AH

Boroughmuir High School

Mathematics Department

Advanced Higher Mechanics
Prelim Paper

Thursday 28 January 2016

2 hours

1.30 p.m. – 3.30 p.m.

Total marks — 69

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

Round to 3 significant figures, or 1 decimal place for angles

State the units for your answer where appropriate.

FORMULAE LIST

Newton's inverse square law of gravitation

$$F = \frac{GMm}{r^2}$$

Simple harmonic motion

$$v^2 = \omega^2(a^2 - x^2)$$

$$x = a\sin(\omega t + \alpha)$$

Centre of mass

Triangle: $\frac{2}{3}$ along median from vertex.

Semicircle: $\frac{4r}{3\pi}$ along the axis of symmetry from the diameter.

Standard derivatives	
$f(x)$	$f'(x)$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Answer all the questions

Candidates should observe that g ms^{-2} denotes the magnitude of the acceleration due to gravity.

Where appropriate, take its magnitude to be 9.8ms^{-2}

1. A body accelerates uniformly from 5 ms^{-1} and after 100 metres it has a velocity of 20 ms^{-1}.

   a) What is the velocity after travelling another 100 metres? **3**

   b) How long will it take to cover a total distance of 1 kilometre? **2**

2. A plane is flying due East. It can fly at 100ms^{-1} in still air, but experiences a wind of 20 ms^{-1} blowing from the South

   a) Find the resultant speed of the plane. **2**

   b) The wind starts to shift direction so that it blows from a more westerly direction. At what bearing will it start to increase the plane's resultant speed above 100ms^{-1}? **5**

3. Express $\frac{2x-3}{x^2-5x+6}$ in partial fractions. **3**

4. A machine which fires tennis balls can reach a maximum height of 80 metres when fired straight up into the air.

   a) Show that $R=\frac{v^2\sin 2\theta}{g}$ where R is the horizontal range, v is the initial velocity, θ is the angle of projection and g is the acceleration due to gravity. **3**

   b) Find the maximum horizontal range of the machine. **3**

   c) List three assumptions you have made in doing this calculation. **1**

5. a) Differentiate $y=7xe^{\tan 3x}$ **3**

   b) Use implicit differentiation to find the gradient of the curve $x^2+y^2=25$ when $x=3$ **3**

6. Find the maximum speed of a 50 000kg train pulled by an 80kW engine against a constant resistance of 3000N along a horizontal track. **2**

   The train leaves a station at sea-level at 10am. At what time would it reach a mountain station at an altitude of 1500m if the constant gradient of the track is such that $\sin\theta=\frac{1}{60}$? Assume that the engine works at full power all of the time, and ignore the time taken for the train to accelerate to full speed. **4**

7. Use the substitution $u = 5 - x^2$ to evaluate $\int_0^2 \frac{x}{5-x^2}\,dx$ **5**

8. A ball of mass 2kg travelling at $5\mathbf{i}\,\text{ms}^{-1}$ hits a stationary ball of mass 1kg which then moves off at a velocity of $(2\mathbf{i}+\mathbf{j})\,\text{ms}^{-1}$.

Calculate the velocity of the 2kg ball after the collision, and find the overall loss of energy. **5**

9. Evaluate $\int_0^{\frac{\pi}{6}} 3x\sin 2x\,dx$ **5**

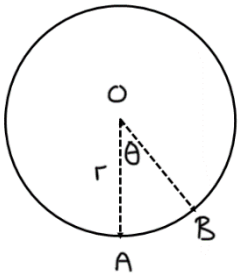
10. A new motorway slip road has the shape of an arc of a circle of radius 100 metres, banked at an angle of 8^0. The coefficient of friction for rubber on wet tarmac is $0{\cdot}3$.

Suggest a speed limit for this section of road (in m.p.h.), justifying your answer with appropriate calculations. (You may use the conversion 1mile = 1.6km) **6**

11. A student wants to calculate the volume of a hemisphere using the formula $V = \frac{2}{3}\pi r^3$. She measures the radius of a sphere to be 9 cm, but thinks that there could be an error of 0.5 cm in this measurement.

Differentiate the formula, and use dv in terms of dr to calculate an approximation for the largest possible error in the calculated volume. Express this as a percentage of the correct volume. **3**

12. A small mass is attached to a light inextensible string of length 60cm. The other end of the string is attached to a fixed point O and the mass is able to move along part of a vertical circle, as shown in the diagram.



Show that $v^2 = u^2 - 2gr(1-\cos\theta)$ where u is the initial velocity at point A, v is the final velocity at point B, and θ is the angle of the string to the vertical at B **3**

If the mass is given an initial velocity of 4 ms^{-1} find the angle at which the string goes slack. **4**

13. A particle obeys these parametric equations

$x = 3t^2 + 1,\ y = 2t - 1$

Calculate the second derivative $\frac{d^2y}{dx^2}$. **4**

THIS IS THE END OF THE PAPER