Revision C2

1. $5\sin x + 4\cos x = K\sin(x+\alpha)^\circ$

$= K\sin x\cos\alpha + K\cos x\sin\alpha$

$= 5\sin x + 4\cos x$

$K\sin\alpha = 5$

$K\cos\alpha = 4$

$K^2 = 5^2 + 4^2$

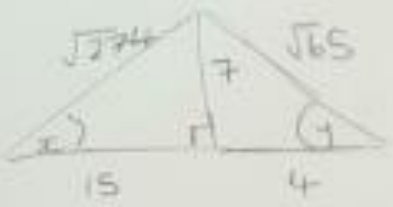
$K = \sqrt{41}$

$\tan\alpha = \frac{5}{4}$

$\alpha = 85.8^\circ$

$5\sin x + 4\cos x = \sqrt{41}\sin(x + 85.8^\circ)$

2.



$\cos(x+y)$

$= \cos x\cos y - \sin x\sin y$

$\cos x = \frac{15}{\sqrt{274}} \times \frac{4}{\sqrt{65}} - \frac{7}{\sqrt{274}} \times \frac{4}{\sqrt{65}}$

$= \frac{60}{\sqrt{17810}} - \frac{28}{\sqrt{17810}}$

$= \frac{32}{\sqrt{17810}}$

3. $(3+2\cos x)(3-2\cos x)$

$= 9 - 4\cos^2 x$

$= 9 - 4(1 - \sin^2 x)$

$= 9 - 4 + 4\sin^2 x = 5 + 4\sin^2 x$

4. $\int \left(4x^{\frac{1}{3}} + \frac{1}{x^5}\right) dx = \int \left(4x^{\frac{1}{3}} + x^{-3}\right) dx$

$= \frac{4x^{\frac{4}{3}}}{4/3} + \frac{x^{-2}}{-2} + C$

$= 3x^{\frac{4}{3}} - \frac{1}{2}x^{-2} + C$

$= 3x^{\frac{4}{3}} - \frac{1}{2x^2} + C$

5. $h'(x) = (x+5)^{-4} dx$

$h(x) = \int (x+5)^{-4} dx$

$= \frac{(x+5)^{-3}}{-3} + C$

$= -\frac{1}{3(x+5)^3} + C$

$\int 4\cos\theta \, d\theta$

$= 4\sin\theta + C$

$\int_{-3}^{2} (x^2 - 8x + 16) dx$

$= \left[\frac{x^3}{3} - \frac{8x^2}{2} + 16x\right]_{-3}^{2}$

$= \left(\frac{2^3}{3} - 4(2)^2 + 16(2)\right) - \left(\frac{(-3)^3}{3} - 4(-3)^2 + 16(-3)\right)$

$= \left(\frac{8}{3} - 16 + 32\right) - \left(-\frac{27}{3} - 36 - 48\right)$

$= \left(\frac{8}{3} + 16\right) - \left(-9 - 36 - 48\right)$

$= \left(\frac{8}{3} + \frac{48}{3}\right) - \left(-93\right)$

$= \frac{56}{3} - -\frac{279}{3}$

$= \frac{335}{3}$ sq units

8. a) $f(x) = \sqrt{3}\cos x + \sin x$ $\quad K\cos(x-\alpha)$

$= K\cos x \cos\alpha + K\sin x \sin\alpha$

$= \sqrt{3}\cos x + \sin x$

$K\cos\alpha = \sqrt{3}$

$K\sin\alpha = 1$

$\tan\alpha = \frac{1}{\sqrt{3}}$

S | A
T | C

$K^2 = \sqrt{3}^2 + 1^2$

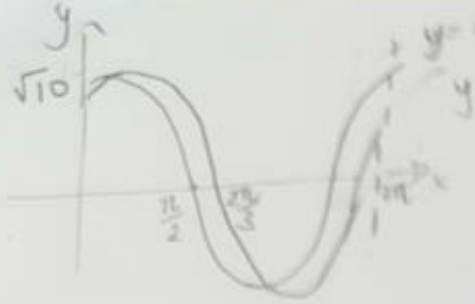
$= 10$

$K = \sqrt{10}$

$\alpha = \frac{\pi}{6}$

$\sqrt{3}\cos x + \sin x = \sqrt{10}\cos\left(x - \frac{\pi}{6}\right)$

b)



9a) $\sqrt{3}\sin x + \cos x$ $\quad K\sin(x+\alpha)^\circ$

$K\sin x\cos\alpha + K\cos x\sin\alpha$

$\sqrt{3}\sin x + \cos x$

$K\cos\alpha = \sqrt{3}$

$K\sin\alpha = 1$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$= 30^\circ$

S | A ✓✓

T | C ✓

$K^2 = \sqrt{3}^2 + 1^2$

$= 10$

$K = \sqrt{10}$

$\sqrt{3}\sin x + \cos x \equiv \sqrt{10}\sin(x + 30)^\circ$

b) max when

$\sqrt{10}\sin(x+30)^\circ = 1$

$x + 30^\circ = 90$

max at $\sqrt{13}$, $x = 120^\circ$

10 $y = \sin 2x + \frac{2}{\sqrt{x}}$

$= \sin 2x + 2x^{-\frac{1}{2}}$

$\frac{dy}{dx} = 2\cos 2x - x^{-\frac{3}{2}}$

$= 2\cos 2x - \frac{1}{\sqrt{x^3}}$

11. $f(x) = \cos^2 x - \sin^2 x$

$f'(x) = -2\sin x \cos x - 2\sin x \cos x$

$= -4\sin x \cos x$

2. $f(x) = 5(7-2x)^3$

$f'(x) = 15(7-2x)^2 \times -2$

$= -30(7-2x)^2$

3. $f(x) = 2x^{\frac{3}{2}} + \sin^2 x$

$f'(x) = 3x^{\frac{1}{2}} + 2\sin x \cos x$