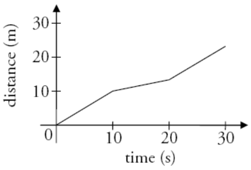
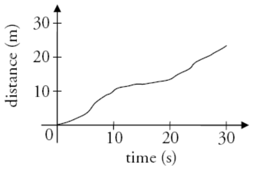
The Derived Function

Learning Intention
By the end of this lesson, you should be able to understand where we use differentiation and differentiate basic functions.

Differentiation belongs to an area of Mathematics called Calculus. Calculus helps us to solve problems involving motion.





Instantaneous speed is the speed of an object at an exact moment in time. It is also known as the **rate of change** of distance with respect to time.

Can you spot the link between A and B?

Does it work in every case?

A	B
x^3	$3x^2$
x^6	$6x^5$
x^{10}	$10x^9$
x^1	1
$3x^0$	0

$x^1 \to 1x^0$

$x^7 \to 7x^6$

$x^3 \to$

$x^{12} \to 12x^{11}$

$x^{-2} \to -2x^{3}$

1

Chapter 6 - Differentiation

The derivative of x^n

Today's Learning:
By the end of this lesson, you should be able to differentiate **any** basic function using the correct notation.

$f'(x)$ is called the derived function or the derivative of $f(x)$.
The derivative of a function represents:

- ☆ the rate of change of a function
- ☆ the gradient of the tangent to the graph of the function.

If $f(x) = x^n$, then $f'(x) = nx^{n-1}$, where n is a rational number.

× by power, reduce power by 1

Examples

For each of the following find $f'(x)$.

1) $f(x) = x^7$

$f'(x) = 7x^6$

2) $f(x) = x^{-3}$

$f'(x) = -3x^{-4}$

Starter

When $2x^2 + 8x - 3$ is written in the form $a(x+b)^2 + c$ the values of a, b and c are

$2(x^2 + 4x - 3/2)$

	a	b	c
A	2	2	−3
B	2	2	−7
C	2	2	−11
D	2	2	7

$2((x+2)^2 - 5.5)$

$2(x+2)^2 - 11$

$2(x^2+4x+4) - 11$

$2x^2 + 8x - 3$

3) $f(x) = x^{\frac{7}{2}}$

$f'(x) = \frac{7}{2}x^{\frac{5}{2}}$

4) $f(x) = \frac{3}{x^2}$

$f(x) = 3x^{-2}$

$f'(x) = -6x^{-3}$

$f'(x) = -2(3)x^{-3}$

$f'(x) = \frac{-6}{x^3}$

5) $f(x) = \sqrt{x}$

$= x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$= \frac{1}{2x^{1/2}}$

$= \frac{1}{2\sqrt{x}}$

Ex 6D

2. The first rule: $a^m \times a^n = a^{m+n}$
3. The second rule: $(a^m)^n = a^{mn}$
4. The third rule: $a^m \div a^n = a^{m-n}$
5. The fourth rule: $a^0 = 1$
6. The fifth rule: $a^{-1} = \frac{1}{a}$ and $a^{-m} = \frac{1}{a^m}$
7. The sixth rule: $a^{\frac{1}{2}} = \sqrt{a}$ and $a^{\frac{1}{3}} = \sqrt[3]{a}$ $\frac{3}{3}$
8. A final result: $a^{\frac{p}{q}} = (a^p)^{\frac{1}{q}} = \sqrt[q]{a^p}$.

$x^{\frac{2}{3}} \rightarrow \frac{2}{3}x^{-1/3}$

$= \frac{2}{3x^{1/3}}$

$= \frac{2}{3\sqrt[3]{x}}$

$x \rightarrow 1$

$5 \rightarrow 0$

$6x \rightarrow 6$

$17 \rightarrow 0$

Other Notation

Today's Learning:
By the end of this lesson, you should able to recognise other notation involved with differentiation.

For an expression in the form $y = ...$, the derivative with respect to x is expressed as $\frac{dy}{dx}$.

To find the derivative of an expression in x with respect to x, the notation used is $\frac{d}{dx}$

Examples

1) $y = x^{-\frac{1}{3}}$

$\frac{dy}{dx} = -\frac{1}{3}x^{-4/3}$

$= \frac{-1}{3x^{4/3}}$

$= \frac{-1}{3\sqrt[3]{x^4}}$

$x^{-\frac{4}{3}} = \frac{1}{x^{4/3}}$

2) Find the derivative of $x^{\frac{3}{2}}$ with respect to x.

$\frac{d}{dx}\left(x^{\frac{3}{2}}\right) = \frac{3}{2}x^{1/2}$

Rate of Change

Today's Learning:
By the end of this lesson, you should be able to calculate rate of change.

The rate of change of a function, $f'(x)$, can be evaluated for any value of x.

Examples

1) Given $f(x) = x^5$ for $x \in$ R, find the rate of change of f when $x = 3$.

$f(x) = x^5$

$f'(x) = 5x^4$

$f'(3) = 5 \times 3^4$

$= 5 \times 81$

$= 405$

2) If $f(x) = x^{-4}$ for $x \in$ R, find $f'(2)$.

$f(x) = x^{-4}$

$f'(x) = -4x^{-5}$

$f'(2) = -4 \times 2^{-5}$

$= -4 \times \frac{1}{2^5}$

$= -4 \times \frac{1}{32}$

$= -\frac{1}{8}$

$f'(x) = \frac{-4}{x^5}$

$f'(2) = \frac{-4}{2^5}$

$= \frac{-4}{32}$

$= \frac{-1}{8}$

3) The value of an investment is calculated using $V(t) = t^2$. V is value and t is time. Calculate the growth rate (rate of change) after 9 years.

$V(t) = t^2$

$V'(t) = 2t$

$V'(9) = 2 \times 9 = 18$

4) Smoke from a factory chimney travels $\sqrt{t}$ km in t hours. Calculate the speed (rate of change) of the smoke after 4 hours.

$d(t) = \sqrt{t}$

$= t^{1/2}$

$d'(t) = \frac{1}{2}t^{-1/2}$

$= \frac{1}{2t^{1/2}}$

$= \frac{1}{2\sqrt{t}}$

$d'(4) = \frac{1}{2\sqrt{4}}$

$= \frac{1}{2 \times 2} = \frac{1}{4}$ km/hour

Differentiating more than one term and ax^n

Today's Learning:
By the end of this lesson, you should be able to differentiate expressions with more than one term.

To differentiate more than one term, just differentiate each term separately.

Examples

1) A function is defined for $x \in$ R by $f(x) = 3x^4$. Find $f'(x)$.

$f'(x) = 12x^3$

2) A function is defined for $x \in \mathbb{R}$ by $f(x) = 3x^3 - 2x^2 + 5x$

Find $f'(x)$.

$f'(x) = 9x^2 - 4x + 5$

3) Differentiate $y = 2x^4 - 4x^3 + 3x^2 + 6x + 2$ with respect to x.

$\frac{dy}{dx} = 8x^3 - 12x^2 + 6x + 6$

4) Differentiate $y = \frac{1}{2x^3}$ with respect to x.

$y = \frac{1}{2}x^{-3}$

$\frac{dy}{dx} = \frac{-3}{2}x^{-4}$

$= \frac{-3}{2x^4}$

Starter

Write the following in the form $a(x + b)^2 + c$

1) $2x^2 - 8x + 20$

2) $2x^2 + 12x + 9$

3) $3x^2 - 6x + 1$

4) $2x^2 - 6x + 7$

Differentiating more complex expressions

Today's Learning:
By the end of this lesson, you should be able to differentiate more complex expressions.

Before differentiating a function we have to express it as a sum of individual terms.

Examples

1) Find $\frac{dy}{dx}$ when $y = (x - 3)(x + 2)$

$y = x^2 + 2x - 3x - 6$

$= x^2 - x^1 - 6$

$\frac{dy}{dx} = 2x - 1$

2) Differentiate $\frac{x^4 - 3x^2}{5x}$ with respect to x.

$\frac{x^4 - 3x^2}{5x} = \frac{x^4}{5x} - \frac{3x^2}{5x} = \frac{x^3}{5} - \frac{3x}{5}$

$= \frac{1}{5}x^3 - \frac{3}{5}x$

$\frac{d}{dx} = \frac{3}{5}x^2 - \frac{3}{5}$

Starter

When $\frac{x^2}{\sqrt{x}}$ is differentiated with respect to x, the result is

A. $\frac{5}{2}x^{\frac{3}{2}}$

B. $\frac{3}{2}x^{\frac{1}{2}}$

C. $\frac{1}{2}x^{\frac{1}{2}}$

D. $\frac{2}{5}x^{\frac{5}{2}}$

$\frac{x^2}{x^{1/2}}$

$x^{3/2}$

$\frac{d}{dx} = \frac{3}{2}x^{1/2}$

3) Differentiate $\frac{x^3 + 3x^2 - 6x}{\sqrt{x}}$

$\frac{x^3+3x^2-6x}{x^{1/2}}$

$$\frac{x^3+3x^2-6x}{\sqrt{x}} = \frac{x^3}{x^{\frac{1}{2}}} + \frac{3x^2}{x^{\frac{1}{2}}} - \frac{6x}{x^{\frac{1}{2}}}$$
$$= x^{3-\frac{1}{2}} + 3x^{2-\frac{1}{2}} - 6x^{1-\frac{1}{2}}$$
$$= x^{\frac{5}{2}} + 3x^{\frac{3}{2}} - 6x^{\frac{1}{2}}$$
$$\frac{d}{dx} = \frac{5}{2}x^{\frac{3}{2}} + \frac{9}{2}x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$$
$$= \frac{5}{2}\sqrt{x}^3 + \frac{9}{2}\sqrt{x} - \frac{3}{\sqrt{x}}$$

$-\frac{1}{2}$

4) Find the derivative of $y = \sqrt{x}(x^2 + \sqrt[3]{x})$ wrt x.

$y = x^{\frac{1}{2}}(x^2 + x^{\frac{1}{3}})$

$$y = \sqrt{x}(x^2 + \sqrt[3]{x})$$
$$y = x^{\frac{1}{2}}\left(x^2 + x^{\frac{1}{3}}\right)$$
$$= x^{\frac{5}{2}} + x^{\frac{5}{6}}$$
$$\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}} + \frac{5}{6}x^{-\frac{1}{6}}$$
$$= \frac{5}{2}\sqrt{x}^3 + \frac{5}{6\sqrt[6]{x}}$$

$\frac{1}{2} + \frac{1}{3}$

$= \frac{3}{6} + \frac{2}{6}$

$\frac{dy}{dx} = \frac{5}{2}x^{3/2} + \frac{5}{6}x^{-1/6}$

$= \frac{5}{2}\sqrt{x^3} + \frac{5}{6\sqrt[6]{x}}$

Starter

Differentiate the following with respect to x:

1) $f(x) = 3x^2 + \frac{1}{x} = 3x^2 + x^{-1}$

$f'(x) = 6x - x^{-2}$

$= 6x - \frac{1}{x^2}$

2) $y = 2x^4 - 5x^3 + \frac{1}{2}x^2 + 3x - 12$

$\frac{dy}{dx} = 8x^3 - 15x^2 + x + 3$

3) $f(x) = x + \frac{1}{x} + \frac{3}{x^2} - \frac{1}{5x^3}$

$= x + x^{-1} + 3x^{-2} - \frac{1}{5}x^{-3}$

$= 1 - x^{-2} - 6x^{-3} + \frac{3}{5}x^{-4}$

$= 1 - \frac{1}{x^2} - \frac{6}{x^3} + \frac{3}{5x^4}$

Applications of Derivatives

(More complex Rate of Change)

Today's Learning:

By the end of this lesson, you should be able to apply differentiation to solve real life problems.

Examples

1) A ball is thrown so that its displacement s after t seconds is given by $s(t) = 12t - 5t^2$.

Find the rate of change after 2 seconds.

$s'(t) = 12 - 10t$ ← rate of change

$s'(2) = 12 - 10 \times 2$

$= 12 - 20$

$= -8$

2) If $y=\frac{1}{x^{\frac{2}{3}}}$, calculate the rate of change of y when $x=8$.

$y = x^{-2/3}$

$\frac{dy}{dx} = -\frac{2}{3}x^{-5/3}$

$= -\frac{2}{3}\cdot\frac{1}{x^{5/3}}$

$= \frac{-2}{3x^{5/3}}$

$= \frac{-2}{3\sqrt[3]{x^5}}$

rate of change for $x=8$:

$\frac{-2}{3\sqrt[3]{8^5}} = \frac{-2}{3\times 2^5} = \frac{-2}{3\times 32}$

$= \frac{-2}{96} = \frac{-1}{48}$

3) The pulse rate, in beats per minute of a runner t minutes after starting to run is given by

$p(t) = 60 + \frac{1}{2}t^2 - t$, for $t \le 10$.

Find the rate of change of the pulse rate of the runner 6 minutes after starting.

$p'(t) = \frac{2}{2}t - 1$

$= t - 1$

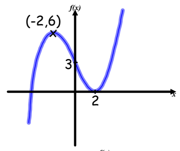
$p'(6) = 6 - 1$

$= 5$

page 96 Ex 6H Q1, 2, 6, 8, 12

Starter

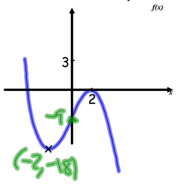
Given the graph f(x), draw the graph

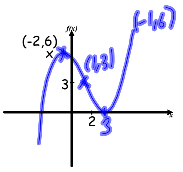


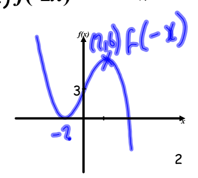
a) $-3f(x)$ flip in x axis stretch ↕ ×3

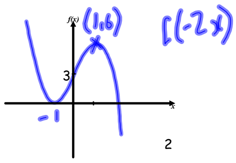
b) $f(x-1)$ moves → 1

c) $f(-2x)$ flip in y axis stretch ↔ by $\frac{1}{2}$







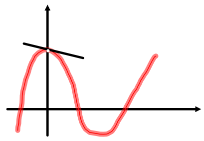


f(-2x)

Today's Learning:
By the end of this lesson, you should be able to determine the equation of a tangent.

Gradients of Tangents

We can determine the equation of a curve, at a given point by considering the straight line which touches the curve at this point (the tangent).



The equation of the tangent is $y - b = m(x - a)$

To find the equation of the tangent we need

☆ the co-ordinates (a,b)

☆ the gradient, m, at that point.

Examples

1) Find the equation of the tangent to the curve with equation $y = x^2 - 3$ at the point $(2,1)$

$\frac{dy}{dx} = 2x$

At $(2,1)$, $m = 2\times 2 = 4$

$y - b = m(x - a)$

$y - 1 = 4(x - 2)$

$y - 1 = 4x - 8$

$y - 4x + 7 = 0$

2) Find the equation of the tangent to the curve with equation $y = x^3 - 2x$ at the point $x = -1$

$\frac{dy}{dx} = 3x^2 - 2$

where $x = -1$, $m = 3(-1)^2 - 2$

$= 1$

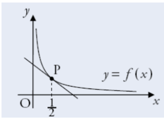
$y = x^3 - 2x$

$= (-1)^3 - 2(-1)$

$= -1 + 2 = 1$ $(-1,1)$

3) A function f is defined for $x > 0$ by $f(x) = \frac{1}{x}$.

Find the equation of the tangent to the curve $y = f(x)$ at P.



where $x = \frac{1}{2}$

At P, $x = \frac{1}{2}$

$f(x) = \frac{1}{x}$

$f(\frac{1}{2}) = \frac{1}{\frac{1}{2}} = 2$

$(\frac{1}{2}, 2)$

$f(x) = x^{-1}$

$f'(x) = -x^{-2} = \frac{-1}{x^2}$

$f'(\frac{1}{2}) = -(\frac{1}{2})^{-2}$

$= \frac{-1}{(\frac{1}{2})^2}$

$= \frac{-1}{\frac{1}{4}}$

$= -4$

$y - b = m(x - a)$

$y - 2 = -4(x - \frac{1}{2})$

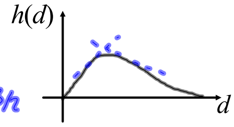
$y - 2 = -4x + 2$

$y + 4x - 4 = 0$

Exercise 6J

A designer of an artificial ski slope describes the shape of the slope by the function

$$h(d) = \frac{4d - 2d^{\frac{3}{2}}}{\sqrt{d}}$$



$= \frac{4d - 2d^{3/2}}{d^{1/2}}$

$= \frac{4d^1}{d^{1/2}} - \frac{2d^{3/2}}{d^{1/2}}$

d is the horizontal distance

$h(d)$ is the height in metres.

Calculate the gradient of the slope 4 m horizontally from the start of the slope.

$= 4d^{1/2} - 2d$

$h'(d) = 2d^{-1/2} - 2$

$= \frac{2}{d^{1/2}} - 2$

$= \frac{2}{\sqrt{d}} - 2$

$h'(4) = \frac{2}{\sqrt{4}} - 2$

$= \frac{2}{2} - 2$

$= -1$

4) The gradient of a tangent to the curve $y = x^4 + 1$ is 32

Find the point of contact of the tangent.

$\frac{dy}{dx} = 4x^3 = 32$

$x^3 = 8$

$x = 2$

$y = 2^4 + 1$

$= 16 + 1$

$= 17$

$(2, 17)$

Starter

A curve has equation $y = 3x^2 - 7x - 2$.

What is the gradient of the tangent at the point where $x = 3$?

A 3

B 4

C 9

D 11

$\frac{dy}{dx} = 6x - 7$

where $x = 3$

$m = 18 - 7$

$= 11$

Continue Ex 6J

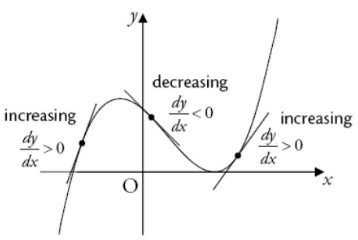
Increasing and Decreasing Functions

Today's Learning:

By the end of this lesson, you should be able to determine whether a function is increasing or decreasing.

When a curve is **increasing**, the tangents will slope upwards, the gradients are positive, so $\frac{dy}{dx} > 0$

When a curve is **decreasing**, the tangents will slope downwards, the gradients are negative, so $\frac{dy}{dx} < 0$



Examples

1) Find the intervals for which the function $y = 3x^2 + 2x - 5$ is increasing and decreasing.

Function is increasing when $\frac{dy}{dx} > 0$

$\frac{dy}{dx} = 6x + 2$

$6x + 2 > 0$

$6x > -2$

$x > -\frac{1}{3}$

Function is decreasing when $\frac{dy}{dx} < 0$

$x = -\frac{1}{3}$

$x < -\frac{1}{3}$

2) Find the intervals for which the function $f(x) = x^3 - 3x^2 + 8$ is increasing and decreasing.

Function is increasing when $f'(x) > 0$

$f'(x) = 3x^2 - 6x$

$3x^2 - 6x > 0$

$3x(x - 2) > 0$

Function is decreasing when $f'(x) < 0$

$f'(x) = 3x^2 - 6x$

$3x^2 - 6x < 0$

$3x(x - 2) < 0$

increasing

$x < 0$ or $x > 2$

decreasing

$0 < x < 2$

3) Show that the function $f(x) = x^3 - 3x^2 + 3x - 10$ is never decreasing.

Function is **not** decreasing if $f'(x) \geq 0$

$f'(x) = 3x^2 - 6x + 3$ $= 3(x^2 - 2x + 1)$

$= 3(x^2 - 2x + 1)$ $= 3(x-1)(x-1)$

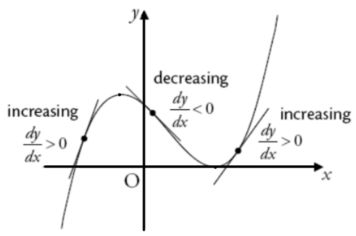
$= 3(x - 1)^2$ $= 3(x-1)^2$

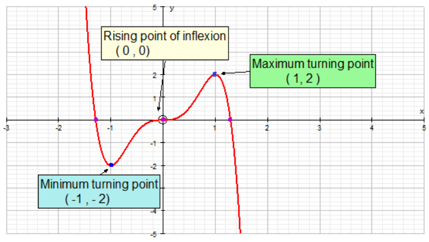
$3(x - 1)^2 \geq 0$ for all values of x, so the function is never decreasing.

ex 6L

Stationary Points

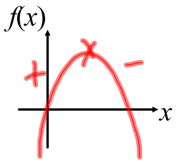
Today's Learning:
By the end of this lesson, you should be able to find stationary points and determine their nature.



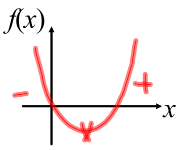


Stationary points occur when $f'(x) = 0$.

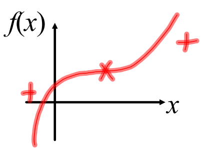
The nature of a stationary point depends on the gradient on either side of it.



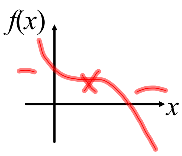
Maximum turning point



Minimum turning point



Rising point of inflection



Falling point of inflection

Example

1) Find the stationary points and determine the nature of $y = 4x^3 - x^4$

SPs occur when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 12x^2 - 4x^3$

$12x^2 - 4x^3 = 0$

$4x^2(3 - x) = 0$

Either $4x^2 = 0$ or $3 - x = 0$

$x = 0$ $\quad$ $x = 3$

When $x = 0$, $y = 4 \times 0^3 - 0^4$ (0,0)

$y = 0$

When $x = 3$, $y = 4 \times 3^3 - 3^4$ (3,27)

$y = 27$

$y = 4x^3 - x^4$ $\quad \frac{dy}{dx} = 12x^2 - 4x^3$ $\quad$ (0,0) (3,27)

x	0^-	0	0^+
$\frac{dy}{dx}$	+	0	+
Slope	/	—	/

x	3^-	3	3^+
$\frac{dy}{dx}$	+	0	−
Slope	/	—	\

$x = -2$

$\frac{dy}{dx} = 12 \times 4 - 4 \times -8$

$= 48 + 32$

$x = 1$

$dy/dx = 12 \times 1 - 4 \times 1$

$= 12 - 4 = 8$

$x = 5$

$\frac{dy}{dx} = 12 \times 25 - 4 \times 125$

$= 300 - 500$

so, (0,0) is a rising point of inflection.

(3,27) is a max turning point.

Ex 6M

Step 1 Differentiate

Step 2 Set equal to 0 and solve

Step 3 Sub in the x coordinate into original equation (y = ...) and find the y coordinate.

Step 4 Find the nature

Step 5 State coordinates and their nature

Curve Sketching

Today's Learning:

By the end of this lesson, you should be able to sketch and fully annotate a curve.

To sketch a curve we need:

- ☆ the y intercept
- ☆ the x intercept
- ☆ the stationary points and their nature.

Examples

1) Sketch the curve $y = x^3 - 3x^2$

Step 1 - Find where the graph cuts the y axis.

When $x = 0$, $\quad y = 0^3 - 3 \times 0^2$

$y = 0$ $\quad$ (0,0)

Step 2 - Find where the graph cuts the x axis.

When $y = 0$, $\quad x^3 - 3x^2 = 0$

$x^2(x - 3) = 0$

Either $x^2 = 0$ or $x - 3 = 0$

$x = 0$ $\quad x = 3$

(0,0) and (3,0)

Step 3 - Find the stationary points $\quad y = x^3 - 3x^2$

SPs occur when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 3x^2 - 6x$

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

Either $3x = 0$ or $x - 2 = 0$

$x = 0$ $\quad x = 2$

When $x = 0$, $\quad y = 0^3 - 3 \times 0^2$

$y = 0$ $\quad$ (0,0)

When $x = 2$, $\quad y = 2^3 - 3 \times 2^2$

Step 4 - Determine the nature of the SPs. $\quad$ (0,0) $\quad$ (2,-4)

$\frac{dy}{dx} = 3x^2 - 6x$

x			
$\frac{dy}{dx}$			
Slope			

x			
$\frac{dy}{dx}$			
Slope			

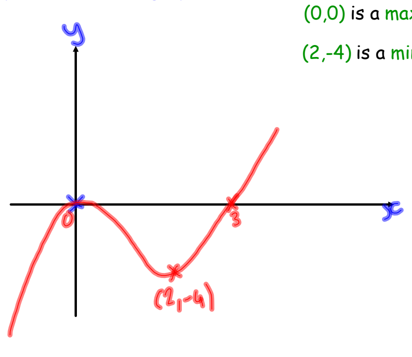
so, (0,0) is a max turning point

Step 5 - Sketch the graph.

(0,0) is a max turning point

(2,-4) is a min turning point

(0,0) and (3,0)



ex 6N
pg 107

Ex 6N

2) Sketch the curve $y = 2x^3 - 3x^2 - 12x + 5$

Step 1 - Find where the graph cuts the y axis.

When $x = 0$, $\quad y = 0^3 - 3 \times 0^2 - 12 \times 0 + 5$

$y = 5$ (0,5)

Step 2 - Find where the graph cuts the x axis.

When $y = 0$, $\quad 2x^3 - 3x^2 - 12x + 5 = 0$

We can't factorise this until Unit 2, so we can't find the x intercept just yet.

Step 3 - Find the stationary points.

SPs occur when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 6x^2 - 6x - 12$

$6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$6(x + 1)(x - 2) = 0$

Either $x + 1 = 0$ or $x - 2 = 0$

$x = -1 \qquad x = 2$

When $x = -1$, $y = (2 \times (-1)^3) - (3 \times (-1)^2) - (12 \times -1) + 5$

$y = 12$

When $x = 2$, $y = (2 \times 2^3) - (3 \times 2^2) - (12 \times 2) + 5$

$y = -15$

(-1,12) and (2,-15)

Step 4 - Determine the nature of the SPs.

x	
$\frac{dy}{dx}$	
Slope	

so, (-1,12) is a max turning point

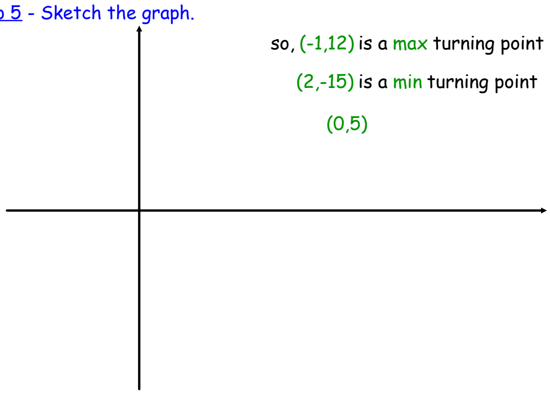
(2,-15) is a min turning point

Step 5 - Sketch the graph.

so, (-1,12) is a max turning point

(2,-15) is a min turning point

(0,5)



Ex 6N page 107

Starter

03/10/17

1. Given $f(x) = 3x^2(2x - 1)$, find $f'(-1)$. (3)

$= 6x^3 - 3x^2$

$f'(x) = 18x^2 - 6x$

$f'(-1) = 18(-1)^2 - 6(-1)$

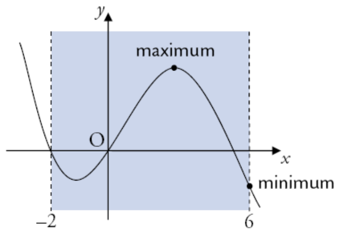
$= 18 - (-6) = 24$

03/10/17 Closed Intervals

Learning Intention
By the end of this lesson, you should be able to find the maximum and miniumum values of a function within a closed interval.

We can restrict the part of a graph we are looking at using a closed interval. The maximum and minimum values can be at stationary points or at end points of the closed interval.

Below is a sketch of a curve, with the closed interval $-2 \le x \le 6$ shaded.



Example

1)
A function f is defined for $x \in R$ by $f(x) = 2x^3 - 5x^2 - 4x + 1$.

Find the maximum and minimum value of $f(x)$ where $-1 \le x \le 4$.

Step 1: Find the stationary points

SPs occur when $f'(x) = 0$

$f'(x) = 6x^2 - 10x - 4$

$6x^2 - 10x - 4 = 0$

$2(3x^2 - 5x - 2) = 0$

$2(3x + 1)(x - 2) = 0$

Either $3x + 1 = 0$ or $x - 2 = 0$

$-1 \div 3$

$x = -\frac{1}{3}$ $x = 2$

When $x = -\frac{1}{3}$, $y = 2x^3 - 5x^2 - 4x + 1$

$= -\frac{2}{27} - \frac{5}{9} + \frac{4}{3} + 1$

$= \frac{46}{27}$

When $x = 2$, $y = 2x^3 - 5x^2 - 4x + 1$

$= (2 \times 2^3) - (5 \times 2^2) - (4 \times 2) + 1$

$= -11$

$(-\frac{1}{3}, \frac{46}{27})$ and $(2,-11)$

$(-\frac{1}{3}, \frac{46}{27})$ and $(2,-11)$ $f'(x) = 6x^2 - 10x - 4$

Step 2: Find the nature of the stationary points

x	$-\frac{1}{3}^-$	$-\frac{1}{3}$	$-\frac{1}{3}^+$	2^-	2	2^+
$f'(x)$	+	0	−	−	0	
Slope	/	—	\	\	—	/

$6(-1)^2 - 10(-1) - 4 = 12$
$6(0)^2 - 10(0) - 4 = -4$

$6(3)^2 - 10(3) - 4 = 20$

so, $(-\frac{1}{3}, \frac{46}{27})$ is a max turning point

$(2,-11)$ is a min turning point

Step 3: Find the extremities by substituting in x coordinates

$f(x) = 2x^3 - 5x^2 - 4x + 1$ $-1 \le x \le 4$.

Points at extremities of closed interval

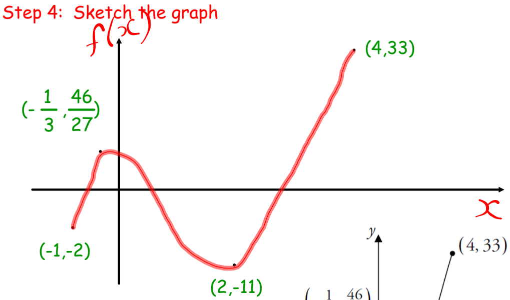
$f(-1) = (2 \times (-1)^3) - (5 \times (-1)^2) - (4 \times (-1)) + 1 = -2$

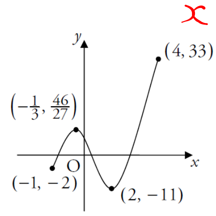
$(-1,-2)$

$f(4) = (2 \times 4^3) - (5 \times 4^2) - (4 \times 4) + 1 = 33$

$(4,33)$

Step 4: Sketch the graph



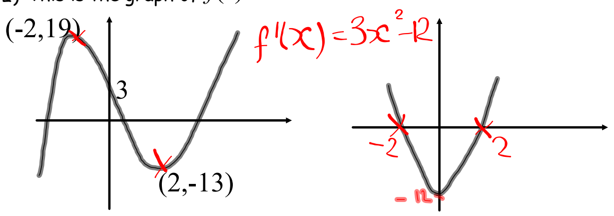


The max value is 33 when $x = 4$.

The min value is -11 when $x = 2$.

Ex 6O Q2

2) This is the graph of $f(x) = x^3 - 12x + 3$



Find $f'(x)$ and sketch the graph of $f'(x)$.

03/10/17 Graph of the Derived Function

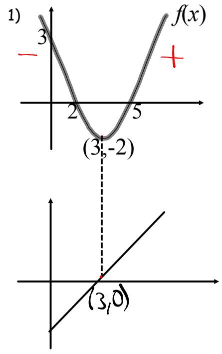
Learning Intention

By the end of this lesson, given the graph of $f(x)$, you should be able to draw the graph of $f'(x)$.

Examples

Sketch the graph of $f'(x)$

1)



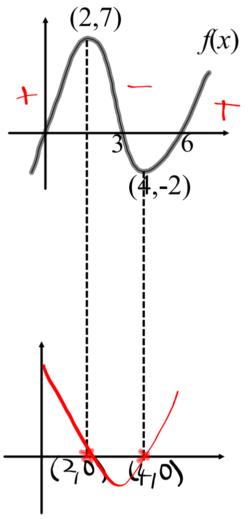
Negative gradient - below x axis

Positive gradient - above x axis

Steps

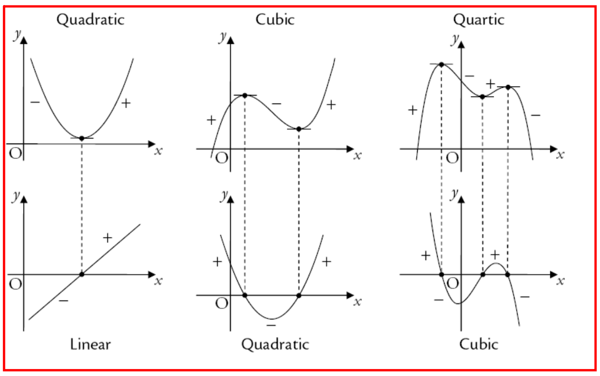
1. Plot the x coordinate of the stationary point onto the new graph.
2. Identify if first part of the graph is +ve or -ve
3. Identify if second part of the graph is +ve or -ve

2) Sketch the graph of $f'(x)$



Steps

1. Plot the x coordinate of the stationary point onto the new graph.
2. Identify if first part of the graph is +ve or -ve
3. Identify if second part of the graph is +ve or -ve



Differentiation of $\sin x$ and $\cos x$

Learning Intention

By the end of this lesson, you should be able to differentiate $\sin x$ and $\cos x$.

There are two new rules which are used to differentiate expressions involving trigonometric functions:

$\frac{d}{dx}(\sin x) = \cos x$ $\quad\quad$ $\frac{d}{dx}(\cos x) = -\sin x$

Examples

1) Differentiate $y = 3\sin x$ with respect to x.

$$y = 3\sin x$$

$$\frac{dy}{dx} = 3\cos x$$

$\frac{d}{dx}(\sin x) = \cos x$ $\quad\quad$ $\frac{d}{dx}(\cos x) = -\sin x$

2) A function f is defined by $f(x) = \sin x - 2\cos x$ for $x \in$ R.
Find $f'(\frac{\pi}{3})$.

$$f(x) = \sin x - 2\cos x$$

$$f'(x) = \cos x - (-2\sin x)$$

$$= \cos x + 2\sin x$$

$$f'(\tfrac{\pi}{3}) = \cos\frac{\pi}{3} + 2\sin\frac{\pi}{3}$$

Using Exact Values

$$= \frac{1}{2} + 2\frac{\sqrt{3}}{2}$$

$$= \frac{1}{2} + \sqrt{3}$$

3) Find the equation of the tangent to the curve $y = \sin x$ when $x = \frac{\pi}{6}$.

Remember we need a point and the gradient.

At $x = \frac{\pi}{6}$ $\quad y = \sin\frac{\pi}{6} = \frac{1}{2}$ $\quad (\frac{\pi}{6}, \frac{1}{2})$

$$\frac{dy}{dx} = \cos x$$

$$= \cos\frac{\pi}{6}$$

$$= \frac{\sqrt{3}}{2}$$

$$y - b = m(x - a)$$

$$y - \frac{1}{2} = \frac{\sqrt{3}}{2}(x - \frac{\pi}{6})$$

×2 ×2

$$2y - 1 = \sqrt{3}(x - \frac{\pi}{6})$$

$$2y - 1 = \sqrt{3}x - \frac{\sqrt{3}\pi}{6}$$

$$0 = \sqrt{3}x - 2y - \frac{\sqrt{3}\pi}{6} + 1$$

4) Find the derivative of $f(x) = \dfrac{1 - x\cos x}{2x}$

$$f(x) = \frac{1}{2x} - \frac{x\cos x}{2x}$$

$$= \frac{1}{2x} - \frac{\cos x}{2}$$

$$= \frac{1}{2}x^{-1} - \frac{1}{2}\cos x$$

$$f'(x) = -\frac{1}{2}x^{-2} + \frac{1}{2}\sin x$$

$$= -\frac{1}{2x^2} + \frac{1}{2}\sin x$$

5) Find the gradient of the tangent to the curve $y = 2\cos x$ at $x = \frac{2\pi}{3}$

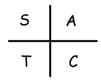
$$\frac{dy}{dx} = -2\sin x$$

At $x = \frac{2\pi}{3}$

$$\frac{dy}{dx} = -2\sin\frac{2\pi}{3}$$

$$= -2 \times \frac{\sqrt{3}}{2}$$

$$= -\sqrt{3}$$



At $x = \frac{2\pi}{3}$

$$y = 2\cos\frac{2\pi}{3}$$

$$= 2 \times -\frac{1}{2}$$

$$= -1$$

$(\frac{2\pi}{3}, -1)$

$$y - b = m(x - a)$$

$$y + 1 = -\sqrt{3}(x - \frac{2\pi}{3})$$

Attachments

maths1_2.ppt

maths1_3.ppt