X202/701

NATIONAL QUALIFICATIONS 2006

MONDAY, 22 MAY
1.00 PM – 4.00 PM

APPLIED MATHEMATICS
ADVANCED HIGHER
Statistics

Read carefully

1. Calculators may be used in this paper.
2. Candidates should answer all questions.

   Section A assesses the Units Statistics 1 and 2
   Section B assesses the Unit Mathematics for Applied Mathematics
3. **Full credit will be given only where the solution contains appropriate working.**
4. A booklet of Statistical Formulae and Tables is supplied for all candidates.

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Section A (Statistics 1 and 2)

Marks

Answer all the questions.

A1. An ultrasonic scanner is used to check concrete beams for voids. The scanner is designed to give a signal if a beam contains a void. The following probabilities were determined for beams produced by a particular manufacturer.

		Scanner Response	
		Signal	*No Signal*
Beam Status	*Contains a void*	0·04	0·02
	Void free	0·01	0·93

The table indicates, for example, that P(Void free *and* Signal) = 0·01.

(*a*) Show that Scanner Response and Beam Status are not independent. **2**

(*b*) For a beam selected at random which gives a signal on being scanned, find the probability that it is void free. **2**

(*c*) For a beam selected at random which contains a void, write down the probability that it gives no signal. **1**

A2. Distinguish between stratified and cluster sampling. **2**

Give an example where each could be used effectively. **2**

A3. A remote restaurant receives a delivery of wine every six months. The manager finds that the demand for vintage Gollinber champagne averages 0·3 bottles per month. Assuming that monthly demands for the champagne are independent, calculate the number of bottles of the champagne the manager should endeavour to have in stock at the beginning of a six-month period in order to ensure a probability of no greater than 0·1 that demand for the champagne exceeds supply. **5**

A4. During a particular week, 20% of a large population of companies experienced a drop in their share prices. An investment portfolio includes shares in 40 of the companies. Stating any assumptions required, estimate the probability that at most five of the shares included in the portfolio would drop in price. **6**

Marks

A5. Many statistical software packages provide facilities for generating samples from distributions. In order to simulate 1200 rolls of a cubical die, *MINITAB* was used to generate a sample of 1200 values from the discrete uniform distribution on the integers 1 to 6 inclusive. The data obtained are summarised below.

Value	1	2	3	4	5	6
Frequency	185	191	200	208	209	207

Carry out a goodness-of-fit test and comment. **5**

A6. The government of a country considered withdrawing federal highway funds from those states failing to comply with the speed limit of 55 mph imposed on interstate highways. Compliance was considered to be breached if more than 60% of drivers exceeded the limit. In one particular state a survey of driving speeds at different times and locations determined that 3127 out of 5000 drivers observed on the interstate highways were exceeding the speed limit.

(*a*) Assuming the sample of driving speeds to be random, obtain an approximate 99% confidence interval for the proportion of drivers who exceeded the speed limit. **4**

(*b*) Comment on the state governor's claim that her state satisfied the compliance requirement. **2**

A7. "Door-to-needle" time for thrombolytic therapy for patients admitted to hospital with acute heart attacks is considered to be critical in terms of outcome. Records for a period of many months for one hospital revealed a mean "door-to-needle" time of 50 minutes with standard deviation 12 minutes. Following a clinical improvement project at the hospital the times (minutes) for a sample of acute heart-attack patients were as follows:

48 57 43 36 14 30 46 51 40 55.

(*a*) Stating any assumptions required, conduct an appropriate hypothesis test, using a critical region, to investigate whether "door-to-needle" times have improved. **7**

(*b*) Calculate the p-value of the test and explain how if confirms your earlier conclusion. **2**

(*c*) Give an example of other data you consider it would be of value to collect and analyse, in order to evaluate the effectiveness of the project. **1**

[Turn over

Marks

A8. The process capability index is defined as $C_p = \frac{USL - LSL}{6\sigma}$ where σ is the process standard deviation and USL and LSL are the upper and lower specification limits respectively. A process for the production of automotive components yields components with length which is normally distributed. The specification limits are 9·95 mm and 10·05 mm.

(*a*) Given that the process mean is centred between the specification limits and that $C_p = 1$, calculate the proportion, in parts per million (PPM), of components which fail to meet the specifications. **6**

(*b*) Given that the process mean is centred between the specification limits and that the proportion of components which fail to meet specifications is 1200 PPM, determine the process standard deviation. **4**

A9. A commuter made a number of car journeys, on weekday mornings, from Loanhead to Dunfermline. Journey duration (minutes) and weather conditions (D = Dry and W = Wet) for 14 randomly chosen days are given in the table below.

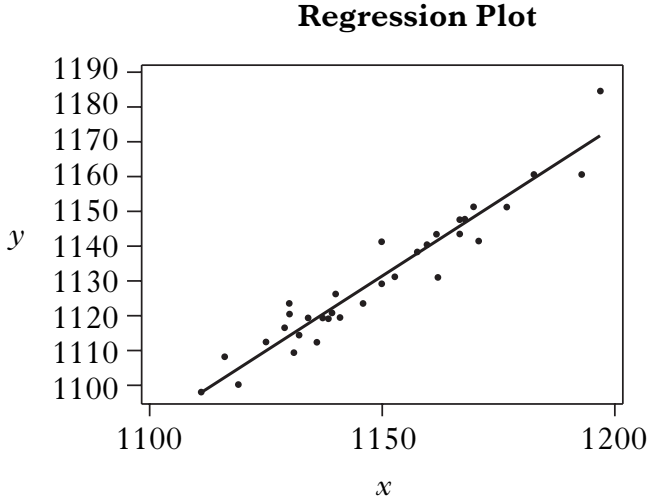
Duration	47	45	51	50	55	61	61	56	57	54	59	60	54	52
Weather	D	W	W	D	D	W	W	W	D	W	D	W	W	D

(*a*) Display the data appropriately. **2**

(*b*) He believed that journeys on wet days would take longer, on average, than journeys on dry days. By performing an appropriate hypothesis test, determine whether or not his belief is supported by the data. **6**

Marks

A10. In the manufacture of glass bottles, gobs of molten glass are poured from the furnace into the moulds in which the containers are formed, by the action of compressed air. The Gob Temperature is of major importance and the manufacturer was interested in being able to predict Gob Temperature from the temperature obtained from a sensor located in the fore-hearth of the furnace. An experiment was conducted from which a series of 35 values of Gob Temperature (y °) and Fore-Hearth Temperature (x °) were obtained. A scatter plot of these data are shown below together with the least squares regression line of y on x.



Given that the equation of the fitted line is $y = 152{\cdot}193 + 0{\cdot}851x$ and that, in the usual notation, $s = 4{\cdot}895$ and $S_{xx} = 15684{\cdot}4$:

(*a*) show that the null hypothesis that the slope is zero would be rejected at the 0·1% level of significance; **3**

(*b*) obtain a 99% prediction interval for Gob Temperature when Fore-Hearth Temperature is 1165 °, given that mean Fore-Hearth Temperature for the experimental results was 1149·6 °; **4**

(*c*) state, with justification, whether or not you would advise the furnace supervisor to pour glass for a container requiring a target Gob Temperature of 1150 ° when Fore-Hearth Temperature is displayed as 1165 ° on the control panel; **1**

(*d*) state, with justification, the value of Fore-Hearth Temperature which would yield the narrowest prediction interval for Gob Temperature. **1**

[*END OF SECTION A*]

[Turn over for Section B on *Page six*

Section B (Mathematics for Applied Mathematics)

Marks

Answer all the questions.

B1. Calculate A^{-1} where $A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \\ 2 & 2 & 1 \end{pmatrix}$.

Hence solve the system of equations

$$\begin{aligned} x + y &= 1 \\ 2x + 3y + z &= 2 \\ 2x + 2y + z &= 1. \end{aligned}$$

5

B2. Given that $y = \ln(1 + \sin x)$, where $0 < x < \frac{\pi}{2}$, show that $\frac{d^2y}{dx^2} = \frac{-1}{1+\sin x}$. **5**

B3. Define $S_n = \sum_{r=1}^{n} r^2$, $n \geq 1$. Write down formulae for S_n and S_{2n+1}. **2**

Obtain a formula for $2^2 + 4^2 + \ldots + (2n)^2$. **1**

B4. Solve the differential equation

$$\cos^2 x \frac{dy}{dx} = y,$$

given that $y > 0$ and that $y = 2$ when $x = 0$. **5**

B5. Use the substitution $1 + x^2 = u$ to obtain $\int \frac{x^3}{\sqrt{1+x^2}} dx$. **5**

B6. (*a*) Evaluate $\int_0^1 xe^{2x} dx$. **4**

(*b*) Use part (*a*) to evaluate $\int_0^1 x^2 e^{2x} dx$. **3**

(*c*) Hence obtain $\int_0^1 (3x^2 + 2x)e^{2x} dx$. **2**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

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