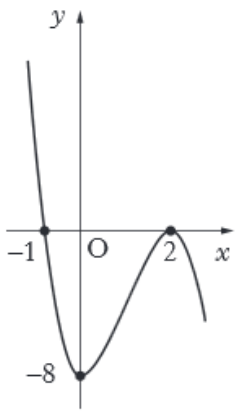
2014 Paper 1

15. The diagram shows a cubic curve passing through (–1, 0), (2, 0) and (0, –8).



What is the equation of the curve?

A $y = -2(x+1)^2(x+2)$

B $y = -2(x+1)(x-2)^2$

C $y = 4(x+1)(x-2)$

D $y = -8(x+1)(x-2)^2$

17. $3x^2 + 12x + 17$ is expressed in the form $3(x+p)^2 + q$.

What is the value of q?

A 1

B 5

C 17

D –19

22. For the polynomial $6x^3 + 7x^2 + ax + b$,

- $x + 1$ is a factor
- 72 is the remainder when it is divided by $x - 2$.

(*a*) Determine the values of a and b.

(*b*) Hence factorise the polynomial completely.

2014 Paper 2

3. Functions f and g are defined on suitable domains by

$$f(x) = x(x-1) + q \text{ and } g(x) = x + 3.$$

(a) Find an expression for $f(g(x))$.

(b) Hence, find the value of q such that the equation $f(g(x)) = 0$ has equal roots.

7. Land enclosed between a path and a railway line is being developed for housing. This land is represented by the shaded area shown in Diagram 1.

- The path is represented by a parabola with equation $y = 6x - x^2$.
- The railway is represented by a line with equation $y = 2x$.
- One square unit in the diagram represents 300 m^2 of land.

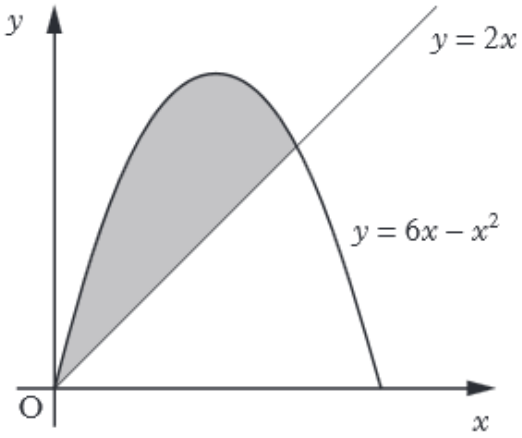


Diagram 1

(a) Calculate the area of land being developed.

(*b*) A road is built parallel to the railway line and is a tangent to the path as shown in Diagram 2.

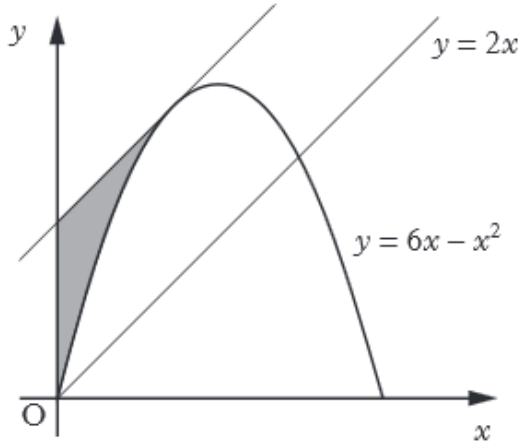


Diagram 2

It is decided that the land, represented by the shaded area in Diagram 2, will become a car park.

Calculate the area of the car park.

2013 Paper 1

3. Calculate the discriminant of the quadratic equation $2x^2 + 4x + 5 = 0$.

A –32

B –24

C 48

D 56

6. What is the remainder when $x^3 + 3x^2 – 5x – 6$ is divided by $(x – 2)$?

A 0

B 3

C 4

D 8

7. Find $\int x(3x + 2)\, dx$.

A $x^3 + c$

B $x^3 + x^2 + c$

C $\frac{1}{2}x^2\left(\frac{3}{2}x^2 + 2x\right) + c$

D $3x^2 + 2x + c$

19. Solve $1 - 2x - 3x^2 > 0$, where x is a real number.

A $x < -1$ or $x > \frac{1}{3}$

B $-1 < x < \frac{1}{3}$

C $x < -\frac{1}{3}$ or $x > 1$

D $-\frac{1}{3} < x < 1$

21. Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$.

2013 Paper 2

3. (*a*) Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully.

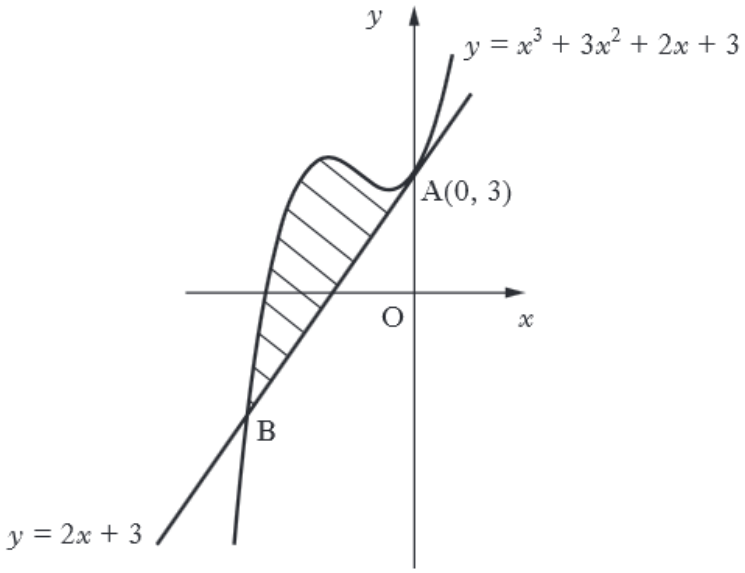
(*b*) Show that the curve with equation

$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

has only one stationary point.

Find the x-coordinate and determine the nature of this point.

4. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown in the diagram.



The line meets the curve again at B.

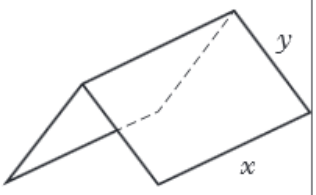
Show that B is the point (–3, –3) and find the area enclosed by the line and the curve.

7. A manufacturer is asked to design an open-ended shelter, as shown, subject to the following conditions.

Condition 1

The frame of a shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.



Condition 2

The frame is to be covered by a rectangular sheet of material.

The total area of the sheet is 24 m^2.

(*a*) Show that the total length, L metres, of the rods used in a shelter is given by

$$L = 3x + \frac{48}{x}.$$

(*b*) These rods cost £8·25 per metre.

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(i) Find the value of x for which L is a minimum.

(ii) Calculate the minimum cost of a frame.

2012 Paper 1

3. If $x^2 - 6x + 14$ is written in the form $(x - p)^2 + q$, what is the value of q?

A -22

B 5

C 14

D 50

11. Find $\int \left(\frac{1}{6x^2} \right) dx$, $x \neq 0$.

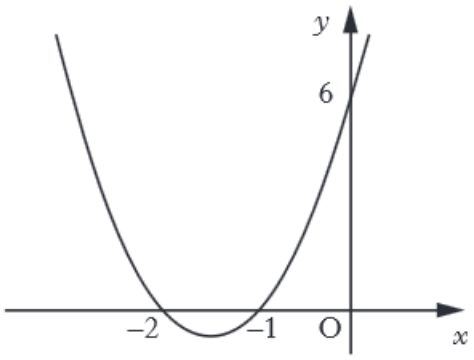
A $-12x^{-3} + c$

B $-6x^{-1} + c$

C $-\frac{1}{3}x^{-3} + c$

D $-\frac{1}{6}x^{-1} + c$

13. A parabola intersects the axes at $x = -2$, $x = -1$ and $y = 6$, as shown in the diagram.



What is the equation of the parabola?

A $y = 6(x - 1)(x - 2)$

B $y = 6(x + 1)(x + 2)$

C $y = 3(x - 1)(x - 2)$

D $y = 3(x + 1)(x + 2)$

19. Solve $6 - x - x^2 < 0$.

A $-3 < x < 2$

B $x < -3, x > 2$

C $-2 < x < 3$

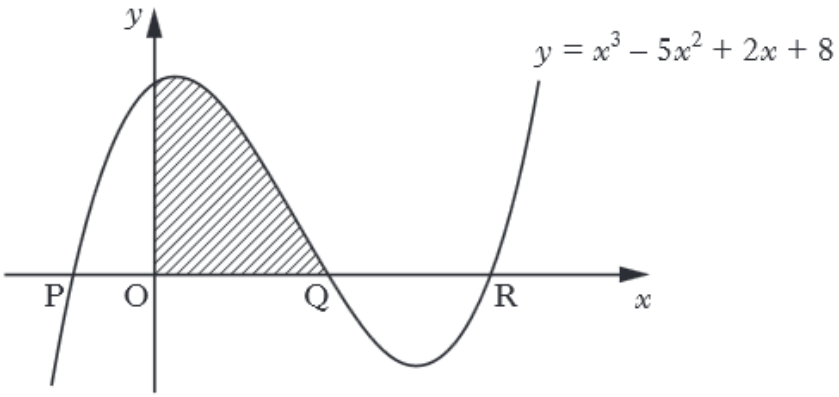
D $x < -2, x > 3$

21. (*a*) (i) Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

(ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

(iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$.

(*b*) The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$.



The curve crosses the x-axis at P, Q and R.

Determine the shaded area.

2012 Paper 2

1. Functions f and g are defined on the set of real numbers by

- $f(x) = x^2 + 3$
- $g(x) = x + 4$.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$.

(*b*) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots.

2011 Paper 1

5. If $x^2 - 8x + 7$ is written in the form $(x - p)^2 + q$, what is the value of q?

A –9

B –1

C 7

D 23

7. A function f is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.

What is the remainder when $f(x)$ is divided by $(x - 1)$?

A 0

B 2

C 3

D 4

9. The discriminant of a quadratic equation is 23.

Here are two statements about this quadratic equation:

(1) the roots are real;

(2) the roots are rational.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

11. Find $\int \left(4x^{\frac{1}{2}} + x^{-3} \right) dx$, where $x > 0$.

A $2x^{-\frac{1}{2}} - 3x^{-4} + c$

B $2x^{-\frac{1}{2}} - \frac{1}{2}x^{-2} + c$

C $\frac{8}{3}x^{\frac{3}{2}} - 3x^{-4} + c$

D $\frac{8}{3}x^{\frac{3}{2}} - \frac{1}{2}x^{-2} + c$

16. Find $\int \frac{1}{3x^4}\,dx$, where $x \neq 0$.

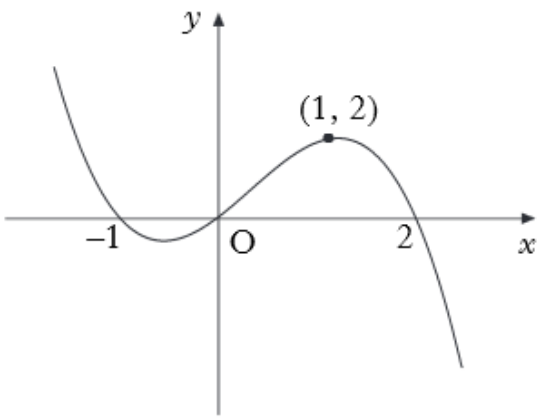
A $-\frac{1}{9x^3}+c$

B $-\frac{1}{x^3}+c$

C $\frac{1}{x^3}+c$

D $\frac{1}{12x^3}+c$

17. The diagram shows the graph of a cubic.



What is the equation of this cubic?

A $y = -x(x+1)(x-2)$

B $y = -x(x-1)(x+2)$

C $y = x(x+1)(x-2)$

D $y = x(x-1)(x+2)$

18. If $f(x) = (x - 3)(x + 5)$, for what values of x is the graph of $y = f(x)$ above the x-axis?

A $-5 < x < 3$

B $-3 < x < 5$

C $x < -5, x > 3$

D $x < -3, x > 5$

2011 Paper 2

2. Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$.

(*a*) Find $g(f(x))$.

(*b*) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$.

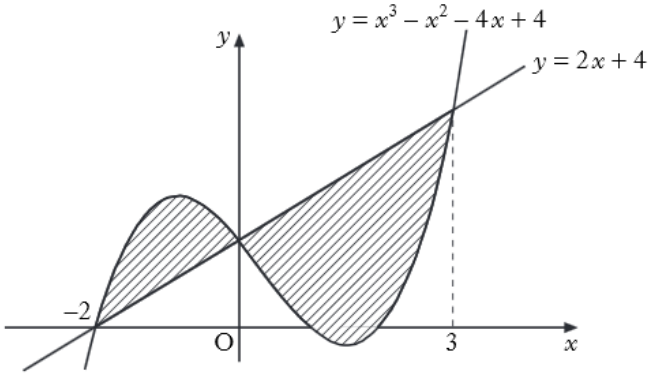
(*c*) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.

(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully.

(*d*) Hence solve $g(f(x)) + xh(x) = 0$.

4. The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$.

The curve and the line intersect at the points $(-2, 0)$, $(0, 4)$ and $(3, 10)$.



Calculate the total shaded area.

5. When $x^2 + 8x + 3$ is written in the form $(x + p)^2 + q$, what is the value of q?

A -19

B -13

C -5

D 19

6. The roots of the equation $kx^2 - 3x + 2 = 0$ are equal.

What is the value of k?

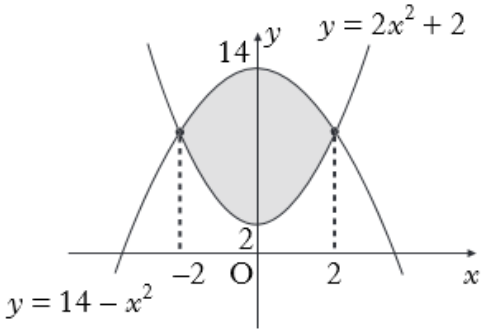
A $-\frac{9}{8}$

B $-\frac{8}{9}$

C $\frac{8}{9}$

D $\frac{9}{8}$

14. The diagram shows graphs with equations $y = 14 - x^2$ and $y = 2x^2 + 2$.



Which of the following represents the shaded area?

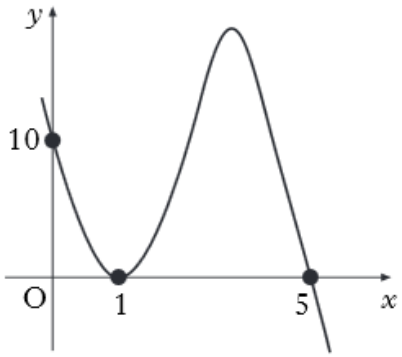
A $\int_{2}^{14} (12 - 3x^2)\,dx$

B $\int_{2}^{14} (3x^2 - 12)\,dx$

C $\int_{-2}^{2} (12 - 3x^2)\,dx$

D $\int_{-2}^{2} (3x^2 - 12)\,dx$

16. The diagram shows the graph with equation $y = k(x - 1)^2(x + t)$.



What are the values of k and t?

	k	t
A	–2	–5
B	–2	5
C	2	–5
D	2	5

18. What is the solution of $x^2 + 4x > 0$, where x is a real number?

A $-4 < x < 0$

B $x < -4, x > 0$

C $0 < x < 4$

D $x < 0, x > 4$

22. (*a*) (i) Show that $(x - 1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.

(ii) Hence factorise $f(x)$ fully.

(*b*) Solve $2x^3 + x^2 - 8x + 5 = 0$.

(*c*) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G.

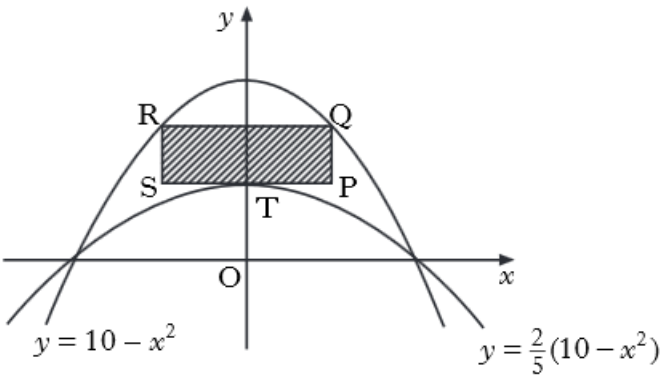
Find the coordinates of G.

(*d*) This tangent meets the curve again at the point H.

Write down the coordinates of H.

2010 Paper 2

5. The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.



A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola;
- RQ and SP are parallel to the x-axis;
- T, the turning point of the lower parabola, lies on SP.

(a) (i) If TP = x units, find an expression for the length of PQ.

(ii) Hence show that the area, A, of rectangle PQRS is given by

$$A(x) = 12x - 2x^3.$$

(b) Find the maximum area of this rectangle.