

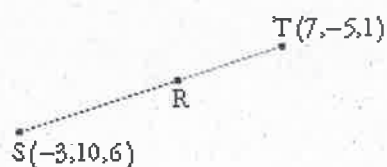
Expressions and Functions Assessment Standard 1.4

1. (a) Points A, B and C have coordinates $(-4, -3, 1)$, $(0, -1, 0)$ and $(4, 1, -1)$ respectively.

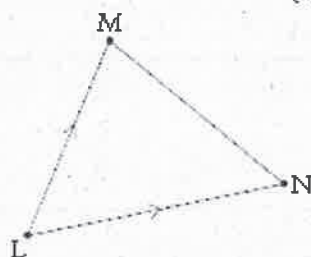
(i) Write down the components of \overrightarrow{AC} .

(ii) Hence show that the points A, B and C are collinear.

(b) The point R divides \overrightarrow{ST} in the ratio 3:2, as shown below. Find the coordinates of R.



2. The diagram shows triangle LMN where $\overrightarrow{LM} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$ and $\overrightarrow{LN} = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}$.



(a) Find the value of $\overrightarrow{LM} \cdot \overrightarrow{LN}$

(b) Use your answer from part (a) to find the size of the angle MLN .

3. (a) Points S, T and U have components $(1, 2, -5)$, $(-3, 4, 1)$ and $(-5, 5, 4)$ respectively.

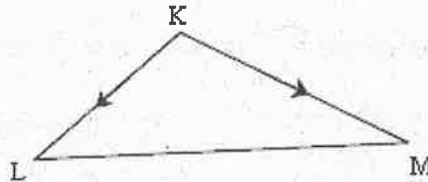
(i) Write down the coordinates of \overrightarrow{ST} .

(ii) Hence show that the points S, T and U are collinear.

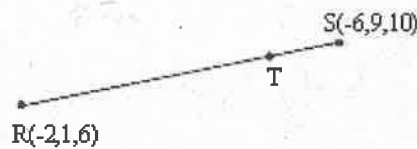
(b) The point P divides \overrightarrow{AB} in the ratio 1:4, as shown below. Find the coordinates of P.



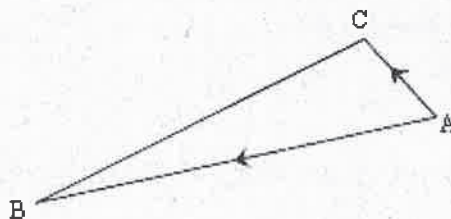
4. The diagram shows triangle KLM where $\overrightarrow{KL} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\overrightarrow{KM} = \begin{pmatrix} -4 \\ -3 \\ 2 \end{pmatrix}$.



- (a) Find the value of $\overrightarrow{KL} \cdot \overrightarrow{KM}$.
- (b) Use your answer from part (a) to find the size of the angle LKM.
- 5.(a) Points P, Q and R have coordinates (2, -3, 3), (6, -2, 0) and (14, 0, -6) respectively.
- (i) Write down the components of \overrightarrow{PQ} .
- (ii) Hence show that the points P, Q and R are collinear.
- (b) The point T divides \overline{RS} in the ratio 3:1, as shown below. Find the coordinates of T.



6. The diagram shows triangle ABC where $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 0 \\ 12 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$.



- (a) Find the value of $\overrightarrow{AB} \cdot \overrightarrow{AC}$

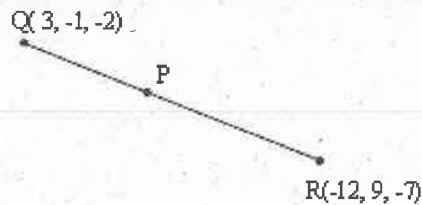
(b) Use your answer from part (a) to find the size of the angle BAC.

7.(a) Points E, F and G have coordinates (1, 4, -2), (-1, 8, -1) and (-5, 16, 1) respectively.

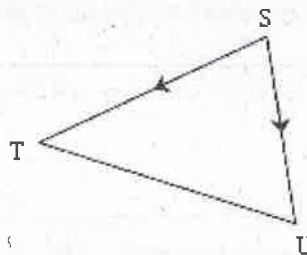
(i) Write down the components of \overrightarrow{EF} .

(ii) Hence show that the points E, F and G are collinear.

(b) The point P divides \overline{QR} in the ratio 2:3, as shown below. Find the coordinates of P.



8. The diagram shows triangle STU where $\overrightarrow{ST} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\overrightarrow{SU} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}$.



(a) Find the value of $\overrightarrow{ST} \cdot \overrightarrow{SU}$

(b) Use your answer from part (a) to find the size of the angle TSU.

9.(a) Given that A(3, t , 5), B(5, 3, 2) and C(9, 13, -4) are collinear, find the value of t .

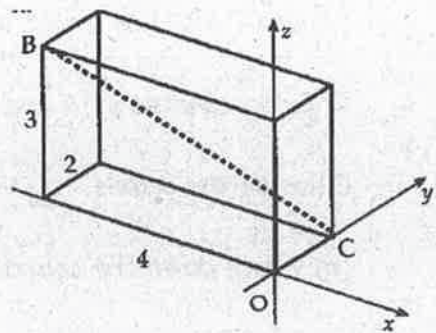
(b) Find the ratio in which B divides AC.

10.(a) Given that P(2, 5, 3), Q(3, 1, p) and R(6, -11, 23) are collinear, find the value of p .

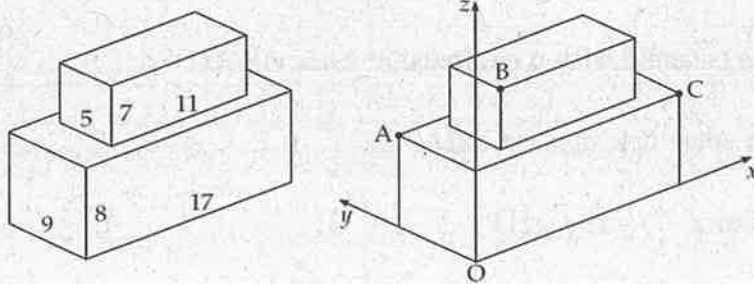
(b) Find the ratio in which Q divides PR,

11. A cuboid crystal is placed relative to the coordinate axes as shown.

Write down the components of \overrightarrow{BC} .



12. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm. Coordinates axes are taken as shown.



The point A has coordinates $(0, 9, 8)$ and C has coordinates $(17, 0, 8)$.

Write down the coordinates of B.

Write down the components of \overrightarrow{BA} and \overrightarrow{BC} .

13. The diagram shows a square-based pyramid of height 8 units.

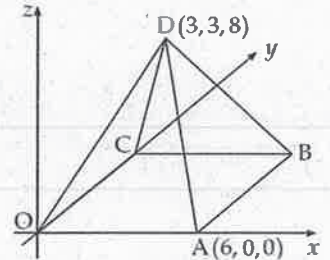
Square $OABC$ has a side length of 6 units.

The coordinates of A and D are $(6, 0, 0)$ and $(3, 3, 8)$.

C lies on the y -axis.

(a) Write down the coordinates of B .

(b) Determine the components of \overrightarrow{DA} and \overrightarrow{DB} .



14. $VABCD$ is a pyramid with a rectangular base $ABCD$.

Relative to some appropriate axes,

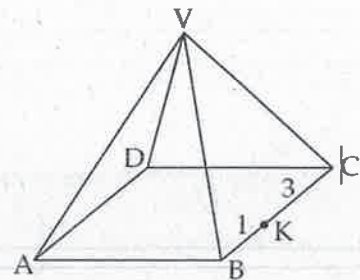
\overrightarrow{VA} represents $-7i - 13j - 11k$

\overrightarrow{AB} represents $6i + 6j - 6k$

\overrightarrow{AD} represents $8i - 4j + 4k$.

K divides BC in the ratio $1 : 3$.

Find \overrightarrow{VK} in component form.



Expressions and Functions Assessment Standard 1.4 Answers

1. (a) (i) $\vec{AC} = \begin{pmatrix} 8 \\ 4 \\ -2 \end{pmatrix} = 2 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$

(ii) $\vec{AB} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$. Since $2\vec{AB} = \vec{AC}$ and A is a common point, A, B and C are collinear.

(b) R(3, 1, 3)

2. (a) $\vec{LM} \cdot \vec{LN} = 20$ (b) Angle MLN = 56.4°

3. (a) (i) $\vec{ST} = \begin{pmatrix} -4 \\ 2 \\ 6 \end{pmatrix}$ (ii) proof (b) P(4, -1, -2)

4. (a) $\vec{KL} \cdot \vec{KM} = -5$ (b) 104.4°

5. (a) (i) $\vec{PQ} = \begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix}$ (ii) proof (b) T(-5, 7, 9)

6. (a) $\vec{AB} \cdot \vec{AC} = 22$ (b) 55.7°

7. (a) (i) $\vec{EF} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$ (ii) proof (b) P(-3, 3, -4)

8. (a) $\vec{ST} \cdot \vec{SU} = 2$ (b) 79.1°

9. (a) $t = -2$ (b) AB : BC = 1 : 2

10. (a) $p = 8$ (b) PQ : QR = 1 : 3

11. B(-4, 0, 3), C(0, 2, 0) and $\vec{BC} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}$

12. B(3, 2, 15), $\vec{BA} = \begin{pmatrix} -3 \\ 7 \\ -7 \end{pmatrix}$ and $\vec{BC} = \begin{pmatrix} 14 \\ -2 \\ -7 \end{pmatrix}$

13. B(6, 6, 0), $\vec{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$ and $\vec{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$

14. $\vec{VK} = \vec{VA} + \vec{AB} + \frac{1}{4}\vec{BC} = \begin{pmatrix} 1 \\ -8 \\ -16 \end{pmatrix}$

Expressions and Functions Assessment Standard 1.1

1.
 - (a) Simplify $\log_a 7 + \log_a 3$.
 - (b) Simplify $\log_3 5 - 3 \log_3 2$.
 - (c) Evaluate $\log_2 2$.

2.
 - (a) Given $x = \frac{\log_e 7}{\log_e 4}$, find an approximation for x .
 - (b) Given $\log_{10} y = 3.1$, write an expression for the **exact** value of y .
 - (c) Given $y = 10^{2.9}$, find an approximation for y .

3.
 - (a) Simplify $\log_x 8 + \log_x 5$.
 - (b) Simplify $5 \log_9 3 - \log_9 27$.

4.
 - (a) Given $x = \frac{\log_e 33}{\log_e 7}$, find an approximation for x .
 - (b) Given $\log_{10} y = 2.5$, write an expression for the **exact** value of y .
 - (c) Given $y = 10^{1.66}$, find an approximation for y .

5.
 - (a) Simplify $\log_p 6 + \log_p 3$.
 - (b) Simplify $2 \log_2 6 - \log_2 9$.

6.
 - (a) Given $x \log_e 9 = \log_e 11$, find an approximation for x .
 - (b) Given $\log_3 y = 1.6$, write an expression for the **exact** value of y .
 - (c) Given $y = 10^{0.8}$, find an approximation for y .

7.
 - (a) Simplify $\log_y 16 - \log_y 8$.
 - (b) Simplify $3 \log_4 2 + \log_4 8$.

8. (a) Given $p \log_e 12 = 2 \log_e 17$, find an approximation for p .
 (b) Given $\log_2 x = 3.4$, write an expression for the exact value of x .
 (c) Given $y = 4^{2.7}$, find an approximation for y .
9. Solve $e^x = 3 \cdot 5$.
10. Solve $e^x = 6 \cdot 1$.
11. Solve $e^x = 2 \cdot 8$.
12. Solve $e^x = 5 \cdot 7$.

Expressions and Functions Assessment Standard 1.1 Answers

- | | | | |
|-----|---------------------------|--------------------------|--------------------|
| 1. | (a) $\log_a 21$ | (b) $\log_3 \frac{5}{8}$ | (c) $\log_2 2 = 1$ |
| 2. | (a) 1.404 | (b) $y = 10^{3.1}$ | (c) 794.3 |
| 3. | (a) $\log_x 40$ | (b) 1 | |
| 4. | (a) 1.80 | (b) $y = 10^{2.5}$ | (c) 45.71 |
| 5. | (a) $\log_p 18$ | (b) 2 | |
| 6. | (a) 1.09 | (b) $3^{1.6}$ | (c) 6.3 |
| 7. | (a) $\log_y 2$ | (b) 3 | |
| 8. | (a) 2.28 | (b) $x = 2^{3.4}$ | (c) 42.2 |
| 9. | 1.253 to 3 decimal places | | |
| 10. | 1.808 to 3 decimal places | | |
| 11. | 1.030 to 3 decimal places | | |
| 12. | 1.740 to 3 decimal places | | |