Starter

Factorise fully:

1) $4a^2b - 6abc$ $2ab(2a - 3c)$

2) $3x^2 + 18x + 24 = 3(x^2 + 6x + 8)$
$= 3(x+4)(x+2)$

3) $2x^2 - 18 = 2(x^2 - 9)$
$= 2(x+3)(x-3)$

4) $2x^2 - x - 6$

$(2x+3)(x-2)$

A surd is a square root which doesn't 'work out' as a rational number.

eg. $\sqrt{5}$ and $\sqrt{3}$ are surds

$\sqrt{4}$ isn't, because $\sqrt{4} = 2$

Today's Learning:

To simplify surds.

$\sqrt{25} = 5$

$\sqrt{27}$ surd

Rules of Surds

A surd is a root that doesn't work out nicely - they are irrational numbers.

1) $\sqrt{m} \times \sqrt{n} = \sqrt{mn}$

2) $\dfrac{\sqrt{m}}{\sqrt{n}} = \sqrt{\dfrac{m}{n}}$

e.g. Simplify:

1) $\sqrt{12}$

$= \sqrt{4 \times 3}$

$= 2 \times \sqrt{3}$

$= 2\sqrt{3}$

2) $\sqrt{5} \times \sqrt{10}$

$= \sqrt{50}$

$= \sqrt{25 \times 2}$

$= 5\sqrt{2}$

3) $5\sqrt{2} \times 3\sqrt{8}$

$= 15\sqrt{16}$

$= 15 \times 4$

$= 60$

4) $\dfrac{\sqrt{3}}{\sqrt{27}}$

$= \sqrt{\dfrac{3}{27}} = \sqrt{\dfrac{1}{9}} = \dfrac{1}{3}$

Starter

Fully factorise the following:

1) $x^2 + 7x + 10$

$(x+5)(x+2)$

10 5,2 10,1

2) $2x^2 - 16x + 14$

$2(x^2 - 8x + 7)$

$2(x-1)(x-7)$

3) $x^2 - 4$

$(x-2)(x+2)$

4) $2x^2 + 13x + 15$

$(2x+3)(x+5)$

Adding/Subtracting Surds

Treat the surd like an x in algebra.

e.g. 1) $\sqrt{18} - \sqrt{8}$

2) $\sqrt{45} + \sqrt{5} - \sqrt{20}$

Multiplying Out Brackets

e.g. Expand and simplify:

1) $2\sqrt{2}(3-\sqrt{2})$

$= 6\sqrt{2} - 2\sqrt{4}$

$= 6\sqrt{2} - 2\times 2$

$= 6\sqrt{2} - 4$

$2\sqrt{2}\times\sqrt{2}$

$2\sqrt{2\times 2}$

$= 2\sqrt{4}$

2) $(6-\sqrt{2})^2$

$= (6-\sqrt{2})(6-\sqrt{2})$

$= 36 - 6\sqrt{2} - 6\sqrt{2} + \sqrt{4}$

$= 36 - 12\sqrt{2} + 2$

$= 38 - 12\sqrt{2}$

$3\times 2x = 6x$

3) $(3+\sqrt{2})(4-2\sqrt{2})$

$= 12 - 2\sqrt{4} - 6\sqrt{2} + 4\sqrt{2}$

$= 12 - 2\times 2 - 2\sqrt{2}$

$= 8 - 2\sqrt{2}$

Practice:

Work through the first column of Q8 and Q9

eg. ① $2\sqrt{3}\times 5 = 10\sqrt{3}$

② $2\sqrt{3}\times\sqrt{5} = 2\sqrt{15}$

$$(3+\sqrt{5})(2-3\sqrt{5})$$
$$= 6 - 9\sqrt{5} + 2\sqrt{5} - 3\sqrt{25}$$
$$= 6 - 7\sqrt{5} - 3 \times 5$$
$$= 6 - 7\sqrt{5} - 15$$
$$= -9 - 7\sqrt{5}$$

Today's Learning

To rationalise the denominator of any surd by using a conjugate surd.

Rationalising the Denominator

We don't like to have surds on the bottom of fractions, so we rationalise the denominator.

e.g. 1) $\frac{\sqrt{6}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{18}}{\sqrt{9}} = \frac{\sqrt{9\times 2}}{3} = \frac{3\sqrt{2}}{3} = \frac{\sqrt{2}}{1} = \sqrt{2}$

2) $\frac{7}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \frac{7\sqrt{8}}{\sqrt{64}} = \frac{7\sqrt{4\times 2}}{8} = \frac{7\times 2\sqrt{2}}{8}$

$= \frac{14\sqrt{2}}{8} = \frac{7\sqrt{2}}{4}$

3) $\frac{1}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{4\sqrt{9}} = \frac{\sqrt{3}}{4\times 3} = \frac{\sqrt{3}}{12}$

Starter

Multiply out the brackets and simplify:

a) $(2+\sqrt{3})(2-\sqrt{3})$ $= 4 - 2\sqrt{3} + 2\sqrt{3} - \sqrt{9}$

$= 4 - 3 = \underline{\underline{1}}$

b) $(2\sqrt{3}+\sqrt{2})(2\sqrt{3}-\sqrt{2})$

$= 4\sqrt{9} - 2\sqrt{6} + 2\sqrt{6} - \sqrt{4}$

$= 4\times 3 - 2$

$= \underline{\underline{10}}$

Rationalising the Denominator

When there is more than one term on the bottom of the fraction, use the **conjugate surd** (the same terms with the opposite sign).

e.g. 1) $\frac{10}{(7+\sqrt{5})} \times \frac{(7-\sqrt{5})}{(7-\sqrt{5})} = \frac{70-10\sqrt{5}}{49-7\sqrt{5}+7\sqrt{5}-\sqrt{25}}$

$= \frac{70-10\sqrt{5}}{49-5}$

$= \frac{70-10\sqrt{5}}{44}$

$= \frac{35-5\sqrt{5}}{22}$

2) $\frac{2}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = \frac{2\sqrt{3}+2\sqrt{2}}{\sqrt{9}+\sqrt{6}-\sqrt{6}-\sqrt{4}}$

$= \frac{2\sqrt{3}+2\sqrt{2}}{3-2}$

$= \frac{2\sqrt{3}+2\sqrt{2}}{1}$

$= 2\sqrt{3}+2\sqrt{2}$

page 5

Q14

Starter

Rationalise the denominators and simplify where possible:

1) $\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{3}}{\sqrt{9}}$

$= \frac{2\sqrt{3}}{3}$

2) $\frac{1}{3\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$= \frac{\sqrt{6}}{3\sqrt{36}}$

$= \frac{\sqrt{6}}{3\times 6}$

$= \frac{\sqrt{6}}{18}$

3) $\frac{5}{4-\sqrt{2}} \times \frac{4+\sqrt{2}}{4+\sqrt{2}}$

$= \frac{20+5\sqrt{2}}{16+4\sqrt{2}-4\sqrt{2}-\sqrt{4}}$

$= \frac{20+5\sqrt{2}}{16-2}$

$= \frac{20+5\sqrt{2}}{14}$

Today's Learning:

To multiply and divide indices and to raise indices to powers.

$m^5 = m \times m \times m \times m \times m$

How else can we write 2 x 2 x 2?

2^3

What is $2^2 \times 2^3$?

$2 \times 2 \times 2 \times 2 \times 2$

2^5

$2^5 \times 2^2 = 2^7$

$3^{10} \times 3^2 = 3^{12}$

$b^4 \times b^4 = b^8$

What is $2^5 \div 2^2$?

$$\frac{2 \times 2 \times 2 \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2}}$$

$$2 \times 2 \times 2 = 2^3$$

What is $(2^3)^2$?

$$2^3 \times 2^3 = 2^6$$

$$(v^2)^4 = v^8$$

$$(b^3)^3 = b^9$$

$$(m^{10})^2 = m^{20}$$

Rules of Indices

5 is the index in a^5. This means a x a x a x a x a

Rules:

1) $a^m \times a^n = a^{m+n}$

2) $\dfrac{a^m}{a^n} = a^{m-n}$

3) $(a^m)^n = a^{mn}$

e.g. Simplify:

1) $\dfrac{g^3 \times g^2}{g^4}$

$= \dfrac{g^5}{g^4}$

$= g^1 = g$

2) $(2h^3)^2$

$= 4h^6$

3) $r^2(r^1 + r^5)$

$= r^3 + r^7$

What is the value of 3^0?

$3^3 = 3 \times 3 \times 3 \times 1$

$3^2 = 3 \times 3 \times 1$

$3^1 = 3 \times 1$

$3^0 = 1$

4) $a^0 = 1$

1) $w^3 \times w^5$ w^8

2) $t^3 \times t$ t^4

3) $z^4 \div z^2$ z^2

4) $\frac{c^8}{c^2}$ c^6

5) $c^7 \div c^7$ 1

6) $(g^2)^4$ g^8

7) $(m^2)^3$ m^6

8) $(ab)^3$ a^3b^3

9) $(3v)^2$ $9v^2$

10) $(5g)^3$ $125g^3$

11) $2a^3 \times 5a^5$ $10a^8$

12) $3y \times (2y^2)^3$

$3y \times 8y^6$

$= 24y^7$

13) $k^2(k^3 + k^5)$

$= k^5 + k^7$

14) $\frac{x^5 \times x^4}{x^6}$ $= \frac{x^9}{x^6}$

$= x^3$

15) $\frac{(3xy^5)^3}{9x^2y}$

$= \frac{27x^3y^{15}}{9x^2y^1}$

$= 3xy^{14}$

Starter

1) Write down the value of:

a) $\sqrt{36}$ $= 6$

b) $\sqrt[3]{8}$ $= 2$

c) 2^4 $= 16$

d) $\sqrt[3]{1000}$ $= 10$

Today's Learning:

To simplify indices with negative and fractional powers.

Rules of Indices

5) $a^{-m} = \dfrac{1}{a^m}$

e.g. rewrite with a positive index:

1) 4^{-4}

$= \dfrac{1}{4^4}$

2) $2b^{-3}$

$= 2 \times \dfrac{1}{b^3}$

$= \dfrac{2}{b^3}$

3) $\dfrac{3}{y^{-5}}$

$= 3y^5$

$x^{-4} = \dfrac{1}{x^4}$

$\dfrac{1}{m^4} = m^{-4}$

$x^{-2} = \dfrac{1}{x^2}$

$\dfrac{1}{y^2} = y^{-2}$

$y^{-4} = \dfrac{1}{y^4}$

$\dfrac{1}{m^{10}} = m^{-10}$

6) $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

e.g. find the value of **a)** $27^{\frac{2}{3}}$

$= \sqrt[3]{27^2}$

$= 3^2$

$= 9$

b) $1000^{\frac{-2}{3}}$

$= \frac{1}{1000^{2/3}}$

$= \frac{1}{\sqrt[3]{1000^2}}$

$= \frac{1}{10^2}$

$= \frac{1}{100}$

Starter

① $y^{-4} = \frac{1}{\boxed{y^4}}$

② $m^{\frac{3}{4}} = \sqrt[\boxed{4}]{m^{\boxed{3}}}$

Write down the value of:

1) $25^{\frac{1}{2}} = \sqrt{25} = 5$

2) $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$

3) $4^{\frac{1}{2}} = \sqrt[2]{4^1} = 2$

4) $27^{\frac{2}{3}} = \sqrt[3]{27^2} = 9$

5) $1000^{\frac{2}{3}} = \sqrt[3]{1000^2}$
$= 10^2$
$= 100$

6) $100^{-\frac{3}{2}}$

$= \frac{1}{100^{3/2}}$

$= \frac{1}{\sqrt[2]{100^3}}$

$= \frac{1}{10^3}$

$= \frac{1}{1000}$

Write in surd form:

1) $x^{\frac{1}{2}} = \sqrt[2]{x^1} = \sqrt{x}$

2) $b^{\frac{1}{4}} = \sqrt[4]{b^1} = \sqrt[4]{b}$

3) $x^{\frac{3}{4}} = \sqrt[4]{x^3}$

4) $c^{\frac{3}{5}} = \sqrt[5]{c^3}$

5) $m^{\frac{-2}{3}} = \frac{1}{m^{2/3}}$

6) $4d^{-\frac{3}{7}} = \frac{1}{\sqrt[3]{m^2}}$

$x^{\frac{1}{2}} = \sqrt{x}$

$\frac{1}{d^{3/7}}$

$= \frac{1}{\sqrt[7]{d^3}}$ $\frac{4}{\sqrt[7]{d^3}}$

Write in index form:

1) $\sqrt{x} = x^{\frac{1}{2}}$

2) $\sqrt{t} = t^{\frac{1}{2}}$

3) $\sqrt[3]{c^2} = c^{2/3}$

4) $\sqrt[3]{p^5} = p^{5/3}$

5) $\frac{1}{\sqrt{a}} = \frac{1}{a^{1/2}} = a^{-\frac{1}{2}}$

6) $\frac{1}{\sqrt[3]{y^4}}$

$= \frac{1}{y^{4/3}}$

$= y^{-4/3}$

$\sqrt[5]{m^3} = m^{3/5}$

$m^3 \times m^4 = m^7$

$\frac{m^6}{m^3} = m^3$

$(m^2)^3 = m^6$

$m^0 = 1$

Today's Learning:

Multiplying out brackets using indices and trying exam type questions.

Multiplying out Brackets

e.g. 1) $\sqrt{x}(x^2-1)$

$= x^{\frac{1}{2}}(x^2-1)$

$= x^{2\frac{1}{2}} - x^{1/2}$

2) $\left(x+\frac{1}{x^2}\right)^2$

$= \left(x+\frac{1}{x^2}\right)\left(x+\frac{1}{x^2}\right)$

$= \left(x^1+x^{-2}\right)\left(x^1+x^{-2}\right)$

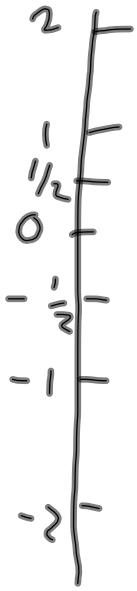
$= x^2+x^{-1}+x^{-1}+x^{-4}$

$= x^2+2x^{-1}+x^{-4}$

$= x^2+\frac{2}{x}+\frac{1}{x^4}$

$$x^{\frac{1}{2}}(x^4+1)$$

$$= x^{4\frac{1}{2}} + x^{\frac{1}{2}}$$



$$\frac{1}{x^1}(\sqrt{x}+x)$$

$$= x^{\boxed{-1}}\left(x^{\boxed{\frac{1}{2}}} + x^1\right)$$

$$= x^{-1/2} + x^0$$

$$= \frac{1}{x^{1/2}} + 1$$

$$= \frac{1}{\sqrt{x}} + 1$$

$$\left(x + \frac{1}{\sqrt{x}}\right)^2$$

$$\left(x + \frac{1}{x^{\frac{1}{2}}}\right)^2$$

$$= \left(x + x^{-\frac{1}{2}}\right)^2$$

$$= \left(x^1 + x^{-1/2}\right)\left(x^1 + x^{-1/2}\right)$$

$$= x^2 + x^{1/2} + x^{1/2} + x^{-1}$$

$$= x^2 + 2x^{1/2} + x^{-1}$$

$$= x^2 + 2\sqrt{x} + \frac{1}{x}$$

$$\frac{1}{x^2}(x^2 + x)$$

Starter

Simplify the following:

1) $\sqrt{x}\left(\frac{1}{x^1}+1\right)$

$= x^{\frac{1}{2}}\left(x^{-1}+1\right)$

$= x^{-\frac{1}{2}} + x^{\frac{1}{2}}$

$= \frac{1}{x^{\frac{1}{2}}} + \sqrt{x}$

$= \frac{1}{\sqrt{x}} + \sqrt{x}$

2) $\frac{3}{x^1}(x+2)$

$= 3x^{-1}(x^1+2)$

$= 3x^0 + 6x^{-1}$

$= 3 + \frac{6}{x}$

Today's Learning:

To use indices in scientific notation.

2×10^9

3×10^{-7}

Rewrite these numbers **in full**:

a) The speed of light: 3×10^8 m/s

300,000,000

b) The diameter of the earth: 1.268×10^4 km

12680

c) The approximate population of the earth: 7.4×10^9

7 4 0000000000

d) The number of dollars Bill Gates is worth: 7.92×10^{10}

79200000000

e) The radius of the orbit of an electron: 5×10^{-8}

0.00000005

Standard Form

A number is expressed as **m x 10^n**, where $1 \leq m < 10$.

Examples. Write your answer in standard form:

1) A space probe can travel at a speed of 3.6×10^6 miles per day. What distance can it travel in a week?

$3.6 \times 10^6 \times 7$

25.2×10^6

$= 2.52 \times 10^7$

3.6

4×7

252

2) $(2 \times 10^{-8}) \times (3.5 \times 10^2)$

$2 \times 10^{-8} \times 3.5 \times 10^2$

$7 \times 10^{-8} \times 10^2$

7×10^{-6}

$3 \times (4 \times 2) \times 5$

2×10^{-8}

Ans $\times 3.5 \times 10^2$