2010 Applied Maths

Advanced Higher – Statistics

Finalised Marking Instructions

General Marking Principles

These principles describe the approach taken when marking Advanced Higher Applied Mathematics papers. For more detailed guidance please refer to the detailed Marking Instructions.

1 The main principle is to give credit for the skills demonstrated and the criteria met. Failure to have a correct method may not preclude a candidate gaining credit for their solution.

2 The answer to one part of a question, even if incorrect, can be accepted as a basis for subsequent dependent parts of the question.

3 The following are not penalised:

- working subsequent to a correct answer (unless it provides firm evidence that the requirements of the question have not been met)
- legitimate variation in numerical values / algebraic expressions.

4 Full credit will only be given where the solution contains appropriate working. Where the correct answer might be obtained by inspection or mentally, credit may be given.

5 Sometimes the method to be used in a particular question is explicitly stated; no credit will be given where a candidate obtains the correct answer by an alternative method.

6 Where the method to be used in a particular question is not explicitly stated in the question paper, full credit is available for an alternative valid method. (Some likely alternatives are included but these should not be assumed to be the only acceptable ones.)

In the detailed Marking Instructions which follow, marks are shown alongside the line for which they are awarded. There are two codes used M and E. The code M indicates a method mark, so in question B1(a), 1M means a method mark for the product rule. The code E refers to ‘error’, so in question B1(b), up to 2 marks can be awarded but 1 mark is lost for each error.

2010 Statistics: Solutions

A1. (a) The reference range is $\mu \pm 1{\cdot}96\sigma$. **1**

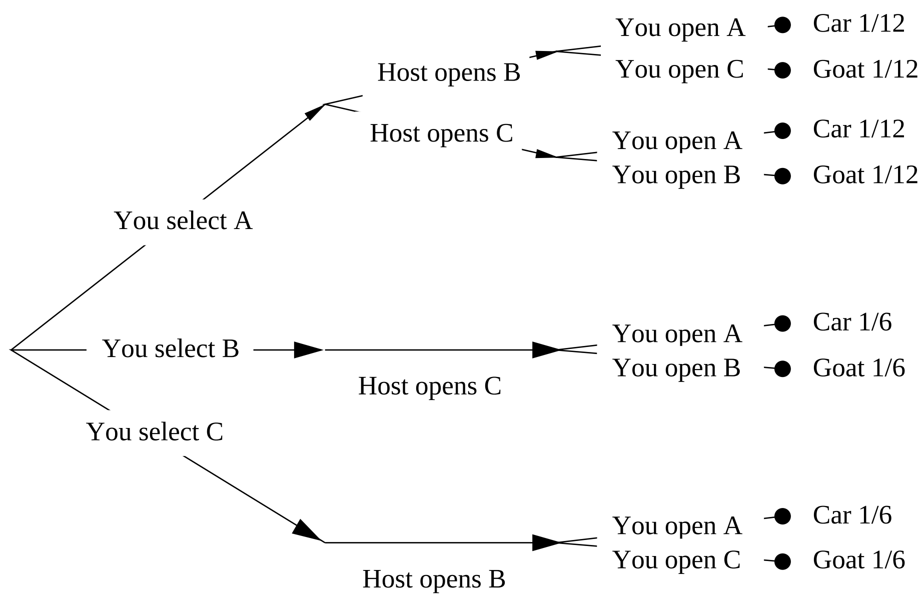
This yields (35·4, 37·0) as the required range. **1**

(b) Mean $= 1{\cdot}8 \times 36{\cdot}2 + 32 \approx 97{\cdot}2$. **1**

Standard deviation $= 1{\cdot}8 \times 0{\cdot}4$ {NB excludes + 32} **1**

$= 0{\cdot}72$. **1**

A2. (a)



Completion of tree **1,1**

Calculation of probabilities **1,1**

(b)

$$P(Car \mid S) = \frac{P(S \cap Car)}{P(S)} = \frac{2/6}{1/2} = \frac{2}{3}$$ **1,1**

Contestants wishing to win a car should switch. **1**

A3. (a) Not every citizen will own a telephone so some citizens will not be represented thus potentially leading to bias. **1**

(b) $y = \hat{p}(1 - \hat{p}) \Rightarrow y' = 1 - 2\hat{p} = 0$ when $\hat{p} = 0{\cdot}5$. **1,1**

$y'' = -2 < 0$ so the stationary point is a maximum with $y_{\max} = 0{\cdot}25$. **1**

With $n = 1000$ the maximum margin of error will be

$1{\cdot}96\sqrt{\frac{0{\cdot}25}{1000}} \approx 0{\cdot}031$ i.e. just over 3%. **1,1**

A4. The observed and expected frequencies are:

	Left-handed	*Right-handed*
Observed	57	343
Expected	40	360

1

$H_0 : p = 0{\cdot}1 \quad H_1 : p \neq 0{\cdot}1$ **1**

$X^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

$x^2 = \frac{(57 - 40)^2}{40} + \frac{(343 - 360)^2}{360} \approx 8{\cdot}028$ **1**

The critical value of chi-squared (1df, 1% level) is $6{\cdot}635 < 8{\cdot}028$ **1**

so the null hypothesis is rejected, at the 1% level, **1**

yielding strong evidence that the proportions of left and right-handed injured nurses are not in the ratio 1:9 . **1**

It would be worth investigating whether or not the hypodermic equipment was designed for right-handed users. **1**

A5. $P(X > Y) = P(X - Y > 0)$ is required. **1**

Assuming that X and Y are independent **1**

$X - Y \sim N(-3{\cdot}0,\ 2{\cdot}5^2)$ **1,1**

$P(X - Y > 0) = P(Z > 1{\cdot}2)$ **1**

$\approx 0{\cdot}1151$ **1**

A6. $SSR = S_{yy} - \frac{(S_{xy})^2}{S_{xx}}$

$\approx 1943{\cdot}76 - \frac{860{\cdot}707^2}{546{\cdot}385} = 587{\cdot}909$ **1**

$s^2 = \frac{SSR}{n - 2} \approx \frac{587{\cdot}909}{18} \approx 5{\cdot}715^2$ **1**

$x = 20 \Rightarrow \hat{Y} \approx 2{\cdot}840 + 1{\cdot}575 \times 20 \approx 34{\cdot}34$ **1**

A 95% confidence interval for $E[Y \mid x = 20]$ is

$\hat{Y} \pm t_{18,\,0{\cdot}975}\, s \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{S_{xx}}}$ **1**

$\approx 34{\cdot}34 \pm 2{\cdot}101 \times 5{\cdot}715 \times \sqrt{\frac{1}{20} + \frac{(20 - 15{\cdot}915)^2}{546{\cdot}385}}$ **1**

$\approx 34{\cdot}34 \pm 3{\cdot}41$

$= (30{\cdot}93,\ 37{\cdot}75)$ **1**

An assumption required is that $\varepsilon_i \sim N(0, \sigma^2)$ in the linear model. **1**

A7. (a) Assuming that waiting times are normally distributed: **1**

$H_0 : \mu = 41$ and $H_1 : \mu < 41$ **1**

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{35{\cdot}2 - 41}{7{\cdot}941/\sqrt{10}} = -2{\cdot}31.$$ **1,1**

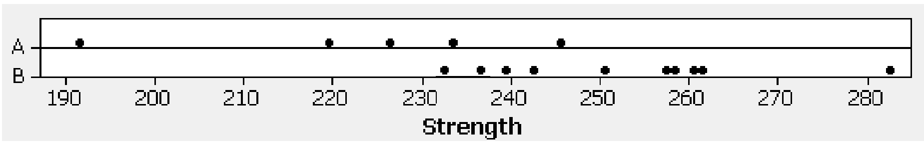
Since $-2{\cdot}31 < -1{\cdot}833$, H_0 is rejected at the 5% level, **1**

furnishing evidence of a reduction in the mean waiting time. **1**

(b) $$\mu \pm 3\frac{\sigma}{\sqrt{n}} = 36 \pm 3 \times \frac{9}{5} = 30{\cdot}6,\ 41{\cdot}4$$ **1,1**

(c) The sample size is large enough for the Central Limit Theorem to apply.
A point plotting below the lower chart limit could signal
a reduction in the mean waiting time. **2E1**

A8. (a)



Choice of display **1**

Labels/ Scales **1**

Appropriate comment on location **1**

(b)

Strength	191	219	226	232	233	236	239	242	245	250	257	258	260	261	283
Formulation	A	A	A	B	A	B	B	B	A	B	B	B	B	B	B
Rank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

H_0 : Median A = Median B H_1 : Median A < Median B **1**

Rank sum for Formulation A is $W = 20$ **1**

$$\mathrm{E}(W) = \frac{1}{2}n(n + m + 1) = \frac{1}{2} \times 5 \times 16 = 40$$

$$\mathrm{V}(W) = \frac{1}{12}nm(n + m + 1) = \frac{1}{12} \times 50 \times 16 \approx 8{\cdot}16^2$$ **1**

$$\mathrm{P}(W \leqslant 20{\cdot}5) = \mathrm{P}\left(Z \leqslant \frac{20{\cdot}5 - 40}{8{\cdot}16}\right) = \mathrm{P}(Z \leqslant -2{\cdot}39)$$ **1**

$$= 1 - 0{\cdot}9916 = 0{\cdot}0084$$ **1**

Since $0{\cdot}0084 < 0{\cdot}01$ we reject H_0 at the 1% level, **1**

indicating that the data provide evidence of greater median
tensile strength for Formulation B. **1**

A9 (a) $F \sim \text{Bin}(300,\ 0{\cdot}002)$ **1**

(b) $P(F \geqslant 4) \approx P(T \geqslant 4)$ where $T \sim \text{Poi}(0{\cdot}6)$ **1**

$= 1 - P(T \leqslant 3)$ **1**

$\approx 1 - \left\{e^{-0\cdot6} + e^{-0\cdot6}0{\cdot}6 + e^{-0\cdot6}\frac{0{\cdot}6^2}{2} + e^{-0\cdot6}\frac{0{\cdot}6^3}{6}\right\}$ **1**

$= 1 - e^{-0\cdot6}\left[1 + 0{\cdot}6 + 0{\cdot}18 + 0{\cdot}036\right]$ **1**

$= 1 - 1{\cdot}816e^{-0\cdot6}$ **1**

(c)

Compensation paid x	Probability $f(x)$
800	0·54881
600	0·32929
400	0·09879
200	0·01976
0	0·00336

1

1

$E(X) = \sum xf(x) \approx 439{\cdot}048 + 197{\cdot}574 + 39{\cdot}516 + 3{\cdot}952 + 0$

$\approx 680{\cdot}09$ **1**

(d) In reality some passengers will travel in family groups so if one member was ill it is unlikely that other members would turn up. **1**

[END OF SECTION A]

Section B

B1. (a) $f(x) = e^{2x}\tan x \quad \Rightarrow \quad f'(x) = 2e^{2x}\tan x + e^{2x}\sec^2 x$ **1M,2E1**

(b) $g(x) = \dfrac{\cos 2x}{x^3} \quad \Rightarrow$

$$g'(x) = \frac{(-2\sin 2x)x^3 - (\cos 2x)(3x^2)}{x^6}$$ **1M,2E1**

$$= \frac{-2x\sin 2x - 3\cos 2x}{x^4}$$ **1**

The Product Rule correctly applied and executed would gain full marks.

B2. The term in a^6 must come from $\dfrac{1}{a^2} \times a^8$. Thus the term is

$$\binom{10}{2} \times \left(\frac{1}{a}\right)^2 \times (3a)^8$$ **1,1**

$$= 45 \times 6561 \times a^6$$ **1**

$$= 295245\,a^6$$ **1**

B3.

$$\frac{3x}{(x+1)^2} = \frac{A}{(x+1)^2} + \frac{B}{(x+1)}$$ **1M**

$$3x = A + B(x+1)$$

$$x = -1 \quad \Rightarrow \quad -3 = A$$ **1**

$$x = 0 \quad \Rightarrow \quad 0 = -3 + B \Rightarrow B = 3$$ **1**

$$\frac{3x}{(x+1)^2} = \frac{-3}{(x+1)^2} + \frac{3}{(x+1)}$$

$$\int \frac{3x}{(x+1)^2}\,dx = \int \frac{-3}{(x+1)^2} + \frac{3}{(x+1)}dx$$

$$= \frac{3}{x+1} + 3\ln(x+1) + c$$ **1,1**

B4.

$$\int y.dy = \int 9te^{3t}.dt$$ **1M,1**

$$\frac{y^2}{2} = 3t\int 3e^{3t}.dt - \int 3e^{3t}.dt$$ **1M,1**

$$= 3te^{3t} - e^{3t} + c$$ **1**

$$(0,2) \Rightarrow 2 = 0 - 1 + c \Rightarrow c = 3$$ **1**

$$y^2 = 2\left(3te^{3t} - e^{3t} + 3\right)$$

$$y = \sqrt{6te^{3t} - 2e^{3t} + 6}$$ **1**

B5. (a)

$$\det A = m(m-0) - 1(0+2) + 1(0-m) \qquad \mathbf{1}$$

$$= m^2 - m - 2 \qquad \mathbf{1}$$

$$= (m+1)(m-2) \Rightarrow m = -1,\ 2 \qquad \mathbf{1}$$

(b)

$$\left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -3 & 0 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right) \qquad \mathbf{1M}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 1 & 1 \end{array}\right) (\mathbf{1}) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 1 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -3 & -2 \\ 0 & 1 & 0 & -1 & 2 & 1 \\ 0 & 0 & -1 & -1 & 1 & 1 \end{array}\right) (\mathbf{1}) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -3 & -2 \\ 0 & 1 & 0 & -1 & 2 & 1 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{array}\right)$$

$$\text{i.e. } B^{-1} = \begin{pmatrix} 3 & -3 & -2 \\ -1 & 2 & 1 \\ 1 & -1 & -1 \end{pmatrix} \qquad \mathbf{1}$$

(c)

$$\begin{pmatrix} 3 & -3 & -2 \\ -1 & 2 & 1 \\ 1 & -1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 & -3 & -2 \\ -1 & 2 & 1 \\ 1 & -1 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} \qquad \mathbf{1}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \qquad \mathbf{1}$$

[END OF SECTION B SOLUTIONS]