Today's Learning:

Solving quadratic equations.

$$a \times b = 0$$

What can you say about a and b?

Hint: Think about some examples that work.

$-1 \times 1 = -1$

How do we solve:

$$5x - 4 = 2(x - 3)$$

$5x - 4 = 2x - 6$
$-2x \quad -2x$
$3x - 4 = -6$
$+4 \quad +4$
$3x = -2$
$x = -\frac{2}{3}$

How do we solve:

$$x^2 + 2x - 3 = 0$$

$(x+3)(x-1) = 0$

3 / 1,3

$x+3=0$ or $x-1=0$
$-3 \quad -3 \qquad +1 \quad +1$
$x=-3 \qquad x=1$

Solving Quadratic Equations

A quadratic equation can be written as **ax^2 + bx + c = 0**.

Then, we can solve by factorising.

Examples:

35 / 5,7 / 1,35

1) $x^2 - 2x - 35 = 0$

$(x+5)(x-7)=0$

$x+5=0$ or $x-7=0$
$-5 \quad -5 \qquad +7 \quad +7$
$x=-5 \qquad x=7$

2) $2x^2 + 10x = 0$

$2x(x+5)=0$

$2x=0$ or $x+5=0$
$-5 \quad -5$
$x=0 \qquad x=-5$

3) $x^2 - 9 = 0$

$(x+3)(x-3)=0$

$x+3=0$ or $x-3=0$

$x=-3 \qquad x=3$

4) $x^2 + 6x + 8 = 0$

8 / 4,2 / 1,8

$(x+4)(x+2)=0$

$x+4=0$ or $x+2=0$
$-4 \quad -4 \qquad -2 \quad -2$
$x=-4 \qquad x=-2$

Example: Solve $2x^2 + 5x + 3 = 0$

$(2x-1)(x+3)=0$

$2x^2+6x-x-3$

$\frac{3}{3,1}$

$(2x+3)(x+1)=0$

$2x^2+2x+3x+3=0$

$2x+3=0$ or $x+1=0$

$-3 \quad -3$ $\qquad -1 \quad -1$

$2x=-3$ $\qquad x=-1$

$\div 2 \quad \div 2$

$x=\frac{-3}{2}$

How would we solve $x^2 + 9x = 10$?

$-10 \qquad -10$

$\cancel{x(x+9)=10}$

$x^2+9x-10=0$

$-x^2+9x-5=0$

$x^2-9x+5=0$

Today's Learning:

To write any quadratic equation in the form **ax^2 + bx + c = 0** and to solve equations that don't factorise by using the **quadratic formula**.

$x^2 + 9x = 2$

$x^2 + 9x - 2 = 0$

$2x^2 + 3x - 2 = 0$

a b c

The Quadratic Formula

If we have an equation ax^2 + bx + c = 0 that we can't factorise, we can use the Quadratic Formula to find solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ (given in exams)

Examples:

1) $x^2 - 5x - 14 = 0$

$a = 1 \quad b = -5 \quad c = -14$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times (-14)}}{2 \times 1}$$

$$= \frac{5 \pm \sqrt{25 + 56}}{2}$$

$$= \frac{5 \pm \sqrt{81}}{2}$$

$$= \frac{5 \pm 9}{2}$$

$x = \frac{5+9}{2} = 7 \qquad x = \frac{5-9}{2} = -2$

2) $x^2 + 4x + 1 = 0$

$a = 1 \quad b = 4 \quad c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 4 \times 1 \times 1}}{2}$$

$$= \frac{-4 \pm \sqrt{16 - 4}}{2}$$

$$= \frac{-4 \pm \sqrt{12}}{2}$$

$x = -0.27$ (2d.p.) $\quad x = -3.73$ (2dp)

Paper 1 Question

Solve

$$x^2 - 11x + 24 = 0.$$

$(x-3)(x-8) = 0$

$x-3=0$ or $x-8=0$

$x=3$ ✓ $x=8$ ✓

Paper 2 Question

Solve the equation $3x^2 + 9x - 2 = 0$.

Give your answers correct to 1 decimal place.

Find the dimensions of the rectangle:

5 -8

Area $= 36\text{cm}^2$

$5-1=4$

$(x-1)$ cm

$-8-1=-9$

$(x+4)$ cm

-4cm

$5+4=9$

a) equation

b) find x

Area = L × B

$36 = (x-1)(x+4)$

$36 = x^2 + 4x - x - 4$

$36 = x^2 + 3x - 4$

$-36 \qquad -36$

$0 = x^2 + 3x - 40$

$0 = (x-5)(x+8)$

$x - 5 = 0$ or $x + 8 = 0$

$+5 \quad +5$

$x = 5 \qquad x = -8$

40

4, 10

5, 8

1, 40

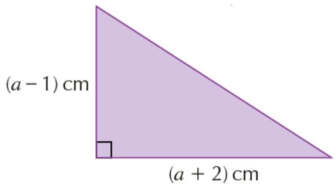
2, 20

$x = -8$ not possible

so $x = 5$

The area of this triangle is 14 cm^2.

Find the value of a.



$$\text{Area} = \frac{1}{2} \times b \times h$$

$$\text{Area} = L \times B \div 2$$

$$14 = (a+2)(a-1) \div 2$$

$$\times 2 \qquad \times 2$$

$$28 = a^2 - a + 2a - 2$$

$$28 = a^2 + a - 2$$

$$-28 \qquad -28$$

$$0 = a^2 + a - 30$$

$$0 = (a-5)(a+6)$$

$a - 5 = 0$ or $a + 6 = 0$

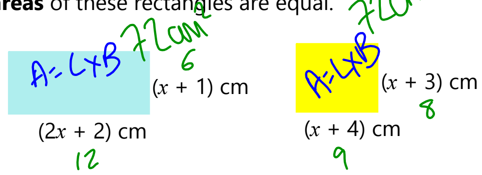
$a = 5$ $\quad$ $a = -6$

-6 not possible

so $a = 5$

Starter

The **areas** of these rectangles are equal.



a) Show that $x^2 - 3x - 10 = 0$.

b) Calculate the area of the rectangles.

$$(2x+2)(x+1) = (x+3)(x+4)$$

$$2x^2 + 2x + 2x + 2 = x^2 + 4x + 3x + 12$$

$$2x^2 + 4x + 2 = x^2 + 7x + 12$$

$$-x^2 \qquad -x^2$$

$$x^2 + 4x + 2 = 7x + 12$$

$$-12 \qquad -12$$

$$x^2 + 4x - 10 = 7x$$

$$-7x \qquad -7x$$

$x^2 - 3x - 10 = 0$ as required.

b) $(x-5)(x+2) = 0$

$x - 5 = 0$ or $x + 2 = 0$

$x = 5$ or -2

-2 not possible

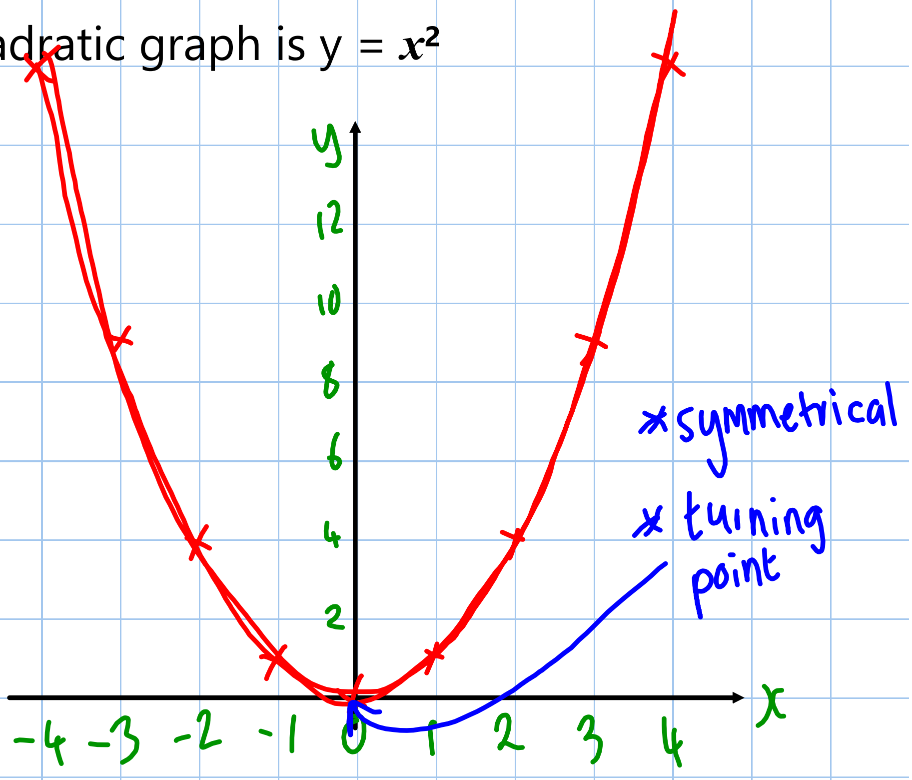
$x = 5$

Today's Learning:

To find the equation of quadratic graphs using substitution of a point.

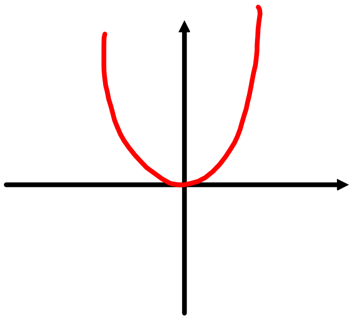
Quadratic Graphs

The simplest quadratic graph is y = x^2

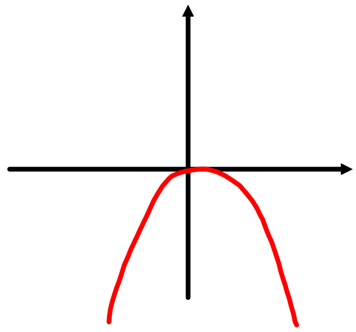


The graph of $y = kx^2$

happy



Positive k

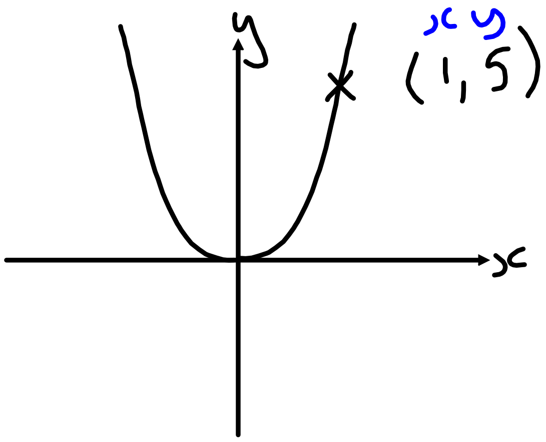


Negative k

sad

$y = kx^2$ graph is the $y = x^2$ graph
stretched ↕ by a factor of k

e.g. Find the equation of the graph of the form $y = kx^2$



$y = kx^2$

$5 = k \times 1^2$

$5 = k \times 1$

$\div 1$ $\div 1$

$5 = k$

$y = 5x^2$

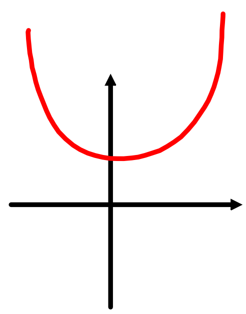
page 2 - Q1

Today's Learning:

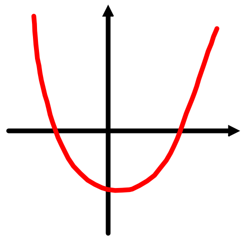
To continue to consider transformations of quadratic graphs.

$y = x^2 + q$

moves up q



positive q



moves down q

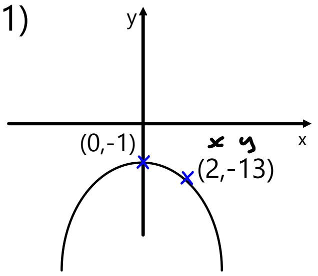
negative q

a) $y = x^2 + 2$

$\boxed{y = x^2 + a \quad \text{pg 3 Q2 a - f}}$

e.g. Find k and q from the graphs of $y = kx^2 + q$:

1)



$y = kx^2 - 1$

$-13 = k \times 2^2 - 1$

$-13 = k \times 4 - 1$

$-13 = 4k - 1$

$+1 \quad +1$

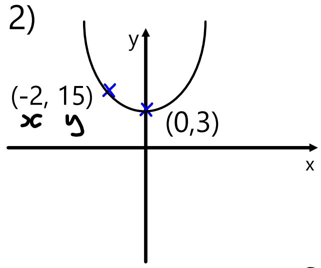
$-12 = 4k$

$\div 4 \quad \div 4$

$-3 = k$

$y = -3x^2 - 1$

2)



$y = kx^2 + 3$

$15 = k \times (-2)^2 + 3$

$15 = k \times 4 + 3$

$15 = 4k + 3$

$-3 \quad -3$

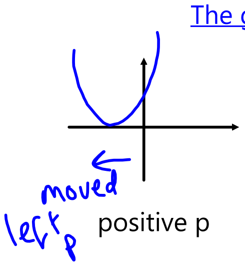
$12 = 4k$

$3 = k$

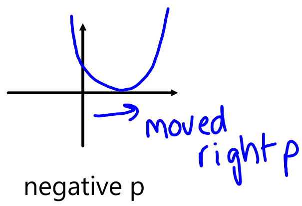
$y = 3x^2 + 3$

pg 3 Q2 g - k

The graph of $y = (x + p)^2$

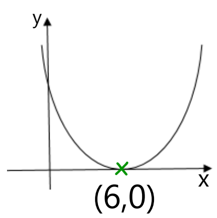


positive p



negative p

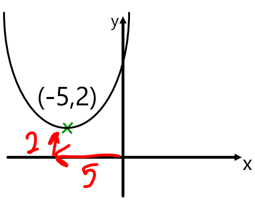
e.g. Find p for the graph of $y = (x + p)^2$:



$p = -6$

$y = (x - 6)^2$

e.g. Find p and q for the graph of $y = (x + p)^2 + q$:



$y = (x + 5)^2 + 2$

page 4 Q 3

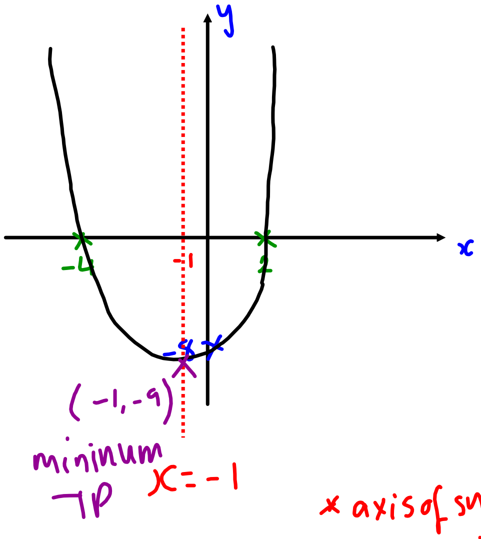
1a) $y = (x - 2)^2 + 1$

Sketching Quadratic Graphs

We can be asked to label:

- Turning Point and its nature
- Roots (where it crosses the x-axis) ✓
- y-intercept ✓
- Equation of the axis of symmetry ✓

e.g.1) Sketch the graph of y = (x - 2)(x + 4)



Set $x = 0$

$y = (0-2)(0+4)$

$y = (-2)(4)$

$= -8$

set $y = 0$

$0 = (x-2)(x+4)$

$x-2=0 \qquad x+4=0$

$x=2 \qquad x=-4$

* axis of symmetry → look halfway between the roots, x = that.

TP : set $x = -1$

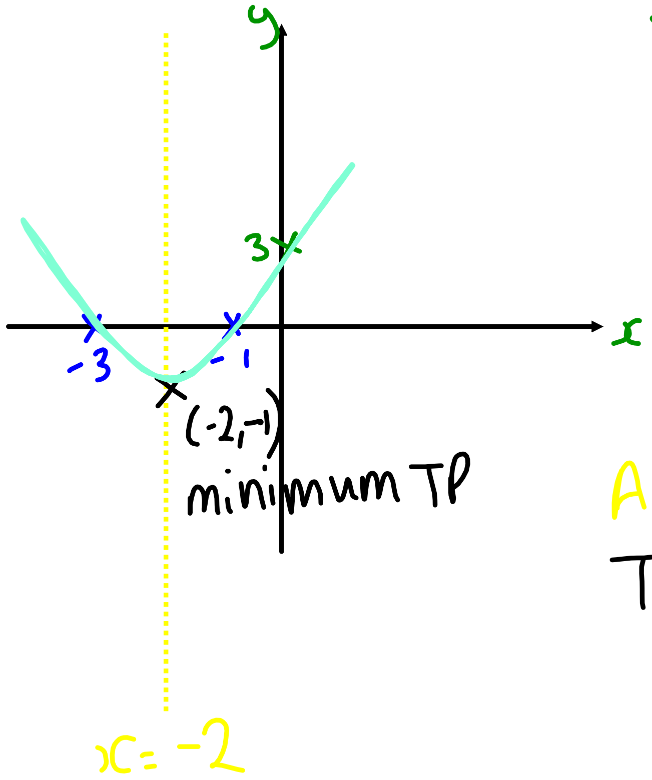
$y = (-1-2)(-1+4)$

$= (-3)(3)$

$= -9$

e.g.2) Sketch the graph of $y = (x + 1)(x + 3)$

page 5 Q2a



set $x = 0$

$y = (0+1)(0+3)$

$= (1)(3)$

$= 3$

set $y = 0$

$0 = (x+1)(x+3)$

$x+1=0$ or $x+3=0$

$x=-1$ or $x=-3$

Axis of symmetry: $x = -2$.

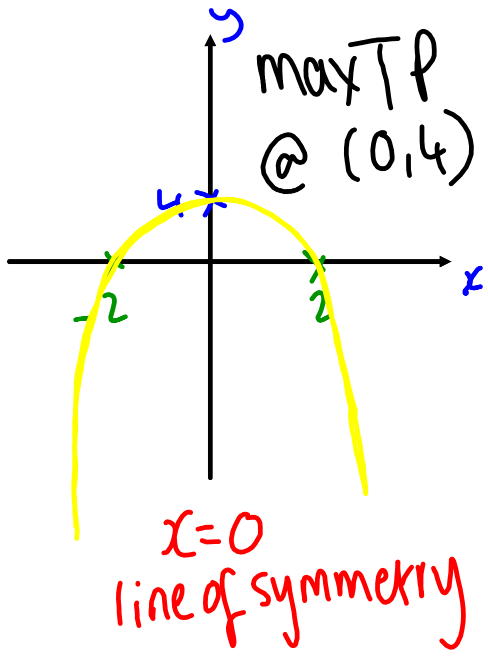
TP: set $x = -2$

$y = (-2+1)(-2+3)$

$= (-1)(1)$

$= -1$

e.g. 3) Sketch the graph of $y = -(x + 2)(x - 2)$



Set $x = 0$

$y = -(0+2)(0-2)$

$= -(2)(-2)$

$= -(-4)$

$= 4$

set $y = 0$

$0 = -(x+2)(x-2)$

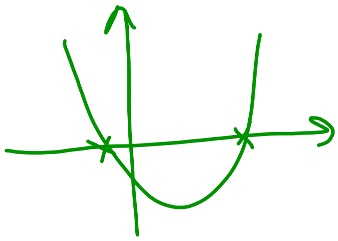
$x+2=0$ or $x-2=0$

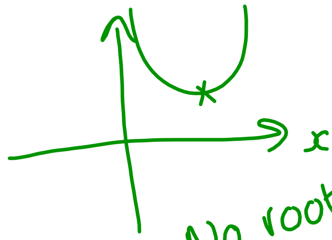
$x=-2$ or $x=2$

Q2 on page 5

How can we tell how many roots an equation has? $y=(x-3)^2+2$

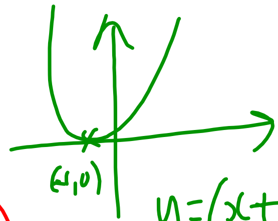
$y=(x-4)(x+2)$





No roots

$0=(x-4)(x+2)$



$y=(x+1)^2$

$$x = \frac{-b \pm \sqrt{\overset{0}{b^2-4ac}}}{2a}$$

$b^2-4ac=0$ one root

$b^2-4ac=-ve$ no roots

$b^2-4ac=+ve.$ 2 solutions

<u>The Discriminant</u>

For a quadratic equation ax^2 + bx + c = 0 the discriminant is $b^2 - 4ac$.

$b^2 - 4ac > 0$ means 2 real, distinct roots

$b^2 - 4ac = 0$ means 2 real, equal roots

$b^2 - 4ac < 0$ means no real roots

e.g. 1) Determine the nature of the roots of $2(x + 1) = x^2 - 3$

$2x+2=x^2-3$
$+3 \quad +3$
$2x+5=x^2$
$-x^2 \quad -x^2$
$-x^2+2x+5=0$
$a=-1 \quad b=2 \quad c=5$
b^2-4ac
$=4-4(-1)(5)$
$=4+20$
$=24$

$24>0$ so
2 real distinct roots