AH Stats Unit 2 NAB Revision : solutions for SI AS 1.1 (Sampling)
SI AS 1.2 (Confidence Intervals and Control Char

SI 1.1 Sampling

1. Let X = annual electric bill for 1 household

~~Assuming regularly selected~~ * CLT

$\bar{X} \sim N\left(120, \frac{25^2}{\cancel{10}\,40}\right)$ approximately

2. Let X = the weight of a randomly selected trout

$\bar{X} \sim N\left(1, \frac{0.25^2}{15}\right)$

3. Let X = weekly wage of a randomly selected worker

$\bar{X} \sim N\left(350, \frac{24^2}{35}\right)$ approximately

4. Let X = length of a randomly selected leaf

$\bar{X} \sim N\left(15, \frac{3.6^2}{60}\right)$

SI 1.2 : Confidence Intervals and Control Charts

a) $\bar{x} = \frac{\Sigma x}{n} = \frac{2226}{10} = 222.6\,g$ s.e. $= \frac{\sigma}{\sqrt{n}} = \frac{8.45}{\sqrt{10}} = 2.672\,g$

$X_i \sim N(\mu, \sigma^2)$, $\therefore$ $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

95% CI is $\left(\bar{X} - 1.96\frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96\frac{\sigma}{\sqrt{n}}\right) = (222.6 - 1.96 \times 2.672, 222.6 + 1.96 \times 2.672)$
$= (217.36, 227.84)$

b) 95% of such intervals will capture the true mean mass, so the true mean mass is very likely to lie between 217.36 g and 227.84 g

2. a) $\bar{x} = \frac{1000.9}{12} = 83.408$ Newtons (2) ~~(Assuming $\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$ approx for $n=12$)~~

(Assuming $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ approx. for $n = 12$)* CLT

~~Est X~~

~~$s.e. = \frac{\sigma}{\sqrt{n}} =$~~

Since σ is unknown we must use the t-dist and estimated standard error in constructing our confidence interval.

$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$ where $s^2 = \frac{1}{n-1}\left(\Sigma x_i^2 - \frac{1}{n}(\Sigma x_i)^2\right)$

$s^2 = \frac{1}{11}\left(83826.27 - \frac{1}{12}(1000.9)^2\right)$

$s^2 = 31.1699$

$s = 5.583$ Newtons

Required 99% CI is: $\bar{x} \pm t_{n-1, 1-\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$

$\Rightarrow \bar{x} \pm t_{11, 0.995}\left(\frac{5.583}{\sqrt{12}}\right)$

$= \overset{83.408}{\cancel{\bar{x}}} \pm 3.106\left(\frac{5.583}{\sqrt{12}}\right)$

$= 83.408 \pm 5.006$

$= (78.402, 88.414)$

b) I would advise the customer that it was very likely that the wire would snap carrying a mass of 89 Newtons as ~~99% of CIs~~ the upper limit of 99% of CIs for the mean breaking strength would be less than this mass.

3. a) Since $n=20$, CLT can be used: Let X = time spent in revision by a randomly selected candidate.

$\bar{x} = 7.2$ hrs, ~~$\sigma =$~~ $s = 1.5$ hrs $t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$: t-dist with $t_{19, 0.975} = 2.093$

Required 95% CI is: $\bar{x} \pm t_{n-1, 1-\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$

$= 7.2 \pm 2.093\left(\frac{1.5}{\sqrt{20}}\right) = 7.2 \pm 0.702$

$= (6.498, 7.902)$ ~~$= (6.5, 8$~~

so the true mean time is very likely to lie between 6.498 hrs and 7.902 hrs.

b) 95% of such CIs will capture the true mean time revising this topic;

$\cancel{4}$ 3

4. $X \sim N(\mu, 240^2)$

$\therefore \overline{X} \sim N\left(\mu, \frac{240^2}{10}\right)$ σ^2 known $\rightarrow$ Z-value is used

$$Z = \frac{\overline{X} - \mu}{240/\sqrt{10}}, \quad Z \sim N(0,1)$$

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{3913}{10} = 391.3 \text{ days}$$

$$s.e. = \frac{\sigma}{\sqrt{n}} = \frac{240}{\sqrt{10}} = 75.895 \text{ days}$$

90% CI is: $\left(\overline{X} - 1.64\frac{\sigma}{\sqrt{n}}, \ \overline{X} + 1.64\frac{\sigma}{\sqrt{n}}\right)$

$= (391.3 - 1.64 \times 75.895, \ 391.3 + 1.64 \times 75.895)$

$= (266.83$[redacted]$, \ 515.77)$

90% of such intervals will contain the true mean of X;

5. $\mu = 500g \quad \sigma = 58g$

Control Limits (3σ) : $\mu \pm \frac{3\sigma}{\sqrt{n}} = 500 \pm 3 \times \frac{58}{\sqrt{4}}$

$= 500 \pm 3 \times 29$

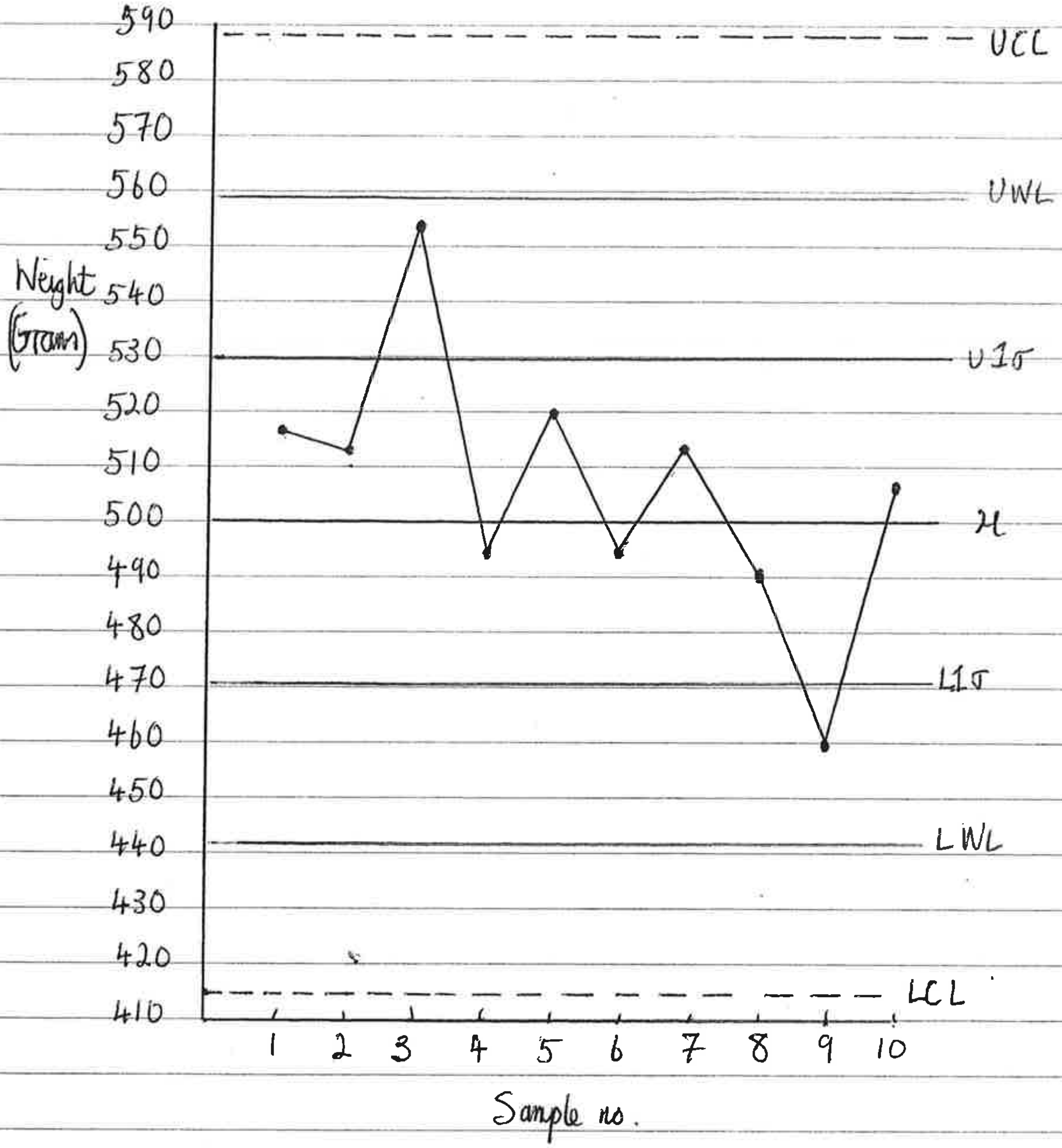
$= 500 \pm 87$

$= (413, 587)$

Warning Limits (2σ): $\mu \pm \frac{2\sigma}{\sqrt{n}} = 500 \pm 2 \times 29 = 500 \pm 58$

$= (442, 558)$

1σ limits: $\mu \pm \frac{\sigma}{\sqrt{n}} = 500 \pm 29 = (471, 529)$



The warning limits (let alone the control limits) are not breached so the process should be allowed to continue.

6. $p = 0.12$, $q = 0.88$

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\hat{p}$	0.11	0.16	0.12	0.12	0.15	0.08	0.04	0.11	0.10	0.11	0.13	0.14	0.16	0.15	0.23

CL: $p \pm 3\sqrt{\frac{pq}{n}} = 0.12 \pm 3\times\sqrt{\frac{0.12\times 0.88}{100}} = 0.12 \pm 3\times 0.0325$

$= 0.12 \pm 0.0975$

$= (0.0225, 0.2175)$

WL: $p \pm 2 \times 0.0325 = p \pm 0.065 = 0.12 \pm 0.065$

$= (0.055, 0.185)$

1σ: $p \pm 0.0325 = 0.12 \pm 0.0325 = (0.0875, 0.1525)$

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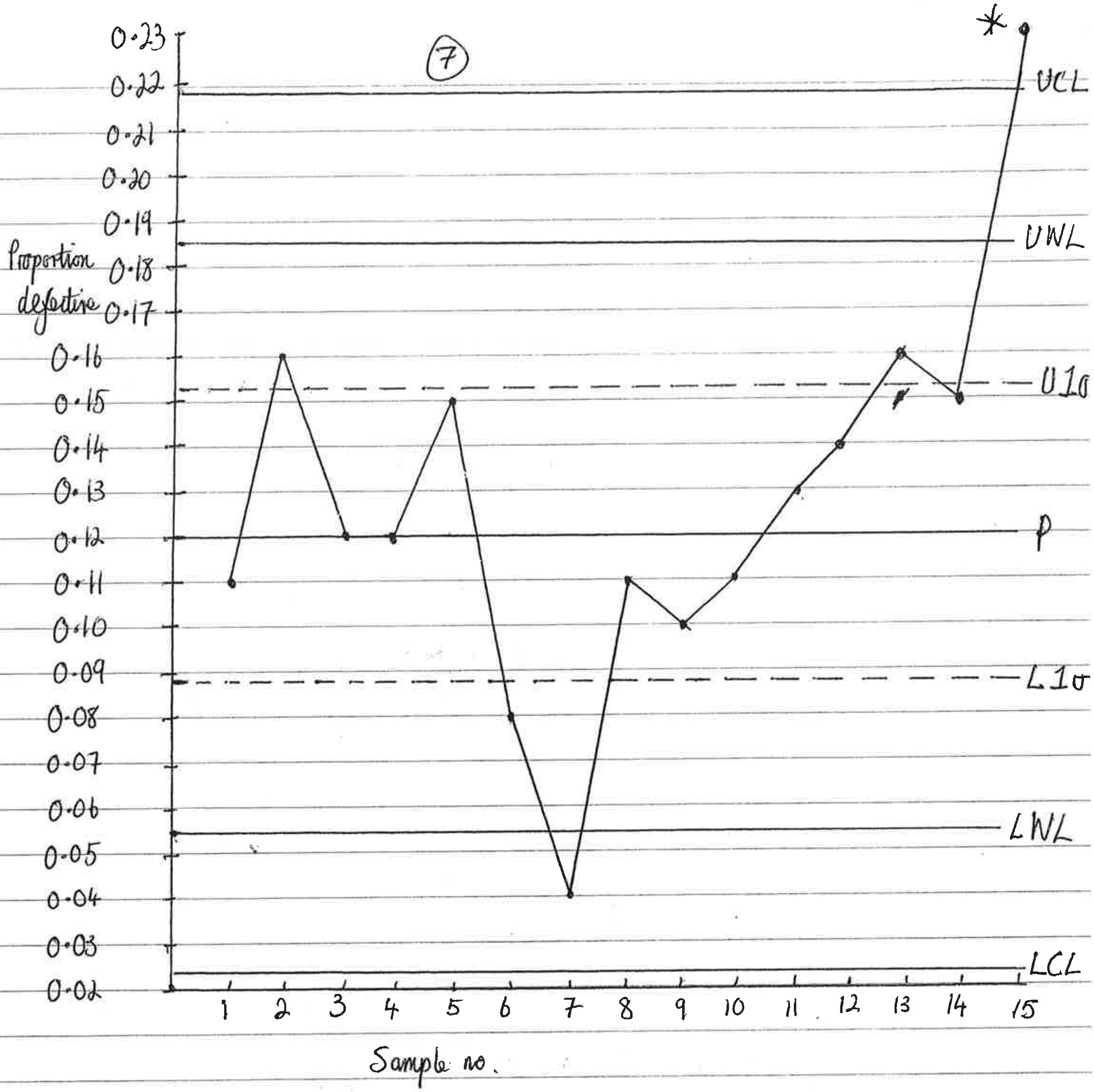

⇒ Process should be halted at sample 15 as it is outwith the Upper Control Limit.