

QUESTION C-1

$$\begin{aligned} 1. a) \vec{wY} &= \underline{y} - \underline{w} = \begin{pmatrix} 3 \\ 13 \\ 6 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 5 \\ 10 \\ 5 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} b) \vec{xY} &= \underline{y} - \underline{x} = \begin{pmatrix} 3 \\ 13 \\ 6 \end{pmatrix} - \begin{pmatrix} -1 \\ 5 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 4 \\ 8 \\ 4 \end{pmatrix} \end{aligned}$$

collinear since Y is common to both \vec{wY} and \vec{xY} and

$$\vec{xY} = \frac{4}{5} \vec{wY}$$

$$\text{or} \\ 5\vec{xY} = 4\vec{wY}$$

$$2. A(5, 3, 1) \quad C(9, 7, 13)$$

$$B = \frac{1}{4} \underline{a} + \frac{3}{4} \underline{c}$$

$$= \frac{1}{4} \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \frac{3}{4} \begin{pmatrix} 9 \\ 7 \\ 13 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{5}{4} \\ \frac{3}{4} \\ \frac{1}{4} \end{pmatrix} + \begin{pmatrix} \frac{27}{4} \\ \frac{21}{4} \\ \frac{39}{4} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{32}{4} \\ \frac{24}{4} \\ \frac{40}{4} \end{pmatrix} = \begin{pmatrix} 8 \\ 6 \\ 10 \end{pmatrix}$$

$$\underline{B(8, 6, 10)}$$

$$3. a) \vec{ST} \cdot \vec{SV} = (-2 \times 1) + (3 \times 1) + (0 \times 3) \\ = -2 + 3 + 0 \\ = 1$$

$$b) \cos \theta = \frac{\vec{ST} \cdot \vec{SV}}{|\vec{ST}| |\vec{SV}|}$$

$$= \frac{1}{\sqrt{13} \sqrt{11}} \\ = 0.0836.$$

$$\theta = 85.2^\circ$$

$$|\vec{ST}| = \sqrt{(-2)^2 + 3^2 + 0^2} \\ = \sqrt{13}$$

$$|\vec{SV}| = \sqrt{1^2 + 1^2 + 3^2} \\ = \sqrt{11}$$

$$4 \quad P(-1, -1, 0) \quad R(5, 2, -3) \quad 2:1$$

$$= \frac{1}{3} \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} + \frac{2}{3} \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{10}{3} \\ \frac{4}{3} \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{9}{3} \\ \frac{3}{3} \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} \quad Q(3, 1, -2)$$



$$5 \quad R(-1, -8, -2) \quad S(2, -5, 4) \quad T(3, -4, 6)$$

$$\begin{aligned} \vec{RS} &= S - R & \vec{ST} &= T - S \\ &= \begin{pmatrix} 2 \\ -5 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ -8 \\ -2 \end{pmatrix} & &= \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 2 \\ -5 \\ 4 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix} & &= \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \end{aligned}$$

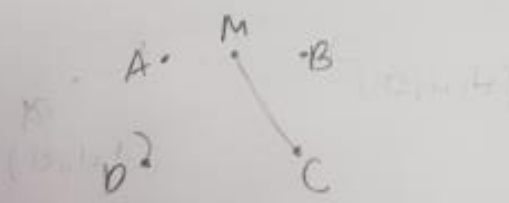
Since $\vec{RS} = 3\vec{ST}$ and \vec{RS} and \vec{ST} share the point S , R, S, T are collinear

$$6. \quad A(4, -1, 3) \quad B(8, 3, -1) \quad C(0, 4, 4) \quad D(-4, 0, 8)$$

$$a) \quad \text{midpoint } AB = \left(\frac{4+8}{2}, \frac{-1+3}{2}, \frac{3+(-1)}{2} \right)$$

$$M = (6, 1, 1)$$

b)



$$= \frac{2}{3} \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix} = T(4, 2, 2)$$

7.



$$= (-1, 3, 4) - (3, 1, -2)$$

$$= (1, 2, 6)$$

$$\vec{AB} = 2\vec{BC}$$

$$\begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$$

$$\vec{AB} = b - a$$

$$= \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -2 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$$

$$\underline{\underline{(5, 0, -5)}}$$

8. a) $(6, 6, 0)$

b) $\vec{DA} = a - d$

$$= \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$$

$$\vec{DB} = b - d$$

$$= \begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$$

c) $\cos(\theta) = \frac{\vec{DA} \cdot \vec{DB}}{|\vec{DA}| |\vec{DB}|}$

$$= \frac{64}{\sqrt{82} \sqrt{82}}$$

$$= 0.78 \dots$$

$$\theta = 38.17^\circ$$

$$\vec{DA} \cdot \vec{DB} =$$

$$(3 \times 3) + (-3 \times 3) + (-8 \times -8)$$

$$= 9 - 9 + 64$$

$$= 64$$

$$|\vec{DA}| = \sqrt{3^2 + (-3)^2 + (-8)^2}$$

$$= \sqrt{82}$$

$$|\vec{DB}|$$

$$= \sqrt{82}$$

$$\begin{aligned}
 9a) \quad & \log_a 40 - \log_a 5 \\
 & = \log_a \left(\frac{40}{5} \right) \\
 & = \underline{\log_a 8}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \log_{12} 3 + 2 \log_{12} 2 \\
 & = \log_{12} 3 + \log_{12} 2^2 \\
 & = \log_{12} 3 + \log_{12} 4 \\
 & = \log_{12} (3 \times 4) \\
 & = \log_{12} 12 \\
 & = \underline{\underline{1}}
 \end{aligned}$$

$$10. \quad e^x = 1.9$$

$$\log_e 1.9 = x$$

$$\underline{\underline{x = 0.64}} \text{ (to 2dp)}$$

$$11. \quad \log_3 (x-5) = 2$$

$$x-5 = 3^2$$

$$x-5 = 9$$

$$\underline{\underline{x = 14}}$$

$$\begin{aligned}
 12. a) \quad & \log_5 6a + \log_5 7b \\
 & = \log_5 (42ab)
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \log_b x^7 - \log_b x^4 \\
 & = \log_b \frac{x^7}{x^4} \\
 & = \log_b x^3 \\
 & = 3 \log_b x
 \end{aligned}$$

$$\begin{aligned}
 13. \quad & \log_4 (x-1) = 3 \\
 & (x-1) = 4^3 \\
 & x-1 = 64 \\
 & \underline{\underline{x = 65}}
 \end{aligned}$$