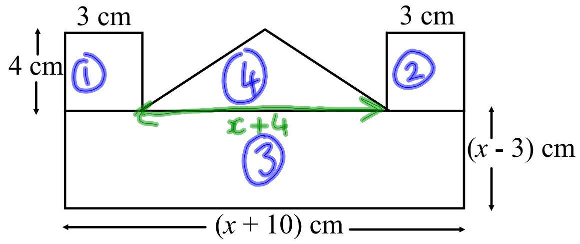
Lesson Starter

Forming Expressions (NAT 5 Revision)

The diagram below shows a logo for a club.



Show that the total area of the logo, in square centimetres, is given by $A = x^2 + 9x + 2$

Today's Learning:
By the end of this lesson, you should be able to solve problems involving differentiation.

Optimisation

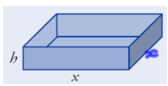
Optimisation means to find the maximum or minimum value using stationary points.

Question Layout

Part a)	Proof using prior knowledge	(max 4 marks)
Part b)	Differentiate and find the stationary points. Prove is maximum or minimum by finding the nature.	(min 6 marks)

Examples

1) Small wooden trays, with open tops and square bases, are being designed. They must have a volume of 108 cm^3.



The internal length of one side of the base is x cm, and the internal height of the tray is h cm.

a) Show that the total internal surface area A, of one tray is given by $A(x) = \frac{432}{x} + x^2$

$A(x) = 4xh + x^2$

$V = l \times b \times h$

$V = x^2 h$

$x^2 h = 108$

$h = \frac{108}{x^2}$

$A(x) = 4x \times \frac{108}{x^2} + x^2$

$= \frac{432x}{x^2} + x^2$

$= \frac{432}{x} + x^2$

b) Find the dimensions of the tray which uses the smallest amount of wood.

Min TP occurs when $A'(x) = 0$

$A(x) = \frac{432}{x} + x^2$

$A(x) = 432x^{-1} + x^2$

$A'(x) = -432x^{-2} + 2x$

$-\frac{432}{x^2} + 2x = 0$

$2x = \frac{432}{x^2}$

$2x^3 = 432$

$x^3 = 216$

$x = 6$

We need to make sure this is a Min TP

x	6^-	6	6^+
$A'(x)$	$-$	0	$+$
Slope	\	—	/

$x = 6$

$A'(x) = -\frac{432}{x^2} + 2x$

$A'(5) = \frac{-432}{25} + 10$

$= -86.4 + 10$

$= -76.4$

$A'(10) = -\frac{432}{100} + 20$

$= -4.32 + 20$

$x = 6$
minimum SP

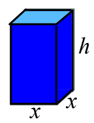
Try page 114 ex 6R Q1

When $x = 6$, $h = \frac{108}{6^2}$

$h = 3$

Dimensions of bottom of box is 6 cm by 6cm.
The depth of the box is 3 cm.

2) An **open** tank is to be designed in the shape of a cuboid with a square base.
The total surface area of the base and the four walls is 100 cm^2.



a) The length of the base is x cm.
Show that the volume of the cuboid is given by

$V(x) = \frac{x}{4}(100 - x^2)$

$V = l \times b \times h$

$= x \times x \times h$

$= x^2 h$

SA = $4xh + x^2 = 100$

$h = \frac{100 - x^2}{4x}$

$V = x^2 \times \frac{100 - x^2}{4x}$

$= \frac{x^2(100 - x^2)}{4x}$

$V(x) = \frac{x}{4}(100 - x^2)$ as required

b) Find the length of the base that makes the volume of the tank a maximum.

Maximum value will occur when $V'(x) = 0$

$$V(x) = \frac{x}{4}(100 - x^2)$$
$$= 25x - \frac{x^3}{4}$$
$$V'(x) = 25 - \frac{3}{4}x^2$$
$$25 - \frac{3}{4}x^2 = 0$$
$$\frac{3}{4}x^2 = 25$$
$$x^2 = \frac{100}{3}$$
$$x = \pm\frac{10}{\sqrt{3}}$$

(length so must be positive)

We need to prove this is a maximum turning point.

$V'(x) = 25 - \frac{3}{4}x^2$

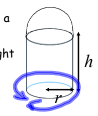
x	
$V'(x)$	
Slope	

The cuboid has a maximum volume if $x = \frac{10}{\sqrt{3}}$ cm

Ex 6R

Q1

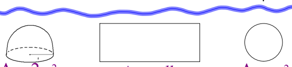
2) A drinking can is in the shape of a cylinder with a hemisphere lid.
The radius of the cylinder is r cm and the height h cm.
The volume of the can is 400 cm^3.



a) Show that the surface area of plastic is given by

$$A(r) = 3\pi r^2 + \frac{800}{r}$$

Curved surface area of hemisphere = $2\pi r^2$



$A = 2\pi r^2$ $A = \pi dh$ $A = \pi r^2$

$A = 2\pi rh$

$$A(r) = 2\pi rh + \pi r^2 + 2\pi r^2$$
$$= 2\pi rh + 3\pi r^2$$

$$V = \pi r^2 h$$
$$400 = \pi r^2 h$$
$$\frac{400}{\pi r^2} = h$$

$$= 2\pi r \times \frac{400}{\pi r^2} + 3\pi r^2$$
$$= \frac{800\pi r}{\pi r^2} + 3\pi r^2$$
$$= \frac{800}{r} + 3\pi r^2$$

b) Find the value of r which ensures that the surface area of the cup is minimised.

$$A(r) = \frac{800}{r} + 3\pi r^2$$

Minimum value will occur when $A'(r) = 0$

$$A(r) = 800r^{-1} + 3\pi r^2$$
$$A'(r) = -800r^{-2} + 6\pi r$$
$$-800r^{-2} + 6\pi r = 0$$
$$-\frac{800}{r^2} + 6\pi r = 0$$
$$6\pi r = \frac{800}{r^2}$$
$$6\pi r^3 = 800$$
$$r^3 = \frac{800}{6\pi}$$
$$r = 3.49$$

$$A'(r) = -\frac{800}{r^2} + 6\pi r$$
$$r = 3.49$$

We need to prove this is a minimum turning point.

r	3.49^-	3.49	3.49^+
$A'(r)$	−	0	+
Slope	\	—	/

$A'(3)$
$A'(4)$

The surface area of the cup is minimised if $r = 3.49$.

Attachments

maths1_2.ppt

maths1_3.ppt