Starter

1) Fully factorise $4y^2 - 5y - 6$ $(4y+3)(y-2)$

2) Expand the brackets and simplify: $(m + 4)(2m - 3)$ $2m^2-3m+8m-12$ $=2m^2+5m-12$

3) Calculate 20% of 340 without a calculator. 68

4) What is 40ml increased by 20% ? 48ml

Today's Learning:

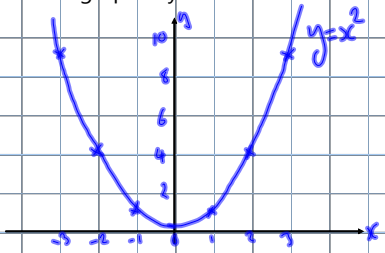
To find the equation of quadratic graphs using substitution of a point.

Quadratic Graphs

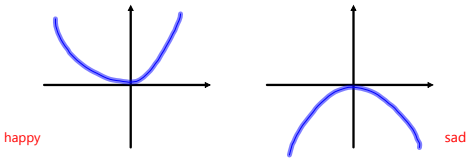
A quadratic equation involves a squared term

e.g. $3x^2 + 2x - 3 = 0$

The simplest quadratic graph is $y = x^2$



The graph of $y = kx^2$



Positive k Negative k

$y = kx^2$ graph is the $y = x^2$ graph stretched ↕ by a factor of k

Starter

1) Factorise fully: $2x^2 + x - 10$ $(2x+5)(x-2)$

2) Without a calculator, find $2.3 \times 10^5 \times 3 \times 10^{-2}$ 6.9×10^3

3) Without a calculator, simplify $\frac{912}{18}$ $= \frac{304}{6} = \frac{152}{3}$ $\frac{152}{3}$

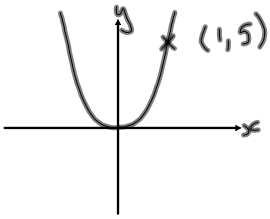
Today's Learning:

To continue to consider transformations of quadratic graphs.

$y=2x^2$

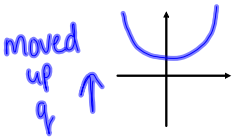
$y=-4x^2$

e.g. Find the equation of the graph of the form $y = kx^2$

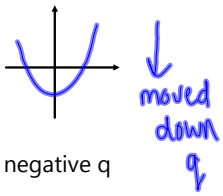


$y = kx^2$
$5 = k \times 1^2$
$5 = 1k$
$k = 5$
$y = 5x^2$

$y = x^2 + q$



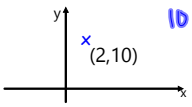
positive q



negative q

Starter

1) Find p for the graph $y = x^2 + p$, shown below:



$10 = 2^2 + p$
$10 = 4 + p$
$6 = p$

2) Without a calculator, find a fifth of 70. 14

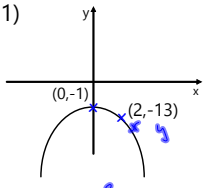
3) Fully factorise: $3g^2 - 23g + 30$ $(3g - 5)(g - 6)$

4) Multiply out the brackets: (e + 2)(e + 3)(e - 1)

$(e^2 + 5e + 6)(e - 1)$
$= e^3 + 5e^2 + 6e - e^2 - 5e - 6$
$= e^3 + 4e^2 + e - 6$

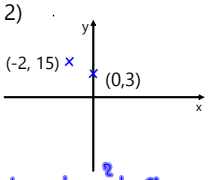
e.g. Find k and q from the graphs of $y = kx^2 + q$:

1)



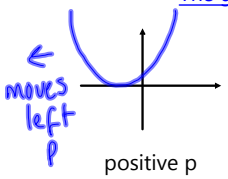
$-1 = k \times 0^2 + q$
$-1 = q$
$y = kx^2 - 1$
$-13 = k \times 4 - 1$
$-13 = 4k - 1$
$-12 = 4k$
$k = -3$
$y = -3x^2 - 1$

2)

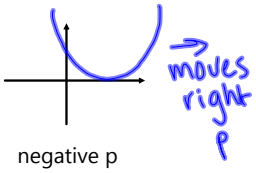


$y = kx^2 + q$
$3 = k \times 0^2 + q$
$3 = q$
$y = kx^2 + 3$
$15 = k \times (-2)^2 + 3$
$15 = 4k + 3$
$12 = 4k$
$k = 3$
$y = 3x^2 + 3$

The graph of $y = (x + p)^2$



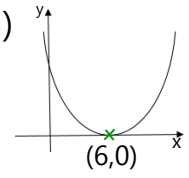
positive p



negative p

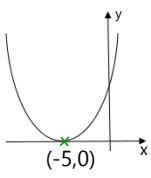
e.g. Find p for these graphs of $y = (x + p)^2$:

a)



$p = -6$

b)



$p = 5$

Starter

1) Fully factorise: $3m^2 + 12m + 9$ $3(m+1)(m+3)$

$(3m+3)(m+3)$

2) Simplify the following:

a) $\sqrt{40} + \sqrt{160}$ b) $\frac{x^4}{x^3 \times x}$

a) $\sqrt{4\times10} + \sqrt{16\times10}$

$= 2\sqrt{10} + 4\sqrt{10}$

$= 6\sqrt{10}$

$= \frac{x^4}{x^4} = 1$

3) Without a calculator, find 53 x 31

53
×31
53
1590
1643

Starter

1) Find a and b, given:

2a - b = 2

a + b = 7

$3a = 9$

$a = 3$

$b = 4$

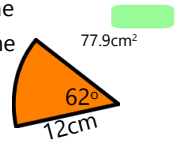
2) Calculate $3 \times 10^4 \times 7 \times 10^2$, giving your answer in scientific notation

21×10^6

$= 2.1 \times 10^7$

3) Round 304.56 to 3 sig. fig. 305

4) Find the area of the sector:



$\frac{62}{360} \times \pi \times 12^2$

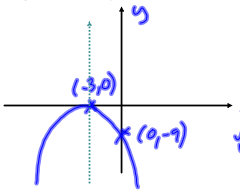
78cm^2

Sketching Quadratic Graphs 23/8/18

We can be asked to label:

- Turning Point and its nature
- Roots (where it crosses the x-axis)
- y-intercept
- Equation of the axis of symmetry

e.g. 1) Sketch $y = -(x + 3)^2$ and label all of the above.



TP: (-3,0)

maximum TP

Roots: (-3,0)

y-int: set $x = 0$

$y = -(x+3)^2$

$= -(0+3)^2$

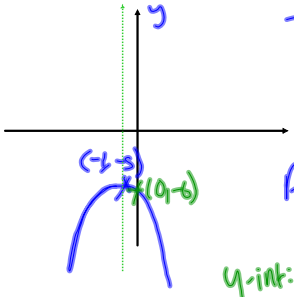
$= -(3)^2$

$= -9$

(0,-9)

axis of symmetry: $x = -3$

e.g. 2) Sketch the graph of $y = -(x + 1)^2 - 5$



TP: maximum TP at (-1,-5)

No roots.

Axis of symmetry $x = -1$

y-int: set $x = 0$

$y = -(x+1)^2 - 5$

$= -(1)^2 - 5$

$= -1 - 5$

$= -6$

(0,-6)

Starter

1) Write down the y-intercept of the line $2y = 3 - 2x$

$y = -x + 1.5$ $y = \frac{3-2x}{2}$

1.5 $y = 1.5 - x$

2) Without a calculator, find a fifth of 22 4.4

3) Simplify $3e^4 \times 2e^{-2}$ $6e^2$

4) What is the difference between -4 and 7? 11

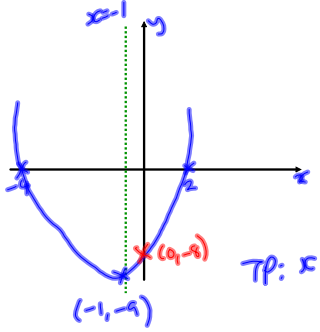
Today's Learning:

Sketching quadratic graphs.

$$a \times b = 0$$

What can you say about a and b?

e.g. 3) Sketch the graph of $y = (x - 2)(x + 4)$



Roots: set $y = 0$

$0 = (x-2)(x+4)$

$x - 2 = 0$ or $x + 4 = 0$

$x = 2$ $x = -4$

Line of symmetry: $x = -1$

TP: $x = -1$

$y = (-1-2)(-1+4)$

$= (-3)(3)$

$= -9$ $(-1,-9)$

minimum TP

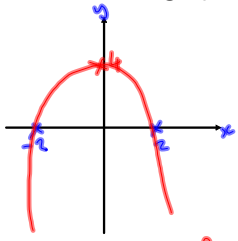
y-intercept: $x = 0$

$y = (0-2)(0+4)$

$= (-2)(4)$

$= -8$

e.g. 4) Sketch the graph of $y = -(x + 2)(x - 2)$



Roots: set $y = 0$

$0 = -(x+2)(x-2)$

$x + 2 = 0$ or $x - 2 = 0$

$x = -2$ $x = 2$

Line of symmetry: $x = 0$

TP: $x = 0$ (from line of symmetry)

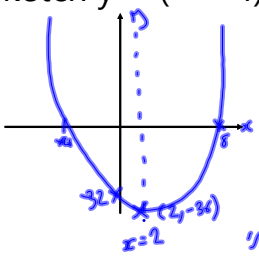
$y = -(0+2)(0-2)$

$= -(2)(-2)$ $(0,4)$

$= 4$

maximum TP

Sketch $y = (x + 4)(x - 8)$



Roots: $(-4,0)$ $(8,0)$

Line of symmetry: $x = 2$

TP: $(2,-36)$

minimum TP

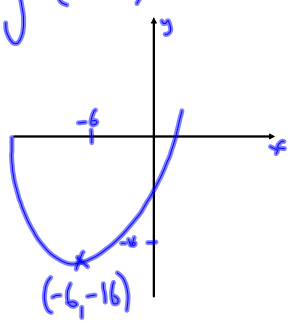
y-int: $(0,-32)$

Starter

a) Write the expression $(x + 10)(x + 2)$ in completed square form. $= x^2 + 12x + 20 = (x+6)^2 - 16$

b) Hence sketch the graph $y = (x + 10)(x + 2)$, marking the coordinates of the turning point and the nature of the turning point.

$y = (x+6)^2 - 16$



$a \times b = 0$

How do we solve $(x + 4)(x - 1) = 0$ for x?

$x + 4 = 0$ or $x - 1 = 0$

$x = -4$ or $x = 1$

How might we solve $x^2 - x - 6 = 0$

6

1, 6

3, 2

$(x-3)(x+2) = 0$

$x = 3$ or -2

Solving Quadratic Equations 4/9/17

A quadratic equation can be written as $ax^2 + bx + c = 0$

Then, we can solve by factorising.

Examples:

1) $x^2 - 2x - 35 = 0$

$(x-7)(x+5) = 0$

$x = 7$ or $x = -5$

2) $2x^2 + 10x = 0$

$2x(x+5) = 0$

$2x = 0$ or $x+5 = 0$

$x = 0$ $x = -5$

Example:

Solve $2x^2 + 5x + 3 = 0$

$(2x+3)(x+1) = 0$

$2x^2 + 2x + 3x + 3$

$2x + 3 = 0$ or $x + 1 = 0$

$-3 \quad -3 \qquad x = -1$

$2x = -3$

$x = -1.5$

Starter

Rewrite with a positive index.

a) u^{-1} b) v^{-5} c) $6p^{\frac{-1}{4}}$ d) $\frac{1}{3}a^{\frac{-3}{4}}$ e) $5f^{-3}$

Rewrite in index form.

a) $\sqrt{p}$ b) $\sqrt[3]{a}$ c) $\sqrt[8]{y}$ d) $\sqrt[5]{n^3}$ e) $\frac{2}{\sqrt[4]{y}}$ f) $\frac{3}{\sqrt[5]{c^6}}$

Today's Learning:

To write any quadratic equation in the form $ax^2 + bx + c = 0$ and to solve equations that don't factorise by using the **quadratic formula**.

The Quadratic Formula

If we have an equation $ax^2 + bx + c = 0$ that we can't factorise, we can use the Quadratic Formula to find solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ (given in exams)

Examples:

1) $x^2 - 5x - 14 = 0$ $a=1$ $b=-5$ $c=-14$

$x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$

$= \frac{5 \pm \sqrt{25 + 56}}{2}$

$= \frac{5 \pm \sqrt{81}}{2}$

$= \frac{5 \pm 9}{2}$

$x = \frac{14}{2} = 7$ or $x = \frac{-4}{2} = -2$

2) $x^2 + 4x + 1 = 0$ $a=1$ $b=4$ $c=1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16 - 4(1)(1)}}{2(1)}$

$= \frac{-4 \pm \sqrt{16-4}}{2}$

$= \frac{-4 \pm \sqrt{12}}{2}$

$x = -0.27$ or -3.73 (2dp)

Starter

1) Given $f(x) = x^2 - 4$, evaluate f(3)

$f(3) = 3^2 - 4$

$= 9 - 4$

$= 5$

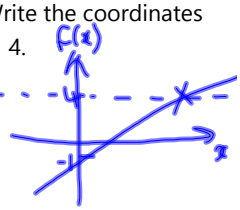
2) Sketch the graph $f(x) = 2x - 1$. Write the coordinates where this line meets the line $f(x) = 4$.

$2x - 1 = 4$

$2x = 5$

$x = 2.5$

$(2.5, 4)$



3) Given $f(x) = 3x + 10$, find x such that $f(x) = 14.8$.

$f(x) = 3x + 10$

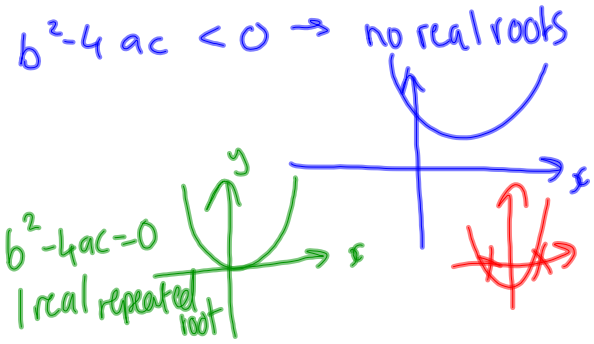
$14.8 = 3x + 10$

$4.8 = 3x$

$x = 1.6$

How can we tell how many roots an equation has?

$b^2-4ac<0 \rightarrow$ no real roots



Starter

Solve using the Quadratic Formula, giving answers to 2 decimal places:

a) $4x^2-11=0$ b) $(x+5)^2=7$ c) $4x(x-3)+2=0$

$(x+5)(x+5)=7$

$x^2+10x+25=7$

$x^2+10x+18=0$

$4x^2-12x+2=0$

$x=\frac{12\pm\sqrt{(-12)^2-4(4)(2)}}{2(4)}$

$=\frac{12\pm\sqrt{112}}{8}$

The Discriminant

For a quadratic equation $ax^2 + bx + c = 0$ the discriminant is $b^2 - 4ac$.

$b^2 - 4ac > 0$ means 2 real, distinct roots

$b^2 - 4ac = 0$ means 2 real, equal roots

$b^2 - 4ac < 0$ means no real roots

e.g. 1) Determine the nature of the roots of $2(x + 1) = x^2 - 3$

$2x+2=x^2-3$

$-x^2+2x+5=0$

$b^2-4ac=2^2-4(-1)(5)$

$=4+20$

$=24$

$24>0$ so 2 real distinct roots.

e.g. 2) Find the range of values for T such that $x^2 + 2x - 2T = 0$ has 2 real, distinct roots.

$a=1\ b=2\ c=-2T$

$b^2-4ac=4-4(1)(-2T)$

$=4+8T$

$4+8T>0$

$-4\quad -4$

$8T>-4$

$T>\frac{-4}{8}$

$T>\frac{-1}{2}$

Starter

Rationalise the denominator:

$\frac{4}{(\sqrt{5}+\sqrt{2})}\times\frac{(\sqrt{5}-\sqrt{2})}{(\sqrt{5}-\sqrt{2})}$

$\frac{4(\sqrt{5}-\sqrt{2})}{5-\sqrt{10}+\sqrt{10}-2}$

$\frac{4(\sqrt{5}-\sqrt{2})}{3}$

The **areas** of these rectangles are equal.

a) Find the value of x. b) Calculate the area of the rectangles.

72cm²

$(2x + 2)$ cm by $(x + 1)$ cm

72cm²

$(x + 4)$ cm by $(x + 3)$ cm

$(2x+2)(x+1)$

$=2x^2+4x+2$

$(x+3)(x+4)$

$=x^2+7x+12$

$2x^2+4x+2=x^2+7x+12$

$x^2+4x+2=7x+12$

$x^2+2=3x+12$

$x^2=3x+10$

$x^2-3x-10=0$

$(x+2)(x-5)=0$

$x=-2$ or 5

not possible

$x=5$

Starter

1) Given the function f(x) = (5 - x)², evaluate:

a) f(3) b) f(-1)

$(5-3)^2$ $(5--1)^2$

4 $6^2 = 36$

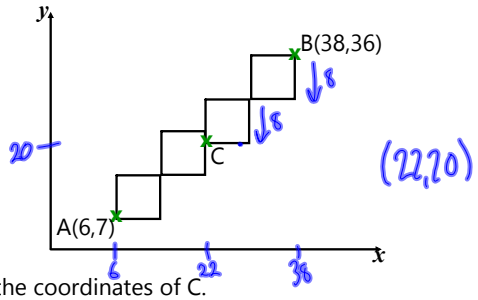
2) Multiply out the brackets and simplify:

(w + 1)(w - 1)(w + 5)

$(w^2-1)(w+5) = w^3+5w^2-w-5$

Starter

A pattern is made from four identical squares. The sides of the squares are parallel to the axes.



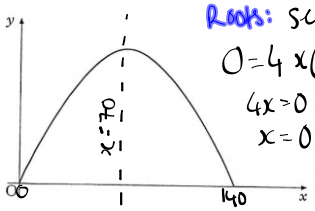
Work out the coordinates of C.

The profit made by a publishing company of a magazine is calculated by the formula

$$y = 4x(140 - x),$$

where y is the profit (in pounds) and x is the selling price (in pence) of the magazine.

The graph below represents the profit y against the selling price x.



Roots: set $y = 0$

$0 = 4x(140 - x)$

$4x = 0$ or $140 - x = 0$

$x = 0$ $x = 140$

Find the maximum profit the company can make from the sale of the magazine.

$y = 4x(140 - x)$

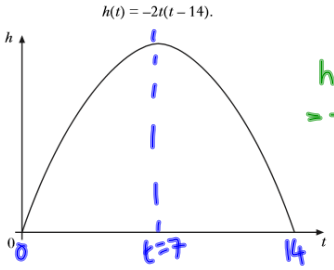
$= 4(70)(70)$

$= 19\,600$ £19 600

The diagram below shows the path of a rocket which is fired into the air.

The height, h metres, of the rocket after t seconds is given by

$$h(t) = -2t(t - 14).$$



$h(t) = -2t(t-14)$

$= -2(7)(-7)$

$= 98$

(a) For how many seconds is the rocket in flight? 14 sec

(b) What is the maximum height reached by the rocket? 98 m

Starter

Simplify:

$$\frac{(a^2)^3 \times a^{-2}}{a^5 \times a^{-5}}$$

$$\frac{a^6 \times a^{-2}}{a^0}$$

$$= \frac{a^6 \times a^{-2}}{1}$$

$$= \frac{a^4}{1} = a^4$$