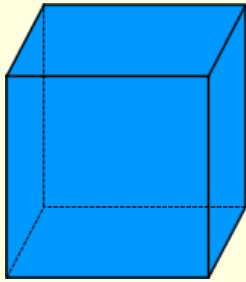


### Starter



1) A small cube is enlarged by a length scale factor of 3.

The small cube has a surface area of  $25\text{cm}^2$  and a volume of  $140\text{cm}^3$ .

What is the new surface area and volume of the enlarged shape?

$$25 \times 3^2 = 25 \times 9 = 225\text{cm}^2$$

$$25 \times 3^3 = 25 \times 27 = 675\text{cm}^3$$

### Similarity



**Today we are learning...**

How to answer a range of exam style questions on similarity.

**I will know if I have been successful if...**

I can quickly calculate the scale factor.

I can find the volume once increased by a scale factor.

I can find the surface area once increased by a scale factor.

### Similarity Question Relay

Answer one question at a time.

Once you have completed a question bring it to me to check your answer and collect the next question.

Write your answer on the question sheet.

Each team has a pass card - use it wisely!

### Plenary

A deflated beach ball has an initial surface area of  $34\text{cm}^2$  and a volume of  $14\text{cm}^3$ .

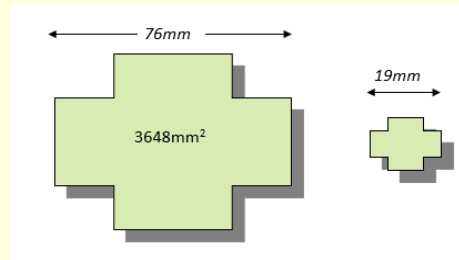
It is inflated until it has increased in size by a scale factor of 4.

What is the new volume and surface area of the ball?

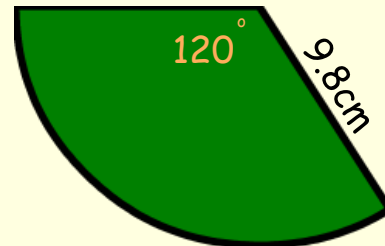


### Starter

1) What is the area of the **smaller** shape?



2) What is the arc length and area of the sector shown?



### CAST Diagram

**Today we are learning...**

How to draw and use the *CAST* diagram to solve trig equations.

**I will know if I have been successful if...**

I know how to draw the *CAST* diagram.

I can identify which quadrants to use.

I can solve to find the required values of theta.

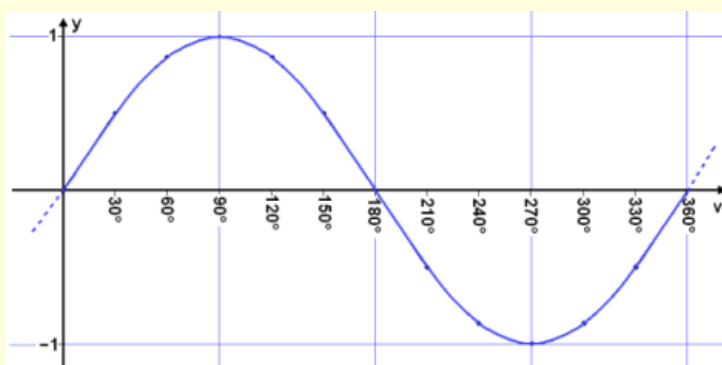


## Examples

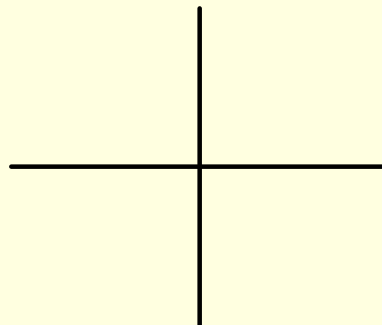
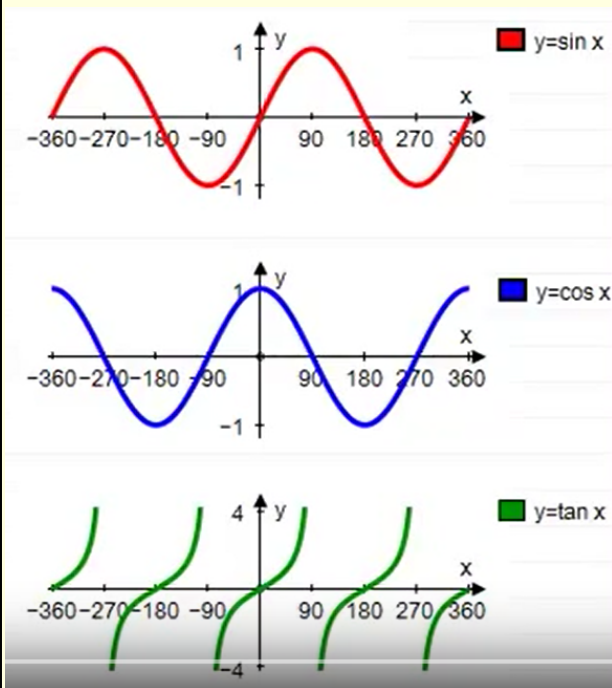
1)  $\sin(x) = 0.5$  for  $0 < x < 360$

## Examples

1)  $\sin(x) = 0.5$  for  $0 < x < 360$

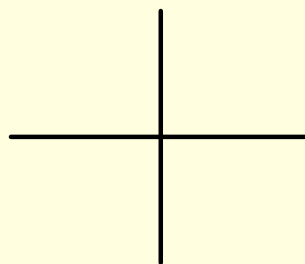


### Drawing the CAST Diagram



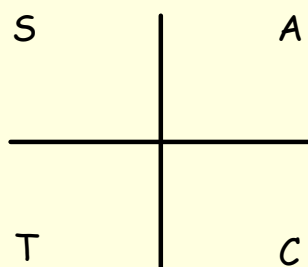
### Examples

1)  $\sin(x) = 0.5$  for  $0 < x < 360$



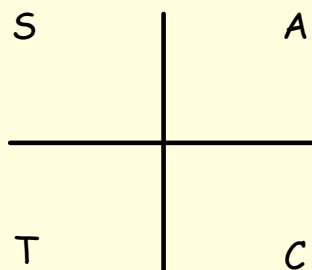
Examples

2)  $\cos(x) = 0.5$  for  $0 < x < 360$



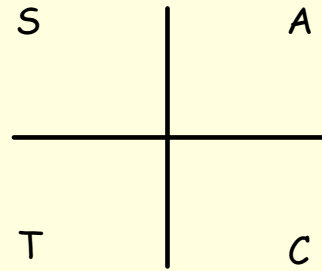
Examples

3)  $\tan(\theta) = \sqrt{3}$  for  $0 < x < 360$



## Examples

4)  $\cos(\theta) = -0.5$  for  $0 < x < 360$



## Practice

1. Solve the following equations where  $0 \leq x \leq 360$ 

- |                             |                             |                             |
|-----------------------------|-----------------------------|-----------------------------|
| (a) $\sin x^\circ = 0.5$    | (b) $\cos x^\circ = 0.866$  | (c) $\tan x^\circ = 1$      |
| (d) $\cos x^\circ = -0.5$   | (e) $\tan x^\circ = -0.577$ | (f) $\sin x^\circ = -0.866$ |
| (g) $\tan x^\circ = 1.732$  | (h) $\sin x^\circ = 0.707$  | (i) $\cos x^\circ = 0.707$  |
| (j) $\sin x^\circ = -0.707$ | (k) $\cos x^\circ = -0.866$ | (l) $\tan x^\circ = -1.732$ |

2. Solve the following equations where  $0 \leq x \leq 360$ 

- |                             |                             |                             |
|-----------------------------|-----------------------------|-----------------------------|
| (a) $\sin x^\circ = 0.313$  | (b) $\cos x^\circ = 0.425$  | (c) $\tan x^\circ = 5.145$  |
| (d) $\cos x^\circ = -0.087$ | (e) $\tan x^\circ = -0.869$ | (f) $\sin x^\circ = -0.191$ |
| (g) $\tan x^\circ = 11.43$  | (h) $\sin x^\circ = 0.695$  | (i) $\cos x^\circ = 0.755$  |
| (j) $\sin x^\circ = -0.358$ | (k) $\cos x^\circ = -0.682$ | (l) $\tan x^\circ = -0.268$ |

3. Solve the following equations where  $0 \leq x \leq 360$ 

- |                             |                            |                            |
|-----------------------------|----------------------------|----------------------------|
| (a) $2 \sin x^\circ = 1$    | (b) $3 \cos x^\circ = 2$   | (c) $3 \tan x^\circ = 5$   |
| (d) $2 \cos x^\circ = -1$   | (e) $2 \tan x^\circ = -8$  | (f) $4 \sin x^\circ = -3$  |
| (g) $5 \tan x^\circ = 23.5$ | (h) $5 \sin x^\circ = 2$   | (i) $6 \cos x^\circ = 1$   |
| (j) $8 \sin x^\circ = -3$   | (k) $11 \cos x^\circ = -9$ | (l) $10 \tan x^\circ = -9$ |

**Answers**

- |           |            |               |            |                |            |                |
|-----------|------------|---------------|------------|----------------|------------|----------------|
| <b>1.</b> | <b>(a)</b> | 30°, 150°     | <b>(b)</b> | 30°, 330°      | <b>(c)</b> | 45°, 225°      |
|           | <b>(d)</b> | 120°, 240°    | <b>(e)</b> | 150°, 330°     | <b>(f)</b> | 240°, 300°     |
|           | <b>(g)</b> | 60°, 240°     | <b>(h)</b> | 45°, 135°      | <b>(i)</b> | 45°, 315°      |
|           | <b>(j)</b> | 225°, 315°    | <b>(k)</b> | 150°, 210°     | <b>(l)</b> | 120°, 300°     |
| <b>2.</b> | <b>(a)</b> | 18·2°, 161·8° | <b>(b)</b> | 64·8°, 295·2°  | <b>(c)</b> | 79°, 259°      |
|           | <b>(d)</b> | 95°, 265°     | <b>(e)</b> | 139°, 319°     | <b>(f)</b> | 191°, 349°     |
|           | <b>(g)</b> | 85°, 265°     | <b>(h)</b> | 44°, 136°      | <b>(i)</b> | 41°, 319°      |
|           | <b>(j)</b> | 201°, 339°    | <b>(k)</b> | 133°, 227°     | <b>(l)</b> | 165°, 345°     |
| <b>3.</b> | <b>(a)</b> | 30°, 150°     | <b>(b)</b> | 48·2°, 311·8°  | <b>(c)</b> | 59°, 239°      |
|           | <b>(d)</b> | 120°, 240°    | <b>(e)</b> | 104°, 284°     | <b>(f)</b> | 228·6°, 311·4° |
|           | <b>(g)</b> | 78°, 258°     | <b>(h)</b> | 23·6°, 156·4°  | <b>(i)</b> | 80·4°, 279·6°  |
|           | <b>(j)</b> | 202°, 338°    | <b>(k)</b> | 144·9°, 215·1° | <b>(l)</b> | 138°, 318°     |

Introduction to next lesson...

a) Solve  $-2 + 3\sin(x) = 0$  for  $0 < x < 360$

b) Solve  $3 - 7\tan(x) = 2$  for  $0 < x < 360$

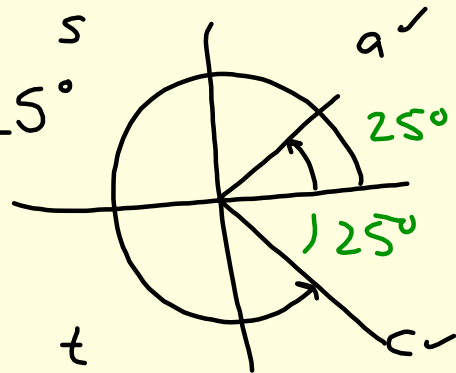


## Starter

1) Solve the following using the CAST diagram for  $0 < x < 360$ .

a)  $\tan(x) = 1.1917$      $\tan^{-1}(1.1917) = 50^\circ$   
 $x = 50^\circ, 230^\circ$

b)  $\cos(x) = 0.9063$   
 $\cos^{-1}(0.9063) = 25^\circ$   
 $x = 335^\circ, 25^\circ$



## Solving Trig Functions

**Today we are learning...**

How to solve trigonometric functions.

**I will know if I have been successful if...**

I can use the CAST diagram as a tool to help me.

I can rearrange equations where required.

I know that there may be more than one solution.



## Example

1) Solve the equation  $5\cos(x) = 1$  for  $0 < x < 360$

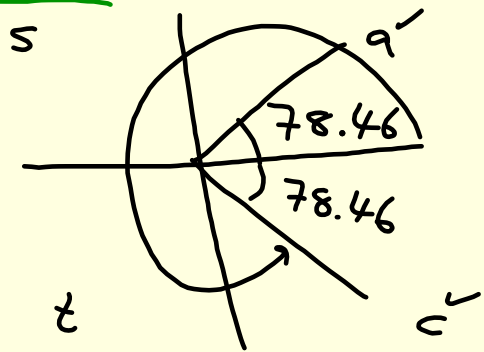
$$5\cos(x) = 1$$

$$\cos(x) = \frac{1}{5} = 0.2$$

$$\cos^{-1}(0.2) = \underline{78.46^\circ}$$

$$x = 360 - 78.46$$

$$= \underline{281.53^\circ}$$



## Example

2) Solve the equation  $5\sin(x) + 4 = 0$  for  $0 < x < 360$

$$5\sin(x) = -4$$

$$\sin(x) = -\frac{4}{5}$$

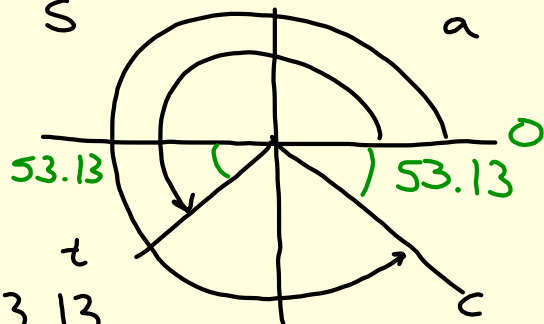
$$\sin^{-1}\left(-\frac{4}{5}\right) = -53.13$$

$$x = 180 + 53.13$$

$$= \underline{233.13^\circ}$$

$$x = 360 - 53.13$$

$$= \underline{306.87^\circ}$$



## Practice

1) Solve the following equations where  $0 \leq x \leq 360$ .

(a)  $2\sin x^\circ = 1$

(b)  $5\sin x^\circ - 4 = 0$

(c)  $5\sin x^\circ = -3$

(d)  $6\sin x^\circ + 1 = 0$

(e)  $4\sin x^\circ + 1 = 1.5$

(f)  $12 + 9\sin x^\circ = 8.$

2) Solve the following equations where  $0 \leq x \leq 360$ .

(a)  $6\cos x^\circ = 3$

(b)  $10\cos x^\circ - 2 = 0$

(c)  $3\cos x^\circ = 2$

(d)  $7\cos x^\circ + 4 = 0$

(e)  $9\cos x^\circ + 7 = 2$

(f)  $18 + 4\cos x^\circ = 15.$

Highlighted in red - negatives!

## Answers

1)

a  $30.0^\circ$   $150^\circ$

b  $53.1^\circ$   $127^\circ$

c  $217^\circ$   $323^\circ$

d  $190^\circ$   $350^\circ$

e  $7.18^\circ$   $173^\circ$

f  $206^\circ$   $334^\circ$

2)

a  $60.0^\circ$   $300^\circ$

b  $78.5^\circ$   $282^\circ$

c  $48.19$   $311.81$

d  $125^\circ$   $235^\circ$

e  $124^\circ$   $236^\circ$

f  $139^\circ$   $221^\circ$

## Plenary

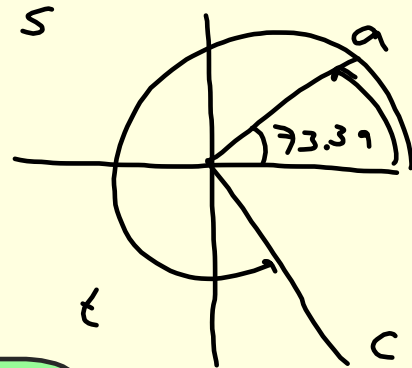
Solve algebraically, the equation  $7 \cos x^\circ - 2 = 0$  for  $0 \leq x \leq 360$ 

3

$$7 \cos(x) = 2$$

$$\cos(x) = \frac{2}{7} \checkmark$$

$$\cos^{-1}\left(\frac{2}{7}\right) = 73.39 \checkmark$$



Answer

$$x = 360 - 73.39$$

$$= 286.61 \checkmark$$

$$x = 73.39$$

---

 3

## Starter

Solve by first rearranging and then using the CAST diagram.

1)  $\cos(x) - 1 = -1$  for  $0 < x < 360$

2)  $3 \tan(x) + 1 = 4$  for  $0 < x < 360$

## Trigonometric Identities

**Today we are learning...**

The key trigonometric identities and how to use them.

**I will know if I have been successful if...**

I can state some of the key identities.

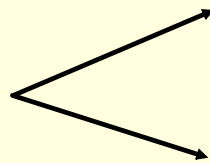
I can substitute expressions involving the key identities.

I can simplify expressions using the key identities.

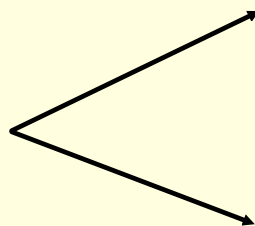


## The Identities

$$\sin^2 A + \cos^2 A = 1$$



$$\tan A = \frac{\sin A}{\cos A}$$



$$\tan A = \frac{\sin A}{\cos A}$$

Using the Identities

$$\sin^2 A + \cos^2 A = 1$$

1) Prove  $3\cos^2(x) + 3\sin^2(x) = 3$

$$\tan A = \frac{\sin A}{\cos A}$$

Using the Identities

$$\sin^2 A + \cos^2 A = 1$$

2) Prove  $\tan(x) \cos(x) = \sin(x)$

$$\tan A = \frac{\sin A}{\cos A}$$

Using the Identities

$$\sin^2 A + \cos^2 A = 1$$

3) Prove  $8\cos^2(x) = 8 - 8\sin^2(x)$

$$\tan A = \frac{\sin A}{\cos A}$$

Using the Identities

$$\sin^2 A + \cos^2 A = 1$$

Prove the following trigonometric **identities** :-

(a)  $5\cos^2 A + 5\sin^2 A = 5$

(b)  $4\cos^2 A = 4 - 4\sin^2 A$

(c)  $2\cos^2 A - 1 = 1 - 2\sin^2 A$

(d)  $6\cos^2 A - 5 = 1 - 6\sin^2 A$

(e)  $(\cos X + \sin X)^2 = 1 + 2\sin X \cos X$

(f)  $(\cos P - \sin P)^2 + 2\sin P \cos P = 1$

(g)  $(\cos X + \sin X)(\cos X - \sin X) = 2\cos^2 X - 1$

(h)  $(\cos X - \sin X)(\cos X + \sin X) = 1 - 2\sin^2 X$

(i)  $\tan P \cos P = \sin P$

(j)  $\frac{1 - \cos^2 \alpha}{\cos^2 \alpha} = \tan^2 \alpha$

(k)  $\frac{1 - \sin^2 \alpha}{\sin^2 \alpha} = \frac{1}{\tan^2 \alpha}$

(l)  $\frac{\sin \beta}{\cos \beta} + \frac{\cos \beta}{\sin \beta} = \frac{1}{\cos \beta \sin \beta}$

## Plenary

(b) Prove that

$$\sin^3 x + \sin x \cos^2 x = \sin x$$

(2 marks)

Hint: Factorise the expression first by taking out a common factor.

## Starter

1) Solve the equation algebraically for  $0 < x < 360$

$$4\tan(x) + 7 = 5$$

2) Simplify

$$\tan(x)\cos(x)$$



## Trigonometric Identities

**Today we are learning...**

The key trigonometric identities and how to use them.

**I will know if I have been successful if...**

I can state some of the key identities.

I can substitute expressions involving the key identities.

I can simplify expressions using the key identities.

