •¹ correct strategy •² calculate interval 	• ¹ $\overline{x} \pm 1 \frac{\sigma}{\sqrt{n}}$ • ² $500 \pm 1 \frac{5 \cdot 73}{\sqrt{5}} = (497 \cdot 4, 502 \cdot 6)$	2
	(b)	(i)	 •³ calculate probability •⁴ correct strategy •⁵ know to double •⁶ calculate probability 	• ³ $P(Z > 1) = 0.1587$ • ⁴ $X \sim B(3, 0.1587)$ • ⁵ $2P(X \ge 2)$ • ⁶ 0.1351	4
		(ii)	• ⁷ appropriate explanation	• ⁷ The probability is considerably larger than normal WECO thresholds so the machine will regularly meet this criteria.	1

Section 2 - Part B

Question		n	Generic scheme	Illustrative scheme	Max mark
13.	(a)		• ¹ appropriate explanation	•1 the lecturers were perhaps not equally good	1
	(b)		 ² deal with small expected frequencies ³ calculate test statistic ⁴ correct critical value ⁵ deal with H₀ ⁶ appropriate conclusion 	• ² combine the last two columns so the E _i are now: 10·4 9·2 7·3 9·2 6·6 5·8 4·7 5·8 • ³ $X^2 = 6.055$ • ⁴ $\chi^2_{3,0.90} = 6.251$ • ⁵ as 6·05 < 6·251 we do not have evidence to reject H ₀ • ⁶ conclude (differently) that Statistics grades awarded are independent of the course	5
	(C)		• ⁷ identification consistent with previous working	• ⁷ Biology grade D+E combined	1

[END OF MARKING INSTRUCTIONS]