

Relationships and Calculus Assessment Standard 1.4

1. Find  $\int 2 + \frac{6}{x^3} dx$ , where  $x \neq 0$ .

2. Find  $\int \frac{1}{x^3} dx$ , where  $x \neq 0$ .

3. Find  $\int \frac{3}{x^4} + 1 dx$ , where  $x \neq 0$ .

4. Find  $\int \frac{12}{x^5} dx$ , where  $x \neq 0$ .

5. (a) Find  $\int \frac{\sqrt{3}}{2} \cos x dx$ .

(b) Integrate  $3 \sin x$  with respect to  $x$ .

(c) Evaluate  $\int_4^6 (x - 3)^3 dx$

6. (a) Find  $\int \frac{1}{2} \cos x dx$ .

(b) Integrate  $\sin 4x$  with respect to  $x$ .

(c) Evaluate  $\int_2^4 (x - 2)^3 dx$

7. (a) Find  $\int 2 \sin x \, dx$ .

(b) Integrate  $\frac{1}{2} \cos x$  with respect to  $x$ .

(c) Evaluate  $\int_1^2 (x + 3)^4 \, dx$

8. (a) Find  $\int -3 \sin x \, dx$ .

(b) Integrate  $\cos 4x$  with respect to  $x$ .

(c) Evaluate  $\int_1^3 (2x + 1)^3 \, dx$

#### Relationships and Calculus Assessment Standard 1.4 Answers

1.  $2x - 3x^{-2} + c$

2.  $-\frac{1}{2}x^{-2} + c$

3.  $-x^{-3} + x + c$

4.  $-3x^{-4} + c$

5. (a)  $\frac{\sqrt{3}}{2} \sin x + c$  (b)  $-3 \cos x + c$  (c) 20

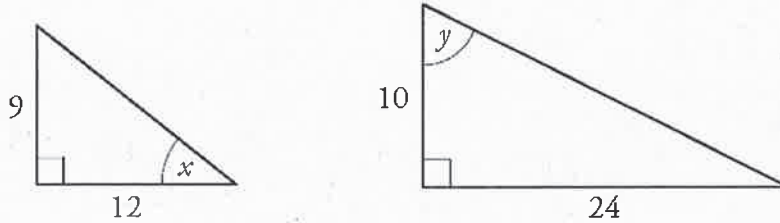
6. (a)  $\frac{1}{2} \sin x + c$  (b)  $-\frac{1}{4} \cos 4x + c$  (c) 4

7. (a)  $-2 \cos x + c$  (b)  $\frac{1}{2} \sin x + c$  (c) 420.2

8. (a)  $3 \cos x + c$  (b)  $\frac{1}{4} \sin 4x + c$  (c) 290

Expressions and Functions Assessment Standard 1.2

1. The diagram below shows two right-angled triangles.

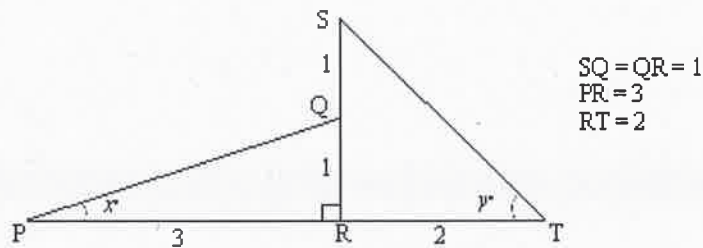


(a) Write down the values of  $\sin x^\circ$  and  $\cos y^\circ$ .

(b) By expanding  $\cos(x + y)^\circ$  show that the exact value of  $\cos(x + y)^\circ$  is  $\frac{-16}{65}$ .

2. Express  $12 \cos x^\circ + 5 \sin x^\circ$  in the form  $k \cos(x - a)^\circ$  where  $k > 0$  and  $0 \leq a \leq 360$ .

3. The diagram below shows two right-angled triangles PQR and SRT.



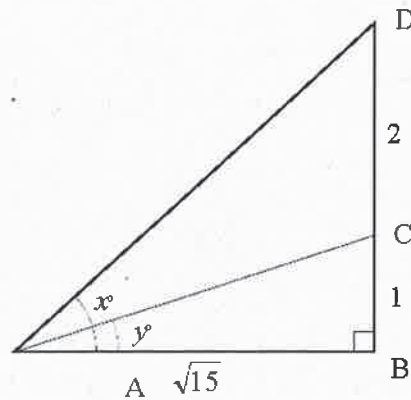
(a) Write down the values of  $\cos x^\circ$  and  $\sin y^\circ$ .

(b) By expanding  $\sin(x + y)^\circ$  show that the exact value of  $\sin(x + y)^\circ$  is  $\frac{8}{\sqrt{80}}$ .

4. Express  $2 \cos x^\circ + 5 \sin x^\circ$  in the form  $k \cos(x - a)^\circ$  where  $k > 0$  and  $0 \leq a \leq 360$ .

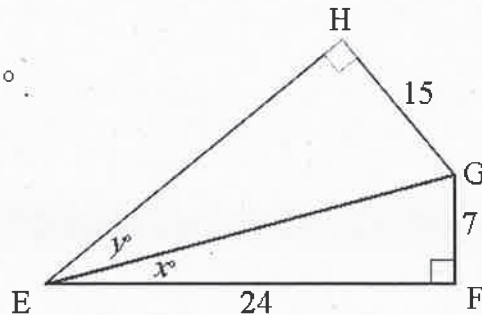
5. The diagram below shows two right-angled triangles ABC and ABD.

$$\angle DAB = x^\circ \text{ and } \angle CAB = y^\circ.$$



- (a) Write down the values of  $\cos x^\circ$  and  $\sin y^\circ$ .
- (b) By expanding  $\cos(x - y)^\circ$  show that the exact value of  $\cos(x - y)^\circ$  is  $\frac{18}{4\sqrt{24}}$ .
6. Express  $4\cos x^\circ + \sin x^\circ$  in the form  $k\cos(x - a)^\circ$  where  $k > 0$  and  $0 \leq a < 360$ .
7. The diagram below shows two right-angled triangles EFG and EHG.

$$\angle FEG = x^\circ \text{ and } \angle HEG = y^\circ.$$



- (a) Write down the values of  $\sin x^\circ$  and  $\cos y^\circ$ .
- (b) By expanding  $\cos(x + y)^\circ$  show that the exact value of  $\cos(x + y)^\circ$  is  $\frac{3}{5}$ .
8. Express  $7\sin x^\circ + 4\cos x^\circ$  in the form  $k\cos(x - a)^\circ$  where  $k > 0$  and  $0 \leq a \leq 360$ .

9. Show that  $(\sin A + \cos A)^2 = 1 + \sin 2A$  and hence state the maximum value of  $4(\sin A + \cos A)^2$ .
10. Show that  $\sin^3 x \cos x + \sin x \cos^3 x = \frac{1}{2} \sin 2x$  and hence state the minimum value of  $8\sin^3 x \cos x + 8\sin x \cos^3 x$ .
11. Show that  $(\cos A + \sin A)(\cos A - \sin A) = \cos 2A$  and hence state the maximum value of  $5(\cos A + \sin A)(\cos A - \sin A)$ .

## Expressions and Functions Assessment Standard 1.2 Answers

1.(a)  $\sin x = \frac{9}{15} = \frac{3}{5}$ ,  $\cos x = \frac{10}{26} = \frac{5}{13}$  (b) Proof

2.  $k = 13$ ,  $\alpha^\circ = 22.6^\circ$

3.(a)  $\sin y = \frac{2}{\sqrt{8}}$ ,  $\cos x = \frac{3}{\sqrt{10}}$  (b) Proof

4.  $k = \sqrt{29}$ ,  $\alpha^\circ = 68.2^\circ$

5.(a)  $\cos x = \frac{\sqrt{15}}{\sqrt{24}}$ ,  $\sin y = \frac{1}{4}$  (b) Proof

6.  $k = \sqrt{17}$ ,  $\alpha^\circ = 14.0^\circ$

7.(a)  $\sin x = \frac{7}{25}$ ,  $\cos y = \frac{20}{25}$  (b) Proof

8.  $k = \sqrt{65}$ ,  $\alpha^\circ = 60.3^\circ$

9. Max value of  $4(\sin A + \cos A)^2 = \max$  value of  $4(1 + \sin 2A) = 4(1 + 1) = 8$ .

10. Min value of  $8\sin^3 x \cos x + 8\sin x \cos^3 x = \min$  value of  $8(\frac{1}{2}\sin 2x) = 8 \times (-\frac{1}{2}) = -4$ .

11. Max value of  $5(\cos A + \sin A)(\cos A - \sin A) = \max$  value of  $5 \cos 2A = 5$ .