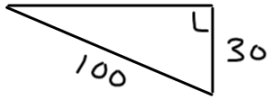
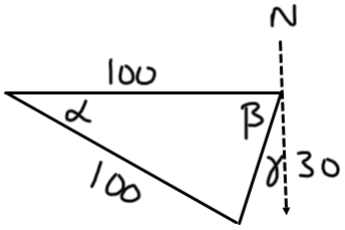
Prelim Marking Scheme Jan 2016

Q1

•1	Substitute into correct formula	•1	$v^2 = u^2 + 2as \Rightarrow a = \dfrac{20^2 - 5^2}{200} = 1.875 \text{ms}^{-2}$
•2	Calculate velocity after 200m	•2	$v^2 = 5^2 + 2 \times 1.875 \times 200 = 775 \Rightarrow v = 27.8 \text{ms}^{-1}$
•3	Substitute into $s = ut + \frac{1}{2}at^2$	•3	$1000 = 5t + \frac{1}{2} \times 1.875 \times t^2$
•4	Re-arrange quadratic	•4	$3t^2 + 16t - 3200 = 0$ (or equivalent)
•5	Solve using quadratic formula and select positive solution	•5	$t = \dfrac{-16 \pm \sqrt{16^2 + 38\,400}}{6}$ =30.1 (disregard -35.4)

Q2

•1	Appropriate diagram drawn or implied	•1	
•2	Solve using Pythagoras	•2	$v^2 = 100^2 - 20^2 \Rightarrow v = 98.0 \text{ms}^{-1}$
•3	Appropriate diagram drawn or implied	•3	
•4	Use cosine rule to calculate α	•4	$\cos\alpha = \dfrac{100^2 + 100^2 - 20^2}{2 \times 100 \times 100}$
•5	Value for α	•5	$\alpha = 11 \cdot 4^\circ$
•6	Calculate β and γ	•6	$\beta = 84 \cdot 3^\circ \Rightarrow \gamma = 5 \cdot 7^\circ$
•7	Interpret inequality using bearing	•7	Wind increases plane's speed for bearings greater than $185 \cdot 7^\circ$

Q3

•1	Factorise and form partial fractions	•1	$\dfrac{A}{x-3} + \dfrac{B}{x-2}$
•2	Re-arrange	•2	$A(x-2) + B(x-3) = 2x - 3$
•3	Solve for A and B	•3	$A = 3, B = -1 \Rightarrow \dfrac{3}{x-3} - \dfrac{1}{x-2}$

Q4

•1	Use $v = u + at$ in vertical direction to find time for half of flight	•1	$0 = v\sin\theta t - gt \Rightarrow t = \dfrac{v\sin\theta}{g}$
•2	Use $s = ut$ for horizontal direction	•2	$R = v\cos\theta \times 2t$
•3	Substitute and use double angle formula	•3	$R = \dfrac{2v^2\sin\theta\cos\theta}{g} = \dfrac{v^2\sin 2\theta}{g}$
•4	Calculate velocity using $v^2 = u^2 + 2as$	•4	$0 = u^2 - 2\times 9\cdot 8\times 80 \Rightarrow u = 39.6\text{ms}^{-1}$
•5	Use 45^0 for maximum range	•5	$\sin 2\theta = \sin 90 = 1 \Rightarrow R = \dfrac{v^2}{g}$
•6	Calculate max range	•6	$R = \dfrac{1568}{9.8} = 160\text{m}$
•7	3 valid assumptions	•7	No air resistance, ball as particle, no spin, no side wind (same vertical plane throughout), uniform g, no earth curvature, initial velocity consistent

Q5

•1	Use product rule	•1	$y' = u'v + uv' = 7e^{\tan 3x} + \ldots$
•2	Differentiates tan3x	•2	$\ldots 3\sec^2 3x$
•3	Completes product rule and factorises	•3	$y' = 7e^{\tan 3x} + (7x)3\sec^2 3xe^{\tan 3x} = 7e^{\tan 3x}(1 + 3x\sec^2 3x)$
•4	Implicit differentiation and re-arrange	•4	$2x + 2y\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{-x}{y}$
•5	Find y when $x = 3$	•5	$y^2 = 5^2 - 3^2 \Rightarrow y = \pm 4$
•6	Evaluate $\frac{dy}{dx}$	•6	$\dfrac{dy}{dx} = \mp\dfrac{3}{4}$

Q6

•1	Equate $F = \dfrac{P}{v}$ with resistive force	•1	$\dfrac{80\,000}{v} = 3000$
•2	Evaluate $v_{\max}$	•2	$v_{\max} = 26.7\text{ms}^{-1}$
•3	Balanced forces to include gravity	•3	$\dfrac{P}{v} = R + \text{mg}\sin\theta$
•4	Calculate new uphill velocity	•4	$\dfrac{P}{v} = 3000 + 50\,000\times 9\cdot 8\times\dfrac{1}{60} \Rightarrow v = 7\cdot 16\text{ms}^{-1}$
•5	Calculate time to ascend	•5	$\dfrac{1500\times 60}{7\cdot 16} = 12\,570\,\text{seconds}$
•6	Convert into hours and minutes	•6	$12\,570 = 209\,\text{minutes} = 3\,\text{hours}\,29\,\text{minutes} \Rightarrow \text{arrives } 1329$

Q7

•1	Express du in terms of dx	•1	$du = -2xdx$
•2	Re-write limits of integration	•2	$\int_5^1$
•3	Substitute	•3	$-\frac{1}{2}\int_5^1 \frac{du}{u}$
•4	Integrate	•4	$-\frac{1}{2}\left[\ln u\right]_5^1$
•5	Evaluate	•5	$\frac{1}{2}\ln 5 = 0\cdot 805$

Q8

•1	Initial momentum in system	•1	$2\begin{pmatrix}5\\0\end{pmatrix} = \begin{pmatrix}10\\0\end{pmatrix}$
•2	Calculate v from final momentum	•2	$\begin{pmatrix}10\\0\end{pmatrix} = 1\begin{pmatrix}2\\1\end{pmatrix} + 2(\mathbf{v}) \Rightarrow \mathbf{v} = \begin{pmatrix}4\\-\frac{1}{2}\end{pmatrix}$
•3	Calculate initial kinetic energy	•3	$\frac{1}{2}\times 2\times 5^2 = 25\text{J}$
•4	Start to calculate final kinetic energy	•4	$\frac{1}{2}\times 1\times(2^2+1^2) + \frac{1}{2}\times 2\times(4^2+\frac{1}{2}^2)$
•5	Final kinetic Energy and loss by subtraction	•5	$= 18\frac{3}{4} \Rightarrow \text{E}_\text{K}\text{loss} = 25 - 18\frac{3}{4} = 6\frac{1}{4}\text{J}$

Q9

•1	Set up integration by parts	•1	$u = 3x, u' = 3$ $v = -\frac{1}{2}\cos 2x, v' = \sin 2x$
•2	Substitute	•2	$\left[-\frac{3}{2}x\cos 2x\right]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}}(-\frac{3}{2}\cos 2x)dx$
•3	Integrate	•3	$\left[-\frac{3}{2}x\cos 2x\right]_0^{\frac{\pi}{6}} + \left[\frac{3}{4}\sin 2x\right]_0^{\frac{\pi}{6}}$
•4	Evaluate	•4	$(-\frac{3}{2}\frac{\pi}{6}\cos\frac{\pi}{3} - 0) + (\frac{3}{4}\sin\frac{\pi}{3} - 0)$
•5	Simplify	•5	$= \frac{3\sqrt{3}}{8} - \frac{\pi}{8}(= 0.257)$

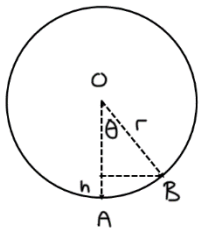
Q10

•1	Horizontal force equal central force	•1	$R\sin 8 + \mu R\cos 8 = \frac{mv^2}{r}$
•2	Vertical forces = weight	•2	$R\cos 8 - \mu R\sin 8 = mg$
•3	Divide equations and cancel R	•3	$\frac{v^2}{rg} = \frac{\sin 8 + \mu\cos 8}{\cos 8 - \mu\sin 8}$
•4	Substitute values and calculate v	•4	$v^2 = 450.7 \Rightarrow v = 21.23 ms^{-1}$
•5	Convert into kph	•5	$21.23\times 60\times 60 \div 1000 =$ 76.4 kph
•6	Convert into mph and suggest limit	•6	$47.8\,\text{mph} \Rightarrow \text{limit of 40 or 45mph}$

Q11

•1	Differentiate	•1	$\frac{dv}{dr} = 2\pi r^2$
•2	Express dv in terms of dr and substitute	•2	$dv = 2\pi r^2 dr = 2\times\pi\times 9^2\times 0\cdot 5$
•3	Evaluate change and express as percentage	•3	$= 254\cdot 5 cm^3 \Rightarrow \frac{254.5}{\frac{2}{3}\times\pi\times 9^3}\times 100 = 16.7\%$

Q12



•1	Express h in terms of r, θ	•1	$h = r - r\cos\theta = r(1-\cos\theta)$
•2	Calculate total energy at B	•2	$\frac{1}{2}mv^2 + mgr(1-\cos\theta)$
•3	Equate initial Energy with energy at B and simplify	•3	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgr(1-\cos\theta) \Rightarrow u^2 = v^2 + 2gr(1-\cos\theta)$
•4	Express central force as unbalanced force along string	•4	$\frac{mv^2}{r} = T - mg\cos\theta$
•5	Set tension to zero when string slack	•5	$T = \frac{mv^2}{r} + mg\cos\theta = 0$
•6	Substitute for v^2 and simplify	•6	$T = m(\frac{u^2 - 2gr(1-\cos\theta)}{r} + g\cos\theta) = 0$ $\Rightarrow \frac{u^2}{r} - 2g + 3g\cos\theta = 0$
•7	Solve for angle	•7	$\cos\theta = \frac{2}{3} - \frac{u^2}{3gr} \Rightarrow \theta = 103\cdot 9^\circ$

Q13

•1	Calculate dy/dt and dx/dt	•1	$\frac{dx}{dt} = 6t, \frac{dy}{dt} = 2$
•2	Combine to find dy/dx	•2	$\frac{dy}{dx} = \frac{2}{6t} = \frac{1}{3t}$
•3	Formula for 2nd derivative	•3	$\frac{d^2y}{dx^2} = \frac{d}{dt}(\frac{dy}{dx}) \times \frac{dt}{dx}$
•4	Complete and simplify	•4	$\frac{1}{3}(-1)t^{-2} \times \frac{1}{6t} = -\frac{1}{18t^3}$