Starter

Solve by first rearranging and then using the CAST diagram.

1) $3\tan(x) + 1 = 4.5$ for $0 < x < 360$

Trigonometric Identities

Today we are learning...

The key trigonometric identities and how to use them.

I will know if I have been successful if...

I can state some of the key identities.

I can substitute expressions involving the key identities.

I can simplify expressions using the key identities.

The Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\tan A = \frac{\sin A}{\cos A}$$

$\tan A = \frac{\sin A}{\cos A}$ Using the Identities $\sin^2 A + \cos^2 A = 1$

1) Prove $3\cos^2(x) + 3\sin^2(x) = 3$

$\tan A = \frac{\sin A}{\cos A}$ Using the Identities $\sin^2 A + \cos^2 A = 1$

2) Prove tan(x) cos(x) = sin(x)

$\tan A = \frac{\sin A}{\cos A}$ Using the Identities $\sin^2 A + \cos^2 A = 1$

3) Prove $8\cos^2(x) = 8 - 8\sin^2(x)$

$\tan A = \dfrac{\sin A}{\cos A}$

Using the Identities

$\sin^2 A + \cos^2 A = 1$

Prove the following trigonometric identities :-

(a) $5\cos^2 A + 5\sin^2 A = 5$

(b) $4\cos^2 A = 4 - 4\sin^2 A$

(c) $2\cos^2 A - 1 = 1 - 2\sin^2 A$

(d) $6\cos^2 A - 5 = 1 - 6\sin^2 A$

(e) $(\cos X + \sin X)^2 = 1 + 2\sin X \cos X$

(f) $(\cos P - \sin P)^2 + 2\sin P \cos P = 1$

(g) $(\cos X + \sin X)(\cos X - \sin X) = 2\cos^2 X - 1$

(h) $(\cos X - \sin X)(\cos X + \sin X) = 1 - 2\sin^2 X$

(i) $\tan P \cos P = \sin P$

(j) $\dfrac{1 - \cos^2 \alpha}{\cos^2 \alpha} = \tan^2 \alpha$

(k) $\dfrac{1 - \sin^2 \alpha}{\sin^2 \alpha} = \dfrac{1}{\tan^2 \alpha}$

(l) $\dfrac{\sin \beta}{\cos \beta} + \dfrac{\cos \beta}{\sin \beta} = \dfrac{1}{\cos \beta \sin \beta}$.

Starter

2018 Paper 2 National 5

Solve the equation $7\sin(x) + 2 = 3$ for $0 < x < 360$

(3 Marks)

Mark Scheme

Question	Generic scheme	Illustrative scheme	Max mark
8.	• 1 rearrange equation • 2 calculate value of x • 3 calculate 2nd value of x	• 1 $\sin x = \frac{1}{7}$ • 2 $8 \cdot 2(1\ldots)$ • 3 $171 \cdot 8$ or $171 \cdot 7(8\ldots)$	3

Notes:

1. Correct answers without working award 1/3 ××✓
2. Accept 8 and 172 with valid working
3. Degree signs are not required
4. Premature rounding: rounded working must be to at least 2 decimal places
   eg (a) $\sin x = \frac{1}{7} = 0 \cdot 14 \rightarrow x = 8(\cdot 04\ldots), 172$ or $171 \cdot (95\ldots)$ award 3/3
   (b) $\sin x = \frac{1}{7} = 0 \cdot 1 \rightarrow x = 6$ or $5(\cdot 73\ldots), 174 \cdot (26\ldots)$ award 2/3 ✓×✓
5. Inappropriate use of RAD or GRAD should only be penalised once in Q8, Q9, Q13 or Q17
   (a) 0·143… ,179·856… (RAD)
   (b) 9·125…. ,170·874… (GRAD)

Trig Exam Questions

Today we are learning...

How to solve an exam question on the CAST Diagram and identities.

I will know if I have been successful if...

I can pick out the key pieces of information.

I can set out my working logically.

I can mark my work against the marking scheme.

2016 Question 14 Paper 2

14. Solve the equation $2\tan x° + 5 = -4$, for $0 \leq x \leq 360$.

(3 Marks)

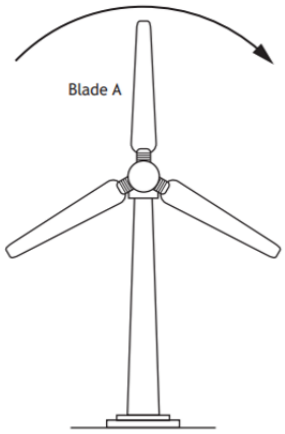
2016 Question 14 Paper 2

Question			Generic Scheme	Illustrative Scheme	Max Mark
14.			Ans: x = 102·5, 282·5		3
			• [1] rearrange equation	• [1] $\tan x = -\frac{9}{2}$	
			• [2] find one value of x	• [2] x = 102·5	
			• [3] find another value of x	• [3] x = 282·5	

Notes:
1. Correct answer without working award 2/3
2. For x = 178·6, 358·6 (uses RAD), award 3/3 (with working), 2/3 (without working)
3. For x = 93·9, 273·9 (uses GRAD), award 3/3 (with working), 2/3 (without working)

2017 Question 15 Paper 2

15. A wind turbine has three blades as shown below.



The height, h metres, of the tip of blade A above the ground in each rotation is given by

$$h = 40 + 23\cos x°, \qquad 0 \leq x < 360$$

where x is the angle blade A has turned clockwise from its vertical position.

(a) Calculate the height of the tip of blade A after it has turned through an angle of 60°.

(b) Find the minimum height of the tip of blade A above the ground.

(c) Calculate the values of x for which the tip of blade A is 61 metres above the ground.

2017 Question 15 Paper 2

The height, h metres, of the tip of blade A above the ground in each rotation is given by

$$h = 40 + 23\cos x°, \qquad 0 \leq x < 360$$

where x is the angle blade A has turned clockwise from its vertical position.

(a) Calculate the height of the tip of blade A after it has turned through an angle of 60°. (1 Mark)

(b) Find the minimum height of the tip of blade A above the ground. (1 Mark)

(c) Calculate the values of x for which the tip of blade A is 61 metres above the ground. (4 Marks)

2017 Question 15 Paper 2

Question			Generic scheme	Illustrative scheme	Max mark
15.	(a)		Ans: 51·5 metres •[1] calculate height	•[1] 51·5	1

2017 Question 15 Paper 2

	(b)		Ans: 17 metres •[1] calculate minimum height	•[1] 17	1

2017 Question 15 Paper 2

	(c)		Ans: 24·1° and 335·9°		4
			•1 substitute 61 correctly into equation	•1 $61 = 40 + 23\cos x$	
			•2 calculate $\cos x$	•2 $\cos x = \frac{21}{23}$	
			•3 calculate value of x	•3 $24(\cdot 07\ldots)$	
			•4 calculate 2nd value of x	•4 $335(\cdot 92\ldots)$	

2018 Question 12 Paper 1

12. Given that $\cos 60° = 0{\cdot}5$, state the value of $\cos 240°$.

(1 Mark)

2018 P1 Q18

18. Express $\sin x° \cos x° \tan x°$ in its simplest form.
Show your working.

(2 Marks)

Question			Generic scheme	Illustrative scheme	Max mark
18.			•1 correct substitution for $\tan x$	•1 $\sin x \cos x \frac{\sin x}{\cos x}$	2
			•2 express in simplest form	•2 $\sin^2 x$	

Spot the Mistakes

Solve simultaneously $4a + 6b = 38$

$3a - 3b = 6$

What should the correct answer be?

Answer

Simultaneous Equations

Today we are learning...

How to quickly and effectively solve simultaneous equations.

I will know if I have been successful if...

I can confidently solve simultaneous equations.

I can choose the best method (Elimination or Substitution)

I check my answers to make sure I am correct.

2016 P1 Q4

4. Charlie is making costumes for a school show.
One day he made 2 cloaks and 3 dresses.
The total amount of material he used was 9·6 square metres.

(a) Write down an equation to illustrate this information.

(1 Mark)

(b) The following day Charlie made 3 cloaks and 4 dresses.
The total amount of material he used was 13·3 square metres.
Write down an equation to illustrate this information.

(1 Mark)

(c) Calculate the amount of material required to make one cloak and the amount of material required to make one dress.

(4 Marks)

2018

2018 P1 Q3

3. Solve, algebraically, the system of equations

$$4x+5y=-3$$
$$6x-2y=5.$$

(3 Marks)

Question	Generic scheme	Illustrative scheme	Max mark
3.	•1 evidence of scaling (match x or y coefficients)	•1 eg $8x+10y=-6$, $30x-10y=25$	3
	•2 follow a valid strategy through to produce values for x and y	•2 values for x and y	
	•3 calculate correct values for x and y	•3 $x=0\cdot5,\ y=-1$	

Notes:

1. Correct answer without working — award 0/3
2. Answer obtained by guess and check — award 0/3