Prelim Marking Scheme Jan 2015 (Revision)

Q1

•1	Integrate to find displacement	•1	$r=\int(4ti-7j)dt=(2t^2+c_1)i+(-7t+c_2)j$
•2	Constants of integration	•2	$t=0\Rightarrow r=0\Rightarrow c_1=c_2=0$ $r=2t^2\mathbf{i}-7t\,\mathbf{j}$
•3	Find displacement after 5 seconds	•3	$r=2\times5^2\mathbf{i}-7\times35\mathbf{j}=50\mathbf{i}-35\mathbf{j}$
•4	Calculate distance	•4	$d=\sqrt{50^2+(-35)^2}=\sqrt{3725}=61\cdot0\,\text{m}$

Q2

•1	Formula for L2 to L3	•1	$ut+\frac{1}{2}at^2=24\Rightarrow 2u+2a=24$
•2	Formula for L2 to L4	•2	$3u+\frac{1}{2}a\times9=48$
•3	Scale for simultaneous equations	•3	$6u+6a=72$ $6u+9a=96$
•4	Solve for a	•4	$3a=24\Rightarrow a=8\text{ms}^{-2}$
•5	Solve for u	•5	$u=4\text{ms}^{-1}$
•6	Use $s=ut+\frac{1}{2}at^2$ for whole journey	•6	$4t+\frac{1}{2}\times8\times t^2=96\Rightarrow 4t^2+4t-96=0$ seconds
•7	Solve quadratic for positive time	•7	$t=4\cdot39$ seconds

Q3

•1	Differentiate with respect to t	•1	$\frac{dy}{dt}=3t^2\qquad\frac{dx}{dt}=2t$
•2	Combine	•2	$\frac{dy}{dx}=\frac{3t}{2}$
•3	Differentiate again	•3	$\frac{d}{dt}\left(\frac{3t}{2}\right)\times\frac{dt}{dx}$
•4	Simplify	•4	$\frac{3}{4t}$

Q4

•1	Position of ship (r_S)	•1	$r_S=5\sqrt{2}t\mathbf{i}+5\sqrt{2}t\mathbf{j}$
•2	Position of lifeboat (r_L)	•2	$r_L=(12-10\sqrt{2}t)\mathbf{i}+10\sqrt{2}t\mathbf{j}$
•3	Position of ship relative to lifeboat	•3	$r_S-r_L=(15\sqrt{2}t-12)\mathbf{i}-5\sqrt{2}t\mathbf{j}$
•4	Square of distance between them	•4	$d^2=450t^2-360\sqrt{2}t+144+50t^2$ $=500t^2-360\sqrt{2}t+144$
•5	Differentiate for minimum value	•5	$(d^2)'=1000t-360\sqrt{2}=0$
•6	Time for closest approach	•6	$t=\frac{360\sqrt{2}}{1000}=0.509$ hours
•7	Closest distance	•7	$d^2=500(0.509)^2-360\sqrt{2}\left(0.509\right)+144=14.4\Rightarrow d=3.79km$

Q5			
•1	Set up partial fractions	•1	$\frac{A}{x+4}+\frac{B}{x+1}$
•2	Equate	•2	A(x+1) + B(x+4) = 2x+3
•3	Solve for A and B	•3	A = $\frac{5}{3}$ B = $\frac{1}{3}$
•4	Re-write integral	•4	$\frac{1}{3}\int(\frac{5}{x+4}+\frac{1}{x+1})dx$
•5	Integrate	•5	$\frac{5}{3}\ln(x+4)+\frac{1}{3}\ln(x+1)+c$
Q6			
•1	reaction	•1	$R=mg\cos\theta+20\sin\theta$
•2	Friction down slope	•2	$F_{\max}=\mu R=0.3(mg\cos\theta+40\sin\theta)$
•3	Equilibrium down slope	•3	$40\cos\theta=10g\sin\theta+0.3(10g\cos\theta+40\sin\theta)$
•4	Divide through by $\cos\theta$, re-arrange	•4	$40=10g\tan\theta+0.3(10g+40\tan\theta)$ $\tan\theta(10g+0\cdot3\times40)=40-0.3\times10g$
•5	solve for $\tan\theta$	•5	$\tan\theta=\frac{40-0.3\times10g}{(10g+0\cdot3\times40)}\Rightarrow\tan\theta=0\cdot0964\Rightarrow\theta=5\cdot5^{o}$
Q7			
•1	Calculate ω	•1	$33rpm=\frac{33\times2\pi}{60}=1.1\pi\, rad\, s^{-1}$
•2	Find central force	•2	$m\omega^2 r=0\cdot05\times(1.1\pi)^2\times0\cdot09=0\cdot0537\text{N}$
•3	Friction Force	•3	$\mu mg=0\cdot15\times0.05\times9.8=0\cdot0735\text{N}$
•4	Decision, reason	•4	No, it will not slide, friction bigger than central force
•5	Equate friction and central force	•5	$m\omega^2 r=\mu mg$
•6	Re-arrange	•6	$r=\frac{\mu g}{\omega^2}$
•7	Substitute and solve	•7	$=0\cdot123\text{m}$
Q8			
•1	set up integral with limits	•1	$\int_0^5\sqrt{25-x^2}\,dx$
•2	make substitution and differentiate	•2	$dx=5cos\theta d\theta$
•3	change limits	•3	$\int_0^{\frac{\pi}{2}}\sqrt{25-25\sin^2\theta}\,cos\theta d\theta$
•4	simplify	•4	$\mathbf{25\int_0^{\frac{\pi}{2}}cosx^2\theta d\theta}$
•5	use double angle formula	•5	$\mathbf{25\int_0^{\frac{\pi}{2}}\frac{1}{2}(cos2\theta+1)d\theta}$
•6	Integrate	•6	$25[\frac{1}{4}\boldsymbol{sin2\theta}+\frac{1}{2}\boldsymbol{\theta}]_0^{\frac{\pi}{2}}$

•7	Process limits	•7	$\frac{25\pi}{4}$

Q9

•1	Tractive Force	•1	$F=\frac{P}{v}$
•2	Forces along slope in equilibrium	•2	$\frac{P}{v}=F_{\max}+mg\sin\theta$
•3	Solve for tractive force	•3	$\frac{P}{v}=400+500\times9\cdot8\times\frac{1}{40}=522.5$
•4	Solve for v	•4	$v=\frac{3000}{522\cdot5}=5\cdot75\text{ms}^{-1}$
•5	Time taken	•5	$t=\frac{s}{v}=\frac{800}{5\cdot74}=139.4$ seconds

Q10

•1	Formula	•1	$\int fg'=fg-\int f'g$
•2	Substitute	•2	$\left[-\tfrac{1}{3}5xe^{-3x}\right]_0^2-\int_0^2 5(-\tfrac{1}{3}e^{-3x})dx$
•3	Integrate	•3	$\left[-\tfrac{1}{3}5xe^{-3x}\right]_0^2+\left[\tfrac{5}{3}\frac{e^{-3x}}{-3}\right]_0^2$
•4	Final answer	•4	$-\frac{10}{3}e^{-6}-\frac{5}{9}e^{-6}+\frac{5}{9}=0\cdot546$

Q11

•1	Consider horizontal component	•1	$3H=v\cos30\times t$
•2	Expression for time	•2	$t=\frac{3H}{\frac{\sqrt{3}}{2}v}=\frac{6H}{\sqrt{3}v}$
•3	Consider vertical component	•3	$s=ut+\tfrac{1}{2}at^2\Rightarrow h=v\sin30t-\tfrac{1}{2}gt^2$
•4	Set up inequality	•4	$\tfrac{1}{2}v(\frac{6H}{\sqrt{3}v})-\tfrac{1}{2}g(\frac{6H}{\sqrt{3}v})^2\geq H$
•5	Simplify, cancel v and H	•5	$\frac{-6gH}{v^2}\geq(1-\sqrt{3})$
•6	Switch sides, change signs and take square root	•6	$v^2(\sqrt{3}-1)\geq6gH\Rightarrow v\geq\sqrt{\frac{6gH}{\sqrt{3}-1}}$

Q12

•1	Differentiate	•1	$\frac{dV}{dr}=4\pi r^2$
•2	Substitute to find dV	•2	$4\pi r^2\times0.1$
•3	Rounded answer	•3	126cm^3