Higher Homework #10
Solutions

1) Line AD has a gradient of 3 as $-\frac{1}{3} \times 3 = -1$

$m = 3 \quad (a, b) = (2, 4)$

$y - 4 = 3(x-2)$
$y - 4 = 3x - 6$
$y - 3x + 2 = 0 \quad \Rightarrow$ Answer A.

OR.

$-y + 3x - 2 = 0$

2) $u_{n+1} = 0.6u_n + 5$

$L = \frac{b}{1-a}$

$a = 0.6 \quad b = 5$

$L = \frac{5}{1-0.6} = 12.5 \Rightarrow$ Answer D.

3) $f(x) = 3x^2 - 1 \quad g(x) = x^2 + 2$

$f(g(x)) = 3(x^2+2)^2 - 1$
$= 3(x^4 + 4x^2 + 4) - 1$
$= 3x^4 + 12x^2 + 12 - 1$
$= 3x^4 + 12x^2 + 11 \Rightarrow$ Option D

4) $f(x) = 2x^3 - 3x^2 + 6$

$f'(x) = 6x^2 - 6x$

$6x^2 - 6x = 0$

$6x(x-1) = 0$

$x = 0$ or $x = 1$

if $x = 0$

$y = 2(0)^3 - 3(0)^2 + 6$

$y = 6$

$\Rightarrow$ Stationary Point at $(0,6)$

$f''(x) = 12x - 6$

$f''(0) = -6 < 0$

$\Rightarrow$ $(0,6)$ is a maximum turning point

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if $x = 1$

$y = 2(1)^3 - 3(1)^2 + 6$

$y = 5$

$\Rightarrow$ Stationary point at $(1,5)$

$f''(1) = 6 > 0$

$\Rightarrow$ $(1,5)$ is a minimum turning point.

6) $y = \frac{24}{\sqrt{x}}$

$y = 24x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -12x^{-\frac{3}{2}}$

$\frac{dy}{dx}(4) = -12(4)^{-\frac{3}{2}}$

$= -\frac{3}{2}$

$\Rightarrow$ gradient of tangent at $x=4$ is -1.5.

if $x=4$

$y = \frac{24}{\sqrt{4}}$

$y = 12$

$(4, 12)$ $\quad m = -1.5$

$y - b = m(x-a)$

$y - 12 = -\frac{3}{2}(x-4)$

$2y - 24 = -3(x-4)$

$2y - 24 = -3x + 12$

$2y + 3x - 12 = 0$ (or equivalent).

7) $f(x) = 4\sqrt{x} + 1$

a) $f(x) - 1 = 4\sqrt{x}$

$\dfrac{f(x) - 1}{4} = \sqrt{x}$

$\left(\dfrac{f(x) - 1}{4}\right)^2 = x$ so

$f^{-1}(x) = \left(\dfrac{x - 1}{4}\right)^2.$

b) $f(x) \in \mathbb{R} : f(x) \geq 1.$ (Range)

$x \in \mathbb{R} : x \geq 0$ (Domain)

c) $f^{-1}(x) \in \mathbb{R} : f^{-1}(x) \geq 0$ (Range)

$x \in \mathbb{R}$: (Domain)

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