

*Prelim Examination 2006 / 2007*  
*(Assessing Units 1 & 2)*

**MATHEMATICS**  
**Advanced Higher Grade**

**Time allowed - 2 hours**

---

**Read Carefully**

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions.
3. **Full credit will be given only where the solution contains appropriate working.**
4. **This examination paper contains questions graded at all levels.**

**All questions should be attempted**

1. (a) Given  $f(x) = e^{-2x}\tan 4x$ ,  $0 < x < \frac{\pi}{8}$ , obtain  $f'(x)$ . 3

(b) For  $y = \frac{\ln 5x}{x-1}$ , where  $x > 1$ , determine  $\frac{dy}{dx}$  in its simplest form. 3

2. For what value of  $t$  does the system of equations

$$\begin{aligned}x + 2y - 3z &= -7 \\4x - y + 2z &= 9 \\3x - 2y + tz &= 13\end{aligned}$$

have no solution? 5

3. Verify that  $1 - 3i$  is a solution of  $z^4 - 4z^3 + 11z^2 - 14z - 30 = 0$ .

Hence express  $z^4 - 4z^3 + 11z^2 - 14z - 30$  in the form  $(z + a)(z + b)(z^2 + cz + d)$ , where  $a, b, c$  and  $d$  are real numbers. 5

4. Use the substitution  $x = 3\cos\theta$  to show that

$$\int_{\frac{3}{2}}^3 \frac{dx}{\sqrt{9-x^2}} = \frac{\pi}{3} \quad \text{6}$$

5. Obtain the binomial expansion of  $\left(3a^2 - \frac{4}{b}\right)^5$ . 3

6. Use integration by parts to evaluate  $\int_0^1 x^2 e^{-x} dx$ . 5

7. Determine whether the function  $f(x) = x^2 \cos x + x^3$  is odd, even or neither.

Justify your answer. 3

8. A spherical balloon is being inflated.

Its volume,  $V \text{ cm}^3$ , is increasing at the rate of  $\frac{30\pi}{7} \text{ cm}^3$  per second.

Find the rate at which the radius is increasing with respect to time when the volume is  $\frac{36\pi}{5} \text{ cm}^3$ .

[Note: The volume of a sphere is given by  $V = \frac{4}{3} \pi r^3$ .] 5

9. Prove that if  $n$  is odd then  $n^4 - 1$  is divisible by 8. 3

10. (a) Obtain partial fractions for

$$\frac{9}{x^2 - 9} \quad \text{2}$$

(b) Hence evaluate

$$\int_0^1 \frac{x^2}{x^2 - 9} dx. \quad \text{4}$$

11. The function  $f$  is defined by

$$f(x) = \frac{x^2}{x+3}, \quad x \neq -3.$$

(a) Obtain algebraically the asymptotes of the graph of  $f$ . 3

(b) Find the stationary points of  $f$  and justify their nature. 5

(c) Sketch the curve showing clearly the features found in (a) and (b). 2

(d) Write down the coordinates of the stationary points of the graph of  $g(x) = 10 + |f(x)|$ . 2

**12.** The first two terms of a series are  $1 + \sqrt{2}$  and  $1 + \frac{1}{\sqrt{2}}$ .

(a) If the series is arithmetic, show that the common difference is  $-\frac{1}{2}\sqrt{2}$ .

Show also that the sum of the first ten terms is  $\frac{5}{2}(4 - 5\sqrt{2})$ . **4**

(b) If the series is geometric, show that the sum to infinity exists.

Show also that  $S_{\infty} = 4 + 3\sqrt{2}$ . **5**

**13.** A solid is formed by rotating the curve  $y = x^2 + 4$  between  $x = 1$  and  $x = t$ ,  $t > 1$ , through  $360^\circ$  about the  $y$  - axis.

Find the value of  $t$  given that the volume of the solid formed is  $40\pi$  units<sup>3</sup>. **6**

*[END OF QUESTION PAPER]*

**Marking Scheme - Advanced Higher Grade 2006/2007  
Prelim (Assessing Units 1 & 2)**

	<b>Give one mark for each •</b>	<b>Illustrations for awarding each mark</b>
1(a)	<b>ans:</b> $f'(x) = 2e^{-2x}(2\sec^2 4x - \tan 4x)$  <b>3 marks</b> <ul style="list-style-type: none"> <li>• knows to use product rule</li> <li>• differentiates <math>e^{-2x}</math> correctly</li> <li>• differentiates <math>\tan 4x</math></li> </ul>	<ul style="list-style-type: none"> <li>•</li> <li>• <math>-2e^{-2x}</math></li> <li>• <math>4\sec^2 4x</math></li> </ul>
1(b)	<b>ans:</b> $\frac{dy}{dx} = \frac{x(1 - \ln 5x) - 1}{x(x-1)^2}$  <b>3 marks</b> <ul style="list-style-type: none"> <li>• knows to use the quotient rule</li> <li>• differentiates correctly</li> <li>• correct simplification for <math>\frac{dy}{dx}</math></li> </ul>	<ul style="list-style-type: none"> <li>•</li> <li>• <math>\frac{x-1}{x} - \ln 5x</math></li> <li>• <math>\frac{x-1}{(x-1)^2}</math></li> <li>• <math>\frac{x-1 - x \ln 5x}{x(x-1)^2}</math></li> </ul>
2	<b>ans:</b> $t = \frac{31}{9}$  <b>5 marks</b> <ul style="list-style-type: none"> <li>• correct augmented matrix</li> <li>• first modified system correct</li> <li>• second modified system correct</li> <li>• third modified system correct</li> <li>• solves for <math>t</math></li> </ul>	<ul style="list-style-type: none"> <li>• <math>\begin{pmatrix} 1 &amp; 2 &amp; -3 &amp; -7 \\ 4 &amp; -1 &amp; 2 &amp; 9 \\ 3 &amp; -2 &amp; t &amp; 13 \end{pmatrix}</math></li> <li>• <math>\begin{pmatrix} 1 &amp; 2 &amp; -3 &amp; -7 \\ 0 &amp; -9 &amp; 14 &amp; 37 \\ 3 &amp; -2 &amp; t &amp; 13 \end{pmatrix}</math></li> <li>• <math>\begin{pmatrix} 1 &amp; 2 &amp; -3 &amp; -7 \\ 0 &amp; -9 &amp; 14 &amp; 37 \\ 0 &amp; -8 &amp; t+9 &amp; 34 \end{pmatrix}</math></li> <li>• <math>\begin{pmatrix} 1 &amp; 2 &amp; -3 &amp; -7 \\ 0 &amp; -9 &amp; 14 &amp; 37 \\ 0 &amp; 0 &amp; t - \frac{31}{9} &amp; \frac{10}{9} \end{pmatrix}</math></li> <li>• <math>t = \frac{31}{9}</math></li> </ul>

	Give one mark for each •	Illustrations for awarding each mark
3	<p><b>ans:</b> Proof, <math>(z - 3)(z + 1)(z^2 - 2z + 10)</math></p> <p style="text-align: right;"><b>5 marks</b></p> <ul style="list-style-type: none"> <li>• verifies that <math>1 - 3i</math> is a solution</li> <li>• knows that <math>1 + 3i</math> is a solution</li> <li>• uses <math>1 + 3i</math> for substitution or synthetic division</li> <li>• finds <math>z^2 - 2z - 3 = (z - 3)(z + 1)</math></li> <li>• finds <math>z^2 - 2z + 10</math> factor</li> </ul>	<ul style="list-style-type: none"> <li>• correct substitution or synthetic division</li> <li>• <math>1 + 3i</math> is a solution</li> <li>• correct substitution or synthetic division</li> <li>• <math>z^2 - 2z - 3 = (z - 3)(z + 1)</math></li> <li>• <math>z^2 - 2z + 10</math></li> </ul>
4	<p><b>ans:</b> Proof</p> <p style="text-align: right;"><b>6 marks</b></p> <ul style="list-style-type: none"> <li>• starts substitution</li> <li>• changes limits correctly</li> <li>• correct substitution</li> <li>• deals with denominator</li> <li>• correctly integrates</li> <li>• substitutes limits correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>dx = -3\sin\theta d\theta</math></li> <li>• <math>\frac{3}{2} \rightarrow \frac{\pi}{3}, 3 \rightarrow 0</math></li> <li>• <math>\int_{\frac{\pi}{3}}^0 \frac{-3\sin\theta d\theta}{\sqrt{9 - 9\cos^2\theta}}</math></li> <li>• <math>\int_{\frac{\pi}{3}}^0 \frac{-3\sin\theta d\theta}{3\sin\theta}</math></li> <li>• <math>- [\theta]_{\frac{\pi}{3}}^0</math></li> <li>• <math>\frac{\pi}{3}</math></li> </ul>
5	<p><b>ans:</b></p> $243a^{10} - \frac{1620a^8}{b} + \frac{4320a^6}{b^2} - \frac{5760a^4}{b^3} + \frac{3840a^2}{b^4} - \frac{1024}{b^6}$ <p style="text-align: right;"><b>3 marks</b></p> <ul style="list-style-type: none"> <li>• correct binomial expression</li> <li>• correct expansion</li> <li>• correct simplification</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\sum_{r=0}^5 \binom{5}{r} (3a^2)^{5-r} \left(\frac{-4}{b}\right)^r</math></li> <li>• <math>(3a^2)^5 + 5(3a^2)^4 \left(\frac{-4}{b}\right) + 10(3a^2)^3 \left(\frac{-4}{b}\right)^2</math></li> <li>• <math>10(3a^2)^2 \left(\frac{-4}{b}\right)^3 + 5(3a^2) \left(\frac{-4}{b}\right)^4 + \left(\frac{-4}{b}\right)^5</math></li> <li>• answer</li> </ul>

	Give one mark for each •	Illustrations for awarding each mark
6	<b>ans:</b> $2 - 5e^{-1}$ <b>5 marks</b> <ul style="list-style-type: none"> <li>• uses integration by parts correctly</li> <li>• uses integration by parts for a second time</li> <li>• integrates correctly</li> <li>• substitutes limits correctly</li> <li>• correct evaluation</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\left[-x^2e^{-x}\right]_0^1 + \int_0^1 2xe^{-x} dx</math></li> <li>• <math>\left[-2xe^{-x}\right]_0^1 + \int_0^1 2e^{-x} dx</math></li> <li>• <math>\left[-2e^{-x}\right]_0^1</math></li> <li>• <math>-e^{-1} - 2e^{-1} - 2e^{-1} + 2e^0</math></li> <li>• <math>2 - \frac{5}{e}</math></li> </ul>
7	<b>ans:</b> Neither <b>3 marks</b> <ul style="list-style-type: none"> <li>• knows to find <math>f(-x)</math></li> <li>• finds <math>f(-x)</math> correctly</li> <li>• correct conclusion</li> </ul>	<ul style="list-style-type: none"> <li>• <math>f(-x) = (-x)^2 \cos(-x) + (-x)^3</math></li> <li>• <math>f(-x) = x^2 \cos x - x^3</math></li> <li>• Neither</li> </ul>
8	<b>ans:</b> 0.35 [cm/s] <b>5 marks</b> <ul style="list-style-type: none"> <li>• knows how find <math>\frac{dr}{dt}</math></li> <li>• finds <math>\frac{dr}{dV}</math> correctly</li> <li>• finds correct formula for <math>\frac{dr}{dt}</math></li> <li>• finds correct radius</li> <li>• evaluates <math>\frac{dr}{dt}</math> correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}</math></li> <li>• <math>\frac{dr}{dV} = \frac{1}{4\pi r^2}</math></li> <li>• <math>\frac{dr}{dt} = \frac{15}{14r^2}</math></li> <li>• <math>r = 1.75</math></li> <li>• <math>\frac{dr}{dt} = 0.35</math></li> </ul>
9	<b>ans:</b> Proof <b>3 marks</b> <ul style="list-style-type: none"> <li>• knows how to start proof : <math>n = 2k \pm 1</math></li> <li>• continues proof : simplifies <math>n = 2k \pm 1</math></li> <li>• completes proof : common factor of 8</li> </ul>	<ul style="list-style-type: none"> <li>• <math>n</math> is odd <math>\Rightarrow n = 2k \pm 1</math> (<math>k \in \mathbb{Z}</math>)</li> <li>• <math>\Rightarrow n^4 - 1 = 16k^4 \pm 32k^3 + 24k^2 \pm 8k</math></li> <li>• <math>\Rightarrow n^4 - 1 = 8(2k^4 \pm 4k^3 + 3k^2 \pm k)</math> which is divisible by 8</li> </ul>

	Give one mark for each •	Illustrations for awarding each mark
10(a)	<p><b>ans:</b> <math>\frac{3}{2(x-3)} - \frac{3}{2(x+3)}</math></p> <p style="text-align: right;"><b>2 marks</b></p> <ul style="list-style-type: none"> <li>• first fraction</li> <li>• second fraction</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\frac{3}{2(x-3)}</math></li> <li>• <math>-\frac{3}{2(x+3)}</math></li> </ul>
10(b)	<p><b>ans:</b> <math>1 + \frac{3}{2} \ln \frac{1}{2}</math></p> <p style="text-align: right;"><b>4 marks</b></p> <ul style="list-style-type: none"> <li>• divides correctly</li> <li>• integrates correctly</li> <li>• substitutes limits correctly</li> <li>• evaluates correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>1 + \frac{9}{x^2 - 9}</math></li> <li>• <math>x + \frac{3}{2} \ln x-3  - \frac{3}{2} \ln x+3 </math></li> <li>• <math>\left(1 + \frac{3}{2} \ln -2  - \frac{3}{2} \ln 4 \right) -</math></li> <li>• <math>\left(0 + \frac{3}{2} \ln -3  - \frac{3}{2} \ln 3 \right)</math></li> <li>• <math>1 + \frac{3}{2} (\ln 2 - \ln 4)</math></li> </ul>
11(a)	<p><b>ans:</b> <math>x = -3</math> &amp; <math>y = x - 3</math></p> <p style="text-align: right;"><b>3 marks</b></p> <ul style="list-style-type: none"> <li>• states equation of vertical asymptote</li> <li>• divides correctly</li> <li>• states equation of oblique asymptote</li> </ul>	<ul style="list-style-type: none"> <li>• <math>x = -3</math></li> <li>• <math>f(x) = x - 3 + \frac{12}{x+3}</math></li> <li>• <math>y = x - 3</math></li> </ul>
11(b)	<p><b>ans:</b> (0,0) → minimum turning point; (-6,-12) → maximum turning point</p> <p style="text-align: right;"><b>5 marks</b></p> <ul style="list-style-type: none"> <li>• differentiates correctly</li> <li>• finds x-coordinates of stationary points</li> <li>• finds y-coordinates of stationary points</li> <li>• finds second derivative or nature table</li> <li>• correct nature of both points</li> </ul>	<ul style="list-style-type: none"> <li>• <math>f'(x) = \frac{x^2 + 6x}{(x+3)^3}</math></li> <li>• <math>f'(x) = 0 \Rightarrow x = 0, -6</math></li> <li>• (0,0) &amp; (-6,-12)</li> <li>• <math>f''(x) = \frac{(2x+6)(x+3)^2 - 2(x^2+6x)(x+3)}{(x+3)^4}</math></li> <li>• <math>f''(0) &gt; 0 \Rightarrow (0,0) \text{Min.T.P.}</math> &amp; <math>f''(-6) &lt; 0 \Rightarrow (-6,-12) \text{Max.T.P.}</math></li> </ul>

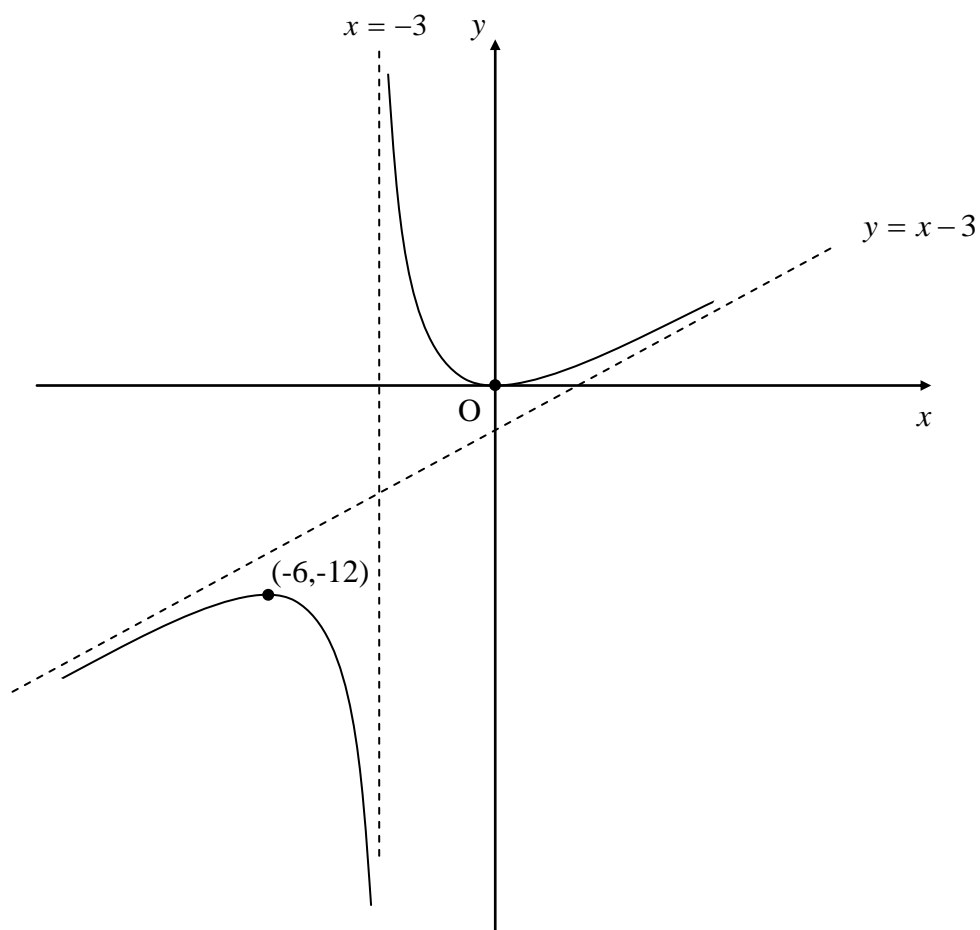


	Give one mark for each •	Illustrations for awarding each mark
11(c)	<b>ans:</b> correct graph <span style="float: right;"><b>2 marks</b></span> <ul style="list-style-type: none"> <li>• turning points shown</li> <li>• completes graph</li> </ul> <p style="text-align: center;"><i>see graph on next page</i></p>	<ul style="list-style-type: none"> <li>• correct turning points</li> <li>• correct behaviour at asymptotes</li> </ul>
11(d)	<b>ans:</b> (0,10) & (-6,22) <span style="float: right;"><b>2 marks</b></span> <ul style="list-style-type: none"> <li>• one point correct</li> <li>• second point correct</li> </ul>	<ul style="list-style-type: none"> <li>• (0,10)</li> <li>• (-6,22)</li> </ul>
12(a)	<b>ans:</b> Proof <span style="float: right;"><b>4 marks</b></span> <ul style="list-style-type: none"> <li>• knows how to find common difference</li> <li>• simplifies correctly</li> <li>• knows how to find sum of first 10 terms</li> <li>• simplifies correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>1 + \frac{1}{\sqrt{2}} - (1 + \sqrt{2})</math></li> <li>• <math>\frac{1}{\sqrt{2}} - \sqrt{2} = \frac{1-2}{\sqrt{2}} = \frac{-1}{\sqrt{2}}</math></li> <li>• <math>\frac{10}{2} \left[ 2(1 + \sqrt{2}) + (10-1) \left( \frac{-1}{\sqrt{2}} \right) \right]</math></li> <li>• <math>5 \left( 2 + 2\sqrt{2} - \frac{9}{\sqrt{2}} \right) = \dots = \frac{5}{2} (4 - 5\sqrt{2})</math></li> </ul>
12(b)	<b>ans:</b> Proof <span style="float: right;"><b>5 marks</b></span> <ul style="list-style-type: none"> <li>• knows to find common ratio</li> <li>• finds common ratio correctly</li> <li>• justifies that sum to infinity exists</li> <li>• knows how to find sum to infinity</li> <li>• simplifies correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\frac{1 + \frac{1}{\sqrt{2}}}{1 + \sqrt{2}}</math></li> <li>• <math>\frac{1}{\sqrt{2}}</math></li> <li>• <math>-1 &lt; \frac{1}{\sqrt{2}} &lt; 1</math></li> <li>• <math>\frac{1 + \sqrt{2}}{1 - \frac{1}{\sqrt{2}}}</math></li> <li>• <math>\frac{\sqrt{2} + 2}{\sqrt{2} - 1} = \dots = 4 + 3\sqrt{2}</math></li> </ul>

	Give one mark for each •	Illustrations for awarding each mark
13	<b>ans: <math>t = 3</math></b> <b>6 marks</b> <ul style="list-style-type: none"> <li>• knows how to find volume of solid</li> <li>• finds limits of integration</li> <li>• integrates correctly</li> <li>• substitutes limits correctly</li> <li>• equates volumes</li> <li>• solves for <math>t</math> correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>V = \int \pi(y - 4)dy</math></li> <li>• <math>1 \rightarrow 5, t \rightarrow t^2 + 4</math></li> <li>• <math>\pi\left[\frac{y^2}{2} - 4y\right]</math></li> <li>• <math>\pi\left\{\left(\frac{(t^2 + 4)^2}{2} - 4(t^2 + 4)\right) - \left(\frac{5^2}{2} - 4(5)\right)\right\}</math></li> <li>• <math>40\pi = \pi\left(\frac{t^4}{2} - \frac{1}{2}\right)</math></li> <li>• <math>t = 3</math></li> </ul>

**TOTAL MARKS = 74**

**Q11 (c)**



*Higher Still - 2006 / 2007*

# **MATHEMATICS**

## **Advanced Higher Grade – Mini Prelim (Unit 3 + Units 1/2 Revision)**

**Time allowed - 1 hour 20 minutes**

---

### **Read Carefully**

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions.
3. **Full credit will be given only where the solution contains appropriate working.**
4. **This test contains questions graded at all levels.**

**All questions should be attempted**

1. Find the general solution of the differential equation

$$x \frac{dy}{dx} + (x-2)y = x^4. \quad 5$$

Given that  $y = 5e^{-1}$  when  $x = 1$ , find the particular solution. 2

2. (a) Show that the matrix  $A = \begin{pmatrix} 2 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 3 & 1 \end{pmatrix}$  is non-singular. 3

(b) Use elementary row operations to find  $A^{-1}$ . 5

(c) **Hence** solve the system of equations

$$\begin{aligned} 2x + y + 4z &= 2 \\ x + 2z &= 3 \\ 2x + 3y + z &= -6. \end{aligned} \quad 2$$

3. (a) Obtain the first five terms in the Maclaurin expansion of  $(1+3x)^{\frac{5}{3}}$ . 4

(b) For what values of  $x$  is this series valid? 2

(c) Use the expansion to find an approximation for  $1 \cdot 9^{\frac{5}{3}}$ . 2

4. (a) Express  $458_6$  in base 8. 3

(b) Prove by induction that  $n(n+1)(n+2)$  is divisible by 6 for all positive integers  $n$ . 6

5. A function  $y(x)$  is defined implicitly by  $x^3 + 4xy = 3$ .

Obtain formulae for  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$  only. 5

Hence evaluate  $\frac{dy}{dx}$  at  $(1, 0)$  and  $\frac{d^2y}{dx^2}$  at  $(2, -1)$ . 2

6. (a) Find the point of intersection of the line  $L_1$

$$\frac{x-6}{2} = \frac{y+2}{1} = \frac{z+7}{-3}$$

and the plane with equation  $3x - y - 2z = 12$ .

4

- (b) Find the point of intersection of the line  $L_1$  and the line  $L_2$

$$\frac{x-6}{-1} = \frac{y+7}{2} = \frac{z}{-2}.$$

4

7. Let  $z = \frac{1}{\cos\theta + i\sin\theta}$ .

- (a) Use de Moivre's theorem to express  $z^5$  in the form  $\cos p\theta - i\sin p\theta$ , where  $p$  is a natural number.

2

- (b) Use the binomial theorem to express  $\sin 5\theta$  in the form

$$q\sin\theta + r\sin^3\theta + t\sin^5\theta,$$

and state the values of  $q$ ,  $r$  and  $t$ .

5

[END OF QUESTION PAPER]

**Mini Prelim (Assessing Unit 3 + Revision)**

	<b>Give one mark for each •</b>	<b>Illustrations for awarding each mark</b>
1	<p><b>ans:</b> <math>y = x^3 - x^2 + \frac{Cx^2}{e^x}</math> <b>5 marks</b></p> <p><math>y = x^3 - x^2 + \frac{5x^2}{e^x}</math> <b>2 marks</b></p> <ul style="list-style-type: none"> <li>• knows to find integrating factor correctly</li> <li>• finds correct integrating factor</li> <li>• uses integrating factor correctly</li> <li>• uses integration by parts correctly</li> <li>• finds general solution correctly</li> <li>• substitutes conditions correctly</li> <li>• finds particular solution correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>e^{\int \frac{x-2}{x} dx}</math></li> <li>• <math>\frac{e^x}{x^2}</math></li> <li>• <math>\frac{e^x}{x^2} y = \int x e^x dx</math></li> <li>• <math>x e^x - \int e^x dx</math></li> <li>• <math>y = x^3 - x^2 + \frac{Cx^2}{e^x}</math></li> <li>• <math>5e^{-1} = 1 - 1 + \frac{C}{e}</math></li> <li>• <math>C = 5</math></li> </ul>
2(a)	<p><b>ans:</b> <math>\det A = 3 \neq 0</math> <b>3 marks</b></p> <ul style="list-style-type: none"> <li>• knows how to find the determinant of a <math>3 \times 3</math> matrix</li> <li>• finds determinant correctly</li> <li>• correct explanation</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\det A = \begin{vmatrix} 2 &amp; 1 &amp; 4 \\ 1 &amp; 0 &amp; 2 \\ 2 &amp; 3 &amp; 1 \end{vmatrix}</math></li> <li>• 3</li> <li>• Since <math> A  \neq 0</math>, <math>A</math> is non-singular</li> </ul>
2(b)		

	<p><b>ans:</b> <math>A^{-1} = \frac{1}{3} \begin{pmatrix} -6 &amp; 11 &amp; 2 \\ 3 &amp; -6 &amp; 0 \\ 3 &amp; -4 &amp; -1 \end{pmatrix}</math></p> <p style="text-align: right;"><b>5 marks</b></p> <ul style="list-style-type: none"> <li>• correct augmented matrix</li> <li>• one correct row</li> <li>• a second correct row</li> <li>• the third row correct</li> <li>• identifies <math>A^{-1}</math></li> </ul>	<ul style="list-style-type: none"> <li>• <math>\begin{pmatrix} 2 &amp; 1 &amp; 41 &amp; 0 &amp; 0 \\ 1 &amp; 0 &amp; 20 &amp; 1 &amp; 0 \\ 2 &amp; 3 &amp; 10 &amp; 0 &amp; 1 \end{pmatrix}</math></li> <li>• <math>\begin{pmatrix} 0 &amp; 1 &amp; 01 &amp; -2 &amp; 0 \end{pmatrix}</math></li> <li>• <math>\begin{pmatrix} 0 &amp; 0 &amp; 11 &amp; \frac{-4}{3} &amp; \frac{-1}{3} \end{pmatrix}</math></li> <li>• <math>\begin{pmatrix} 1 &amp; 0 &amp; 0-2 &amp; \frac{11}{3} &amp; \frac{2}{3} \end{pmatrix}</math></li> <li>• <math>A^{-1} = \begin{pmatrix} -2 &amp; \frac{11}{3} &amp; \frac{2}{3} \\ 1 &amp; -2 &amp; 0 \\ 1 &amp; \frac{-4}{3} &amp; \frac{-1}{3} \end{pmatrix}</math></li> </ul>
2(c)	<p><b>ans:</b> <math>x = 3, y = -4, z = 0</math></p> <p style="text-align: right;"><b>2 marks</b></p> <ul style="list-style-type: none"> <li>• knows to pre-multiply both sides by <math>A^{-1}</math></li> <li>• correct solution</li> </ul>	<ul style="list-style-type: none"> <li>• <math>\frac{1}{3} \begin{pmatrix} -6 &amp; 11 &amp; 2 \\ 3 &amp; -6 &amp; 0 \\ 3 &amp; -4 &amp; -1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}</math></li> <li>• <math>\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}</math></li> </ul>
3(a)	<p><b>ans:</b> <math>(1 + 3x)^{\frac{5}{3}} = 1 + 5x + 5x^2 - \frac{5}{3}x^3 + \frac{5}{3}x^4</math></p> <p style="text-align: right;"><b>4 marks</b></p> <ul style="list-style-type: none"> <li>• evaluates <math>f(0)</math> &amp; <math>f'(0)</math> correctly</li> <li>• evaluates <math>f''(0)</math> correctly</li> <li>• evaluates <math>f'''(0)</math> &amp; <math>f^{iv}(0)</math> correctly</li> </ul>	<ul style="list-style-type: none"> <li>• <math>f(0) = 1</math> &amp; <math>f'(0) = 5</math></li> <li>• <math>f''(0) = 10</math></li> <li>• <math>f'''(0) = -10</math> &amp; <math>f^{iv}(0) = 40</math></li> </ul>

	<ul style="list-style-type: none"> <li>• correct expansion</li> </ul>		<ul style="list-style-type: none"> <li>• Correct expansion</li> </ul>
3(b)	<p><b>ans:</b> <math> x  &lt; \frac{1}{3}</math></p> <ul style="list-style-type: none"> <li>• knows range of validity</li> <li>• solves inequality</li> </ul>	<b>2 marks</b>	<ul style="list-style-type: none"> <li>• <math> 3x  &lt; 1</math></li> <li>• <math> x  &lt; \frac{1}{3}</math></li> </ul>
3(c)	<p><b>ans:</b> 2.9185</p> <ul style="list-style-type: none"> <li>• use expansion correctly</li> <li>• correct approximation</li> </ul>	<b>2 marks</b>	<ul style="list-style-type: none"> <li>• <math>(1 + 3(0.3))^{\frac{5}{3}} = 1 + 5(0.3) + 5(0.3)^2 - \frac{5}{3}(0.3)^3 + \frac{5}{3}(0.3)^4</math></li> <li>• 2.9185</li> </ul>



4(a)	<b>ans:</b> $266_8$ <b>3 marks</b> <ul style="list-style-type: none"> <li>changes to base 10</li> <li>repeated division by 8</li> <li>correct answer in base 8</li> </ul>	<ul style="list-style-type: none"> <li><math>458_6 = 182</math></li> <li><math>182 \div 8 = 22 \text{ r } 6, 22 \div 8 = 2 \text{ r } 6, 2 \div 8 = 0 \text{ r } 2</math></li> <li><math>266_8</math></li> </ul>
4(b)	<b>ans:</b> Proof <b>6 marks</b> <ul style="list-style-type: none"> <li>knows how to start proof; e.g. true for <math>n=1</math></li> <li>assume true for <math>n=k</math></li> <li>statement for <math>n=k+1</math></li> <li>continues proof : consider <math>n</math> odd</li> <li>continues proof : consider <math>n</math> even</li> <li>completes proof</li> </ul>	<ul style="list-style-type: none"> <li><math>n=1:1(1+1)(1+2) = 6</math> which is divisible by 6</li> <li><math>n=k:k(k+1)(k+2) [=6L]</math> is divisible by 6</li> <li><math>n=k+1:(k+1)(k+2)(k+3)</math> is divisible by 6</li> <li><u><math>k</math> odd</u> <math>[=2m+1]</math>-  <math>(k+1)(k+2)(k+3)</math>  <math>=6L+3(k+1)(k+2)</math>  <math>=6L+3(2m+2)(2m+3)</math>  <math>=6[L+(m+1)(2m+3)]</math> which is divisible by 6</li> <li><u><math>k</math> even</u> <math>[=2m]</math>-  <math>(k+1)(k+2)(k+3)</math>  <math>=6L+3(k+1)(k+2)</math>  <math>=6L+3(2m+1)(2m+2)</math>  <math>=6[L+(2m+1)(m+1)]</math> which is divisible by 6</li> <li>Since true for <math>n=1</math> and <math>[\text{true for } n=k \Rightarrow \text{true for } n=k+1]</math>, the result is true for all positive integers <math>n</math>.</li> </ul>
5	<b>ans:</b> $\frac{dy}{dx} = \frac{-3}{4}x - \frac{y}{x}$ <b>5 marks</b> $\frac{d^2y}{dx^2} = \frac{2y}{x^2}$ $\frac{dy}{dx} = \frac{-3}{4}$ <b>2 marks</b> $\frac{d^2y}{dx^2} = \frac{-1}{2}$ <ul style="list-style-type: none"> <li>knows how to use implicit differentiation</li> <li>differentiates correctly</li> <li>knows how to find second derivative</li> <li>differentiates correctly</li> <li>finds simplified answer (in terms of <math>x</math> and <math>y</math> only)</li> <li>evaluates first derivative correctly</li> <li>evaluates second derivative correctly</li> </ul>	<ul style="list-style-type: none"> <li><math>3x^2 + 4y + 4x \frac{dy}{dx} = 0</math></li> <li><math>\frac{dy}{dx} = \frac{-3}{4}x - \frac{y}{x}</math></li> <li><math>\frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right)</math></li> <li><math>\frac{-3}{4} - \left\{ -x^{-2}y + x^{-1} \frac{dy}{dx} \right\}</math></li> <li><math>\frac{d^2y}{dx^2} = \frac{2y}{x^2}</math></li> <li><math>\frac{-3}{4}</math></li> <li><math>\frac{-1}{2}</math></li> </ul>

6(a)	<b>ans:</b> (2,-4,-1) <b>4 marks</b> <ul style="list-style-type: none"> <li>expresses <math>x</math>, <math>y</math> and <math>z</math> in terms of <math>t</math></li> <li>substitutes in plane equation</li> <li>solves for <math>t</math></li> <li>correct point</li> </ul>	<ul style="list-style-type: none"> <li><math>x = 2t + 6, y = t - 2, z = -3t - 7</math></li> <li><math>3(2t + 6) - (t - 2) - 2(-3t - 7) = 12</math></li> <li><math>t = -2</math></li> <li>(2,-4,-1)</li> </ul>
6(b)	<b>ans:</b> (4,-3,-4) <b>4 marks</b> <ul style="list-style-type: none"> <li>correct system of equations</li> <li>integrates correct value for <math>t</math></li> <li>correct value for <math>s</math></li> <li>correct point</li> </ul>	<ul style="list-style-type: none"> <li><math>2t + s = 0, t - 2s = -5, -3t + 2s = 7</math></li> <li><math>t = -1</math></li> <li><math>s = 2</math></li> <li>(4,-3,-4)</li> </ul>
7(a)	<b>ans:</b> $z^5 = \cos 5\theta - i \sin 5\theta$ <b>2 marks</b> <ul style="list-style-type: none"> <li>applies de Moivre's theorem correctly</li> <li>express answer in correct form</li> </ul>	<ul style="list-style-type: none"> <li><math>\cos(-5\theta) + i \sin(-5\theta)</math></li> <li><math>\cos 5\theta - i \sin 5\theta</math></li> </ul>
7(b)	<b>ans:</b> $\sin 5\theta = 5 \sin \theta - 20 \sin^3 \theta + 16 \sin^5 \theta$ $q = 5, r = -20 \text{ \& } t = 16$ <b>5 marks</b> <ul style="list-style-type: none"> <li>uses the binomial theorem correctly</li> <li>equates imaginary parts</li> <li>substitutes correctly</li> <li>simplifies correctly</li> <li>correct values of <math>q, r \text{ \&amp; } t</math>.</li> </ul>	$z^5 = (\cos \theta)^5 + 5(\cos \theta)^4(-i \sin \theta) +$ <ul style="list-style-type: none"> <li><math>10(\cos \theta)^3(-i \sin \theta)^2 + 10(\cos \theta)^2(-i \sin \theta)^3 + 5(\cos \theta)(-i \sin \theta)^4 + (-i \sin \theta)^5</math></li> <li><math>\sin 5\theta = 5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta</math></li> <li><math>5(1 - \sin^2 \theta)^2 \sin \theta - 10(1 - \sin^2 \theta) \sin^3 \theta + \sin^5 \theta</math></li> <li><math>5 \sin \theta - 20 \sin^3 \theta + 16 \sin^5 \theta</math></li> <li><math>q = 5, r = -20 \text{ \&amp; } t = 16</math></li> </ul>

**TOTAL MARKS = 56**