Advanced Higher Mechanics

Prelim Paper

Tuesday 27 January 2015

2 hours

1.35 p.m. – 3.35 p.m.

Total marks — 61

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

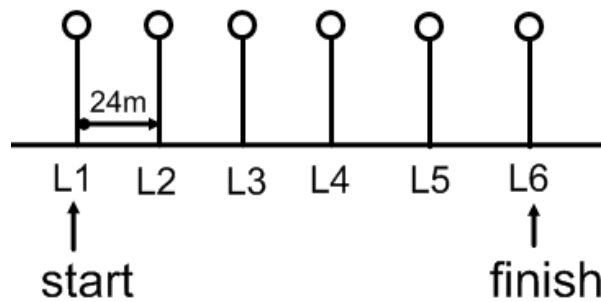Round to 3 significant figures, or 1 decimal place for angles

State the units for your answer where appropriate.

Answer all the questions. Candidates should observe that g ms^{-2} denotes the magnitude of the acceleration due to gravity. Where appropriate, take its magnitude to be 9.8ms^{-2}

1 A body moves from rest with velocity given by $(4t\mathbf{i}-7\mathbf{j})$ ms^{-1} where t is the time in seconds from the start of motion. The body starts at the origin and **i** and **j** define unit vectors at right angles to each other. Find the distance of the body from the origin after 5 seconds.

4

2. There are a series of lamp-posts along the side of the road, all 24m apart from each other, as shown in the diagram below.

A car reaches L2 with a certain speed and then accelerates uniformly between L2 and L6. The car takes 2 seconds to travel from the L2 to L3 and then another 1 second to travel from L3 to L4.

5

Calculate the acceleration of the car, and the total time taken to travel from L2 to L6.
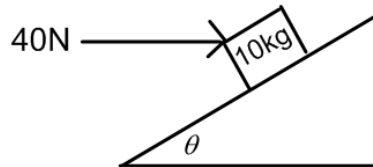

3. A particle moves so that $y = t^3 + 1$ and $x = t^2$

Find $\dfrac{dy}{dx}$ and then $\dfrac{d^2y}{dx^2}$

4

4. B is 12km East of A.
A ship sails North-East from A at a speed of 10km/hr.
At the same time a lifeboat travels North-West from B at a speed of 20km/hour.

Calculate the distance of closest approach, and the time until the two vessels will be closest.
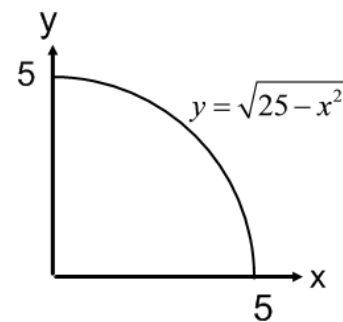
7

5. Use partial fractions to solve $\int \dfrac{2x+3}{x^2+5x+4}$

5

6. A slope is inclined at θ^0 to the horizontal. A 10kg block is on the slope, with $\mu = 0.3$
A force of 40N pushes the block horizontally as shown in the diagram.
Calculate the smallest value of θ for which the block will **not** move up the slope

5

7. An old-fashioned record player turns at 33 rpm (revolutions per minute).
A 50g coin rests on the surface of a record at a distance of 9cm from the centre.
$\mu = 0.15$

a) Calculate the central and friction forces and decide if the coin will slide.

4

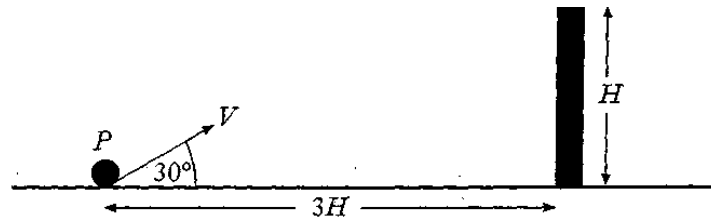b) What is the maximum distance from the centre that a coin can be placed without slipping?

3

8. Find the area of this quarter circle by integration.

Use the substitution $x = 5\sin\theta$.

6

9. How long would a 3kW motor take to drive a small car (mass 500kg) along an 800m road inclined at an angle such that $\sin\theta = \dfrac{1}{40}$ and against a constant friction force of 400N.
(You may assume that the car is already travelling at the appropriate speed.)

5

10 Use integration by parts to find $\int_0^2 5xe^{-3x}dx$

4

11. A football is kicked towards a wall from a point P on a horizontal playing field. The initial speed given to the ball is V ms^{-1} at an angle of 30° to the horizontal, as shown below. The wall is a horizontal distance $3H$ metres from the point P, where H is the height of the wall in metres.

6

Treating the ball as a particle, show that the ball goes **over** the wall if

$$V > \sqrt{\frac{6gH}{\sqrt{3}-1}},$$

where g ms^{-2} is the magnitude of the acceleration due to gravity.

12 Find the rate of change of the volume of a spherical balloon with respect to r.

(Use $V = \frac{4}{3}\pi r^3$)

3

Use your answer when r = 10cm to find the approximate increase in volume when the change in radius (dr) is 0.1cm

End of question Paper.