Chapter 14: Exponential and Logarithmic Functions

Exponential Functions

Today's Learning:
By the end of this lesson, you should know what an exponential function is.

Growth and decay occur all around us.

Bacteria grow.

Radioactive elements decay.

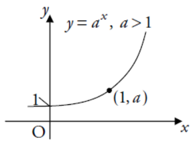
Plants grow and decay.

A function of the form $y = a^x$ is called an exponential function to the base a.

a is constant and $a > 0$

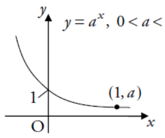
x is a variable and is called the power or the exponent

If $a > 1$ then the graph looks like this:



This is sometimes called a growth function

If $0 < a < 1$ then the graph looks like this:



This is sometimes called a decay function

Examples

1) The number of bacteria present doubles every hour.
There are 500 present at 12 noon.
Find the number present at:

a) 2 pm

After 1 hour 500×2

After 2 hours $500 \times 2 \times 2 = 500 \times 2^2$

b) 3 pm

After 3 hours 500×2^3

c) midnight

After 12 hours 500×2^{12}

d) after n hours

500×2^n

2)

The otter population on an island increases by 16% per year. How many full years will it take the population to double?

Let u_0 be the initial population.

$$u_1 = 1{\cdot}16u_0$$
$$u_2 = 1{\cdot}16u_1 = 1{\cdot}16(1{\cdot}16u_0) = 1{\cdot}16^2 u_0$$
$$u_3 = 1{\cdot}16u_2 = 1{\cdot}16(1{\cdot}16^2 u_0) = 1{\cdot}16^3 u_0$$
$$\vdots$$
$$u_n = 1{\cdot}16^n u_0$$

For the population to double after n years, we require $u_n \geq 2u_0$.

So the coefficient of u_0, which is $1{\cdot}16^n$, must be at least 2, i.e. $1{\cdot}16^n \geq 2$.

Try values of n until this is satisfied.

$u_n = 1{\cdot}16^n u_0$
$u_n = 2u_0$
$u_n > 2u_0$

If $n = 2,\ 1{\cdot}16^2 = 1{\cdot}35 < 2$
If $n = 3,\ 1{\cdot}16^3 = 1{\cdot}56 < 2$
If $n = 4,\ 1{\cdot}16^4 = 1{\cdot}81 < 2$
If $n = 5,\ 1{\cdot}16^5 = 2{\cdot}10 > 2$

Therefore after 5 years the population will double.

3)

The efficiency of a machine decreases by 5% each year. When the efficiency drops below 75%, the machine needs to be serviced. After how many years will the machine need serviced?

Let u_0 be the initial efficiency.

$$u_1 = 0{\cdot}95u_0$$
$$u_2 = 0{\cdot}95u_1 = 0{\cdot}95(0{\cdot}95u_0) = 0{\cdot}95^2 u_0$$
$$u_3 = 0{\cdot}95u_2 = 0{\cdot}95(0{\cdot}95^2 u_0) = 0{\cdot}95^3 u_0$$
$$\vdots$$
$$u_n = 0{\cdot}95^n u_0$$

When the efficiency drops below $0{\cdot}75u_0$ (75% of the initial value) the machine must be serviced. So the machine needs serviced after n years if $0{\cdot}95^n < 0{\cdot}75$.

$u_n = 0{\cdot}95^n u_0$

$u_n = 0{\cdot}75u_0$

$u_n < 0{\cdot}75u_0$

Try values of n until this is satisfied:

If $n = 2$, $0{\cdot}95^2 = 0{\cdot}903 > 0{\cdot}75$

If $n = 3$, $0{\cdot}95^3 = 0{\cdot}857 > 0{\cdot}75$

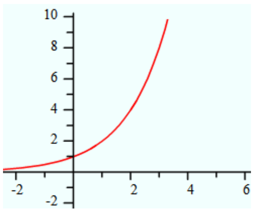
If $n = 4$, $0{\cdot}95^4 = 0{\cdot}815 > 0{\cdot}75$

If $n = 5$, $0{\cdot}95^5 = 0{\cdot}774 > 0{\cdot}75$

If $n = 6$, $0{\cdot}95^6 = 0{\cdot}735 < 0{\cdot}75$

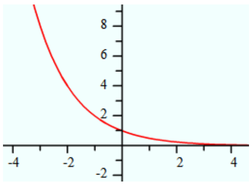
Ex 15C

Therefore after 6 years, the machine will have to be serviced.



- always positive
- never crosses the x axis
- is increasing
- passes through $(0,1)$
- passes through $(1,a)$

$y = a^x$



- always positive
- never crosses the x axis
- is decreasing
- passes through $(0,1)$
- passes through $(1,a)$

$y = a^{-x}$

NOTE

They all pass through $(0,1)$ - anything to the power 0 is always 1.

$y = a^x$ is always positive - raising to a power **always** gives a positive answer.

$y = (\frac{1}{2})^x$ is the same as $y = 2^{-x}$

Starter

$\frac{d}{dx}\sqrt{(x+2)(x-2)}$ equals

A. $x(x^2-4)^{-\frac{1}{2}}$

B. $\frac{1}{2}(x^2-4)^{-\frac{1}{2}}$

C. $\frac{2}{3}(x^2-4)^{\frac{3}{2}}$

D. $(x^2-4)^{\frac{1}{2}}$

$$= \frac{d}{dx}\sqrt{x^2-4}$$
$$= \frac{d}{dx}(x^2-4)^{1/2}$$
$$= \frac{1}{2}(x^2-4)^{-1/2} \times 2x$$
$$= x(x^2-4)^{-1/2}$$

The Exponential Function to the base e

Today's Learning:

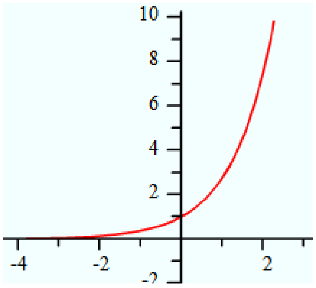
By the end of this lesson, you should know what e stands for.

$\boxed{e \text{ is approximately } 2{\cdot}71828 \ldots\ldots}$

This number occurs often when describing growth and decay.
For example, population growth and radioactive decay.

e^x is called the exponential function to the base e.

What does the graph of e^x look like?



- always positive
- never crosses the x axis
- is increasing
- passes through $(0,1)$
- passes through $(1,e)$

Examples

1) Evaluate e^{-2}

0·1353...

2) The mass of a quantity of radioactive substance decays according to the formula

$$m = 40e^{-0{\cdot}03t}$$

m is the mass in grams

t is the time in years

What is the mass after 12 years?

$$m = 40 \times e^{-0{\cdot}03 \times 12}$$

$$= 40 \times e^{-0{\cdot}36}$$

$$= 27{\cdot}91 \text{ grams}$$

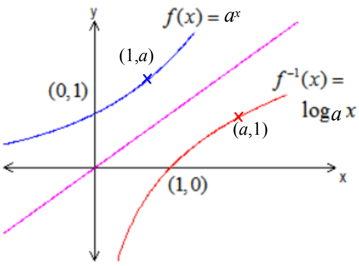
Ex 15D

Q1, 2, 4 and 5.

The Logarithmic Function

Today's Learning:
By the end of this lesson, you should know that the inverse of the exponential function is the logarithmic function and be able to re write exponential functions as logarithmic functions.

In Unit 1 we learned that the exponential function has an inverse function called the logarithmic function.



From the graph

$\log_a 1 = 0$

$\log_a a = 1$

We will revisit this later

If $\log_a x = y$ then $x = a^y$

Examples

Rewrite as a logarithm

$x = a^y$

$\log_a x = y$

1) $10^3 = 1000$

$\log_{10}1000 = 3$

2) $5^4 = 625$

$\log_5 625 = 4$

3) $2^{10} = 1024$

$\log_2 1024 = 10$

$\log_{10}1000 = 3$
$10^3 = 1000$

$\log_2 8 = 3$
$2^{\square} = 8$

Rewrite using a power

4) $\log_2 16 = 4$

$2^4 = 16$

5) $\log_5 25 = 2$

$5^2 = 25$

6) $\log_5 0\cdot2 = -1$

$5^{-1} = 0\cdot2$

$x = a^y$

$\log_a x = y$

7) Change to exponential form

a) $6y = \log_3 5$

$5 = 3^{6y}$

b) $7 = \log_2 4x$

$4x = 2^7$

$4x = 128$

$x = 32$

Ex 15E

07/03/18 Lesson Starter

If $\log_a x = y$ then $x = a^y$

1) Write in logarithmic form.

a) $5^4 = 625$

$\log_5 625 = 4$

b) $y = 9^3$

$\log_9 y = 3$

2) Simplify

a) $\log_4 64$ $= 4^{\square} = 64$

$= 3$

b) $\log_2 256$ $2^{\square} = 256$

$= 8$

Laws of Logarithms

Today's Learning:
By the end of this lesson, you should know and be able to apply the three rules of logarithms.

We should already know:

$\boxed{\log_a a = 1}$ $a^1 = a$

$\boxed{\log_a 1 = 0}$ $a^0 = 1$

There are three laws that apply when using logarithms

Law 1 $\log_a x = p$ $\log_a y = q$

$a^p = x$ $a^q = y$

$x = a^p$ $y = a^q$

$xy = a^p \times a^q$

$xy = a^{p+q}$ Re write as a log

$\log_a xy = p + q$

$\log_a xy = \log_a x + \log_a y$

Law 1 $\boxed{\log_a xy = \log_a x + \log_a y}$

Law 2

$\log_a x = p$ $\log_a y = q$

$a^p = x$ $a^q = y$

$x = a^p$ $y = a^q$

$$\frac{x}{y} = \frac{a^p}{a^q}$$

$$\frac{x}{y} = a^{p-q}$$

Re write as a log

$$\log_a \frac{x}{y} = p - q$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

Law 2 $\boxed{\log_a \frac{x}{y} = \log_a x - \log_a y}$

Law 3

$\log_a x = p$

$a^p = x$

$x = a^p$

$x^n = (a^p)^n$

$x^n = a^{pn}$

Re write as a log

$\log_a x^n = pn$

$\log_a x^n = n \log_a x$

Law 3 $\boxed{\log_a x^n = n \log_a x}$

Law 1 $\boxed{\log_a xy = \log_a x + \log_a y}$

Law 2 $\boxed{\log_a \frac{x}{y} = \log_a x - \log_a y}$

Law 3 $\boxed{\log_a x^n = n \log_a x}$

Examples

Simplify the following

1) $\log_5 2 + \log_5 4$

Use Law 1 $\boxed{\log_a xy = \log_a x + \log_a y}$

$= \log_5 (2 \times 4)$

$= \log_5 8$

2) $\log_4 6 - \log_4 3$

$\boxed{\log_a \frac{x}{y} = \log_a x - \log_a y}$

Use Law 2

$= \log_4 (6 \div 3)$

$= \log_4 2$ $4^{\square} = 2$

$= \frac{1}{2}$

3) $2\log_7 3$

$\boxed{\log_a x^n = n \log_a x}$

Use Law 3

$= \log_7 3^2$

$= \log_7 9$

4) $\log_{10} 2 + \log_{10} 500$

Use Law 1 $\boxed{\log_a xy = \log_a x + \log_a y}$

$= \log_{10} 1000$

$= 3$ $10^{\square} = 1000$

5) $\log_3 \frac{63}{7}$

$= \log_3 9$

$= 2$

$\boxed{\log_a \frac{x}{y} = \log_a x - \log_a y}$

$3^{\square} = 9$

6) $\frac{1}{2}\log_2 16 - \frac{1}{3}\log_2 8$

Use Law 3

$\boxed{\log_a x^n = n \log_a x}$

$= \log_2 16^{\frac{1}{2}} - \log_2 8^{\frac{1}{3}}$

$16^{\frac{1}{2}} = \sqrt{16} = 4$

$8^{1/3} = \sqrt[3]{8} = 2$

$= \log_2 4 - \log_2 2$

$= 2 - 1$

$\log_2\left(\frac{4}{2}\right)$

$= \log_2 2$

$= 1$

$= 1$

7) Evaluate $\log_{12} 10 + \log_{12} 6 - \log_{12} 5$

$= \log_{12} 60 - \log_{12} 5$

$= \log_{12} 12$

$= 1$

8) Evaluate $\log_6 4 + 2\log_6 3$

$= \log_6 4 + \log_6 3^2$

$= \log_6 4 + \log_6 9$

$= \log_6 36$

$= 2$

Ex 15F

All Questions

Logarithmic Equations

Today's Learning:
By the end of this lesson, you should be able to solve logarithmic equations.

Examples

1) Solve $\log_a 13 + \log_a x = \log_a 273$ for $x > 0$

$\log_a 13x = \log_a 273$

$13x = 273$

$x = 21$

2) Solve $\log_{11}(4x + 3) - \log_{11}(2x - 3) = 1$ for $x > \frac{3}{2}$

$\log_{11} \frac{4x + 3}{2x - 3} = 1$

Since,
$\log_a x = y$
$x = a^y$

$11^1 = \frac{4x + 3}{(2x - 3)}$

$11(2x - 3) = 4x + 3$

$22x - 33 = 4x + 3$

$18x = 36$

$x = 2$

3) Solve $\log_a(2p+1)+\log_a(3p-10)=\log_a 11p$ for $p>4$

$$\log_a((2p+1)(3p-10))=\log_a 11p$$

$$(2p+1)(3p-10)=11p$$

$$6p^2-17p-10=11p$$

$$6p^2-28p-10=0$$

$$2(3p^2-14p-5)=0$$

$$2(3p+1)(p-5)=0$$

$p=-\frac{1}{3}$ or $p=5$

$p=5$ as $p>4$

4) Solve $\log_2 7=\log_2 x+3$ for $x>0$

$$\log_2 7=\log_2 x+3$$

$$\log_2 7-\log_2 x=3$$

$$\log_2\frac{7}{x}=3$$

Since, $\log_a x=y$, $x=a^y$

$$2^3=\frac{7}{x}$$

$$8=\frac{7}{x}$$

$8x=7$ $x=\frac{7}{8}$

or Solve $\log_2 7=\log_2 x+3$ for $x>0$

$$\log_2 7=\log_2 x+3\log_2 2$$

$$\log_2 7=\log_2 x+\log_2 2^3$$

$$\log_2 7=\log_2 x+\log_2 8$$

$$\log_2 7=\log_2 8x$$

$$7=8x$$

$$x=\frac{7}{8}$$

5) Solve $5^x=11$

$$5^x=11$$

Take logs of both sides

$$\log_{10}5^x=\log_{10}11$$

Use Law 3

$$x\log_{10}5=\log_{10}11$$

$$x=\frac{\log_{10}11}{\log_{10}5}$$

$x=\frac{\log 11}{\log 5}$

$$x=1.49$$

or

$$5^x=11$$

$$\log_5 11=x$$

$$x=1.49$$

Starter

True or False?

F $log_{10}15=log_{10}3\times log_{10}5$

T $log_{10}4=log_{10}8-log_{10}2$

T $log_{10}8^2=2log_{10}8$

F $log_{10}2^6=6log_9 2$

Natural Logs

Today's Learning:
By the end of this lesson, you should be able to solve logarithmic equations which contain natural logs.

Logarithms to the base e are called natural logarithms, written $\log_e x$ or $\ln x$ for short.

Examples

1) Solve $\ln_e x=9$

$$\log_e x=9$$

$\log_a x=y$, $x=a^y$

$$x=e^9$$

$$x=8103.08$$

2) Solve $e^x = 7$ or $e^x = 7$

$e^x = 7$

Take logs of both sides

$\log_e e^x = \log_e 7$

Use Law 3

$x\log_e e = \log_e 7$

$\log_e e = 1$

$\boxed{\ln e = 1}$

$x = \log_e 7$

$x = \ln 7$

$x = 1 \cdot 946$

$e^x = 7$

$\log_e 7 = x$

$\ln 7 = x$

$x = 1 \cdot 946$

3) Solve $7^{2x+1} = 49$

Take logs of both sides

$\ln 7^{2x+1} = \ln 49$

Use Law 3

$(2x + 1) \times \ln 7 = \ln 49$

$2x + 1 = \dfrac{\ln 49}{\ln 7}$

$2x + 1 = 2$

$x = \dfrac{1}{2}$

4) For the formula $s(t) = 100e^{-3t}$

a) Calculate $s(0)$

$s(0) = 100 \times e^{-3 \times 0}$

$= 100 \times 1$

$= 100$

b) Calculate the value of t where $s(t) = \dfrac{1}{2}s(0)$

$s(t) = 100e^{-3t}$ $s(0) = 100$

$100e^{-3t} = 50$

$e^{-3t} = \dfrac{1}{2}$

$\ln e^{-3t} = \ln\dfrac{1}{2}$

$-3t\underbrace{\ln e}_{=1} = \ln\dfrac{1}{2}$

$-3t = \ln\dfrac{1}{2}$

$t = 0 \cdot 23$

5) The formula $A_t = A_0e^{-kt}$ gives the amount of a radioactive substance after time t minutes,
After 4 minutes 50 g is reduced to 45 g.

a) Find the value of k correct to 2 sig figs.

b) How long does it take for the substance to reduce to half its original weight?

a) $A_t = A_0e^{-kt}$

$45 = 50e^{-k \times 4}$

$0 \cdot 9 = e^{-4k}$

Take natural logs of both sides

$\ln 0 \cdot 9 = \ln e^{-4k}$

Use Law 3

$$\ln 0{\cdot}9 = -4k \times \ln e$$

$\log_e e = 1$

$$\ln 0{\cdot}9 = -4k$$

$$k = \frac{\ln 0{\cdot}9}{-4}$$

$$k = 0{\cdot}026$$

b)

$$A_t = A_0 e^{-0{\cdot}026t}$$

$$25 = 50e^{-0{\cdot}026t}$$

$$e^{-0{\cdot}026t} = \frac{1}{2}$$

$$\ln e^{-0{\cdot}026t} = \ln \frac{1}{2}$$

Use Law 3

$$-0{\cdot}026t \ln e = \ln \frac{1}{2}$$

$\log_e e = 1$

$$-0{\cdot}026t = \ln \frac{1}{2}$$

$$t = 26{\cdot}7 \text{ minutes}$$

ex 15H Q4 →

Formulae from Experimental Data

Today's Learning:
By the end of this lesson, you should be able to use a straight line graph containing $\log x$ and $\log y$ to determine a formula of type $y = kx^n$.

If $y = kx^n$

Take logs of both sides

$$\log y = \log kx^n$$

Use Law 1

$$\log y = \log k + \log x^n$$

Use Law 3

$$\log y = \log k + n\log x$$

$$\log y = n\log x + \log k$$

If $y = kx^n$ then $\boxed{\log y = n\log x + \log k}$

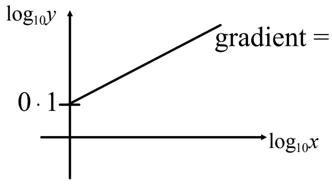
Does this type of equation look familiar?

This is similar to $y = mx + c$

NOTE: $y = \log y$, $x = \log x$, $m = n$ and $c = \log k$

Examples

1) Express y in terms of x.



If $y = kx^n$ then $\log y = n\log x + \log k$

$$\log y = n\log x + \log k$$

Substitute $n = 2$

$$\log y = 2\log x + \log k$$

Substitute a point from the graph

$$0{\cdot}1 = 2 \times 0 + \log k$$

$$0{\cdot}1 = \log k$$

or

We can see from the graph that $0{\cdot}1 = \log k$

$$\log k = 0{\cdot}1$$

$\log_a y = x \longrightarrow y = a^x$

$$10^{0{\cdot}1} = k$$

$$k = 1{\cdot}26$$

Substitute k and n into $y = kx^n$.

$$y = 1{\cdot}26x^2$$

2) The following data was collected during an experiment.

x	$50 \cdot 1$	$194 \cdot 9$	$501 \cdot 2$	$707 \cdot 9$
y	$20 \cdot 9$	$46 \cdot 8$	$83 \cdot 2$	$102 \cdot 3$

a) Show that x and y are related by the formula $y = kx^n$.

b) Find the values of k and n and state the formula that connects x and y.

a) Show that x and y are related by the formula $y = kx^n$.

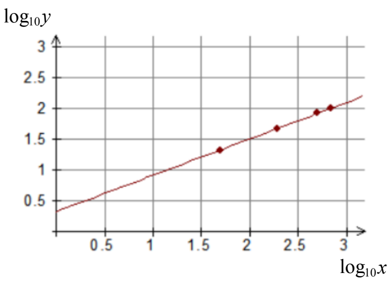
If $y = kx^n$ then $\log y = n\log x + \log k$

To show that x and y are related by the formula $y = kx^n$ we must show that $\log x$ and $\log y$ are related by the formula $\log y = n\log x + \log k$.

Take logs of x and y.

$\log_{10}x$	$1 \cdot 70$	$2 \cdot 30$	$2 \cdot 70$	$2 \cdot 85$
$\log_{10}y$	$1 \cdot 32$	$1 \cdot 67$	$1 \cdot 92$	$2 \cdot 01$

Plot the points from the table.



So, $\log_{10}x$ and $\log_{10}y$ are linked in the form $\log_{10}y = n\log_{10}x + \log_{10}k$.

Hence, x and y are connected in the form $y = kx^n$.

b) Find the values of k and n and state the formula that connects x and y.

Pick two points from the table

$(1 \cdot 70, 1 \cdot 32)$ and $(2 \cdot 85, 2 \cdot 01)$

Find the gradient by using these two points

$$n = \frac{2 \cdot 01 - 1 \cdot 32}{2 \cdot 85 - 1 \cdot 70}$$

$$n = \frac{0 \cdot 69}{1 \cdot 15}$$

$$n = 0 \cdot 6$$

Substitute into $\log y = n\log x + \log k$.

$$\log y = 0 \cdot 6\log x + \log k$$

Substitute a point from the table.

$$1 \cdot 32 = 0 \cdot 6 \times 1 \cdot 70 + \log k$$

$$1 \cdot 32 = 1 \cdot 02 + \log k$$

$$0 \cdot 3 = \log k$$

$$\log k = 0 \cdot 3$$

$$10^{0 \cdot 3} = k$$

$$k = 2 \cdot 00$$

Substitute into $y = kx^n$

$$y = 2 \cdot 00x^{0 \cdot 6}$$

Ex 15I any 2 qs on Q1

any 2 qs on Q2

Further Formulae from Experimental Data

Learning Intention

By the end of this lesson, you should be able to use a straight line graph containing $\log x$ and $\log y$ to determine a formula of type $y = ab^x$.

If $y = ab^x$

$$\log y = \log ab^x$$

Use Law 1

$$\log y = \log a + \log b^x$$

Use Law 3

$$\log y = \log a + x\log b$$

$$\log y = x\log b + \log a$$

If $y = ab^x$ then $\boxed{\log y = x\log b + \log a}$

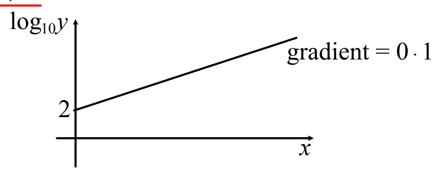
Does this type of equation look familiar?

This is similar to $y = mx + c$

NOTE: $y = \log y,\ x = x,\ m = \log b$ and $c = \log a$

Examples

1)



Express y in terms of x

$$\log_{10} y = 0 \cdot 1x + 2$$

If $y = ab^x$ then $\boxed{\log y = x\log b + \log a}$

$\log_{10} a = 2$ $\qquad$ $\log_{10} b = 0 \cdot 1$

$10^2 = a$ $\qquad$ $10^{0 \cdot 1} = b$

$a = 100$ $\qquad$ $b = 1 \cdot 26$

$$y = a \times b^x$$

$$y = 100 \times 1 \cdot 26^x$$

2) The results from an experiment were noted as follows.

x	$1 \cdot 30$	$2 \cdot 00$	$2 \cdot 30$	$2 \cdot 80$
$\log_{10} y$	$2 \cdot 04$	$2 \cdot 56$	$2 \cdot 78$	$3 \cdot 14$

The relationship between the data can be written in the form $y = ab^x$.

Find the values of a and b, and state the formula for y in terms of x.

If $y = ab^x$ then $\boxed{\log y = x\log b + \log a}$

Find the gradient by using two points from the table.

$$\log_{10} b = \frac{3 \cdot 14 - 2 \cdot 04}{2 \cdot 80 - 1 \cdot 30}$$

$$\log_{10} b = 0 \cdot 73333 \ldots$$

$$b = 5 \cdot 41$$

We will use this later.

$$\log y = 0 \cdot 7333\ldots \times x + \log a$$

$$\log y = 0 \cdot 7333\ldots \times x + \log a$$

Sub in a point from the table.

$$2 \cdot 04 = 0 \cdot 7333.. \times 1 \cdot 3 + \log a$$

$$1 \cdot 09 = \log a$$

$$\log_{10} a = 1 \cdot 09$$

$$10^{1 \cdot 09} = a$$

$$a = 12 \cdot 30$$

Re write with a and b.

$$y = ab^x$$

$$y = 12.30 \times 5.41^x$$

2) Experimental data are given in the table.

x	1.0	1.5	2.2	2.5	3.0
y	6.0	8.5	13.8	16.9	24.0

a) Show that x and y are related by the formula $y = ab^x$.

b) Find the values of a and b, and state the formula that connects x and y.

a) Show that x and y are related by the formula $y = ab^x$.

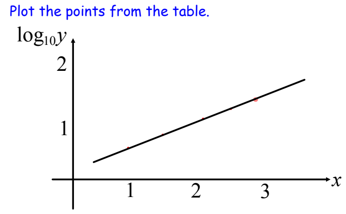
If $y = ab^x$ then $\log y = x\log b + \log a$

To show that x and y are related by the formula $y = ab^x$ we must show that x and $\log y$ are related by the formula $\log y = x\log b + \log a$.

Take logs of y only

x	1.0	1.5	2.2	2.5	3.0
$\log_{10} y$	0.78	0.93	1.12	1.23	1.38

Plot the points from the table.



Since x and $\log y$ are related, then x and y are related by the formula $y = ab^x$.

b) Find the values of a and b, and state the formula that connects x and y.

Take two points from the table and find the gradient.

$(1.0, 0.78)$ and $(3.0, 1.38)$

$$\log b = \frac{1.38 - 0.78}{3.0 - 1.0}$$

$$= 0.3$$

$$b = 10^{0.3}$$

$$= 2.00$$

We will use this later.

Substitute the gradient and a point into $\log y = x\log b + \log a$

$$\log y = x\log b + \log a$$

$$0.78 = 1 \times 0.3 + \log a$$

$$\log a = 0.48$$

$$a = 10^{0.48}$$

$$a = 3.02$$

Substitute a and b into $y = ab^x$

$$y = ab^x$$

$$y = 3.02 \times 2.0^x$$

$y = kx^n$

$\log y = \log kx^n$

$\log y = \log k + \log x^n$

$\log y = \log k + n\log x$

$\underbrace{\log y}_{y} = \underbrace{n}_{m}\underbrace{\log x}_{x} + \underbrace{\log k}_{c}$

$\log y$, $\log x$

$y = ab^x$

$\log y = \log ab^x$

$\log y = \log a + \log b^x$

$\log y = \log a + x\log b$

$\underbrace{\log y}_{y} = \underbrace{x}_{x}\underbrace{\log b}_{m} + \underbrace{\log a}_{c}$

$\log y$, x

$\log y = (\log b)(x) + \log a$

Ex 15J

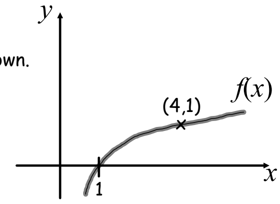
Graph Transformations

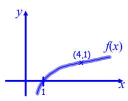
Learning Intention

By the end of this lesson, you should be able to use the techniques learned in Unit 1 (and some new ones!) to sketch a number of related graphs.

Examples

1) The graph of $f(x) = \log_4 x$ is shown.





From our Unit 3 knowledge,

$\log_4 4x = \log_4 4 + \log_4 x$ use law 1

$= 1 + \log_4 x$

when g(x) = 0

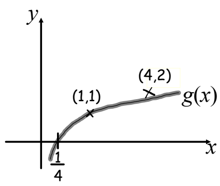
$0 = 1 + \log_4 x$

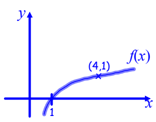
$\log_4 x = -1$

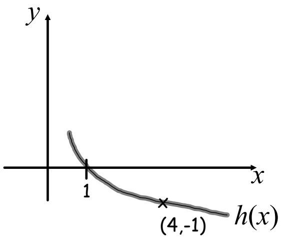
$x = 4^{-1}$

$x = \frac{1}{4}$

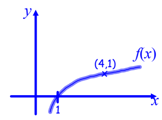
$\log_4 4x$ is the same as $\log_4 x$ but it has been moved up one place.





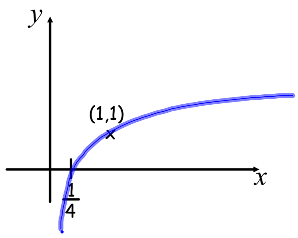


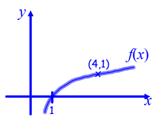
$f(x) = \log_4(x)$



a) Sketch the graph of $g(x) = \log_4(4x)$.

From Unit 1, each x coordinate will be divided by 4.





b) Sketch the graph of $h(x) = \log_4 \frac{1}{x}$.

This is not one of the rules we know from Unit 1.
We need to use our knowledge of logs.

$$\log_4 \frac{1}{x} = \log_4 x^{-1}$$

$$= -\log_4 x$$

$\log_4 \frac{1}{x}$ is the same as $\log_4 x$ but it has been flipped in the x axis.