# Belmont Academy

*Prelim Examination 2005*
*(Assessing Units 1 & 2)*

# MATHEMATICS

## Higher Grade - Paper I (Non~calculator)

**Time allowed - 1 hour 10 minutes**

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Read Carefully

1. **Calculators may not be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

## FORMULAE LIST

**Circle:**

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

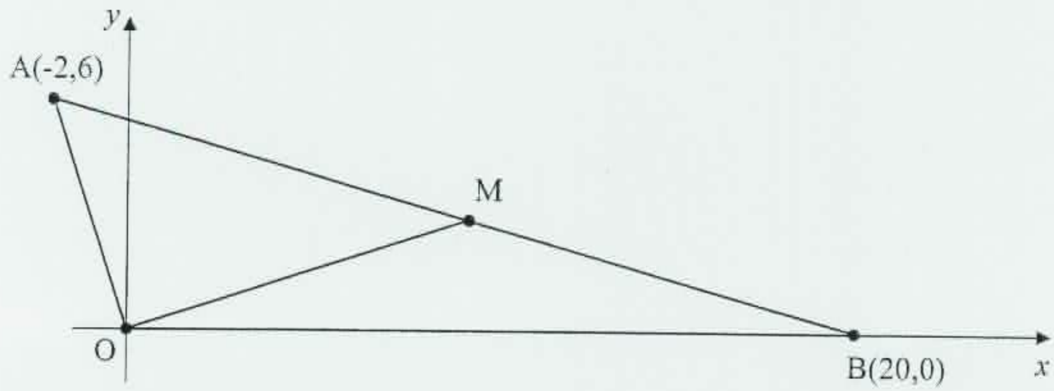
The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

**Trigonometric formulae:**

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

**All questions should be attempted**

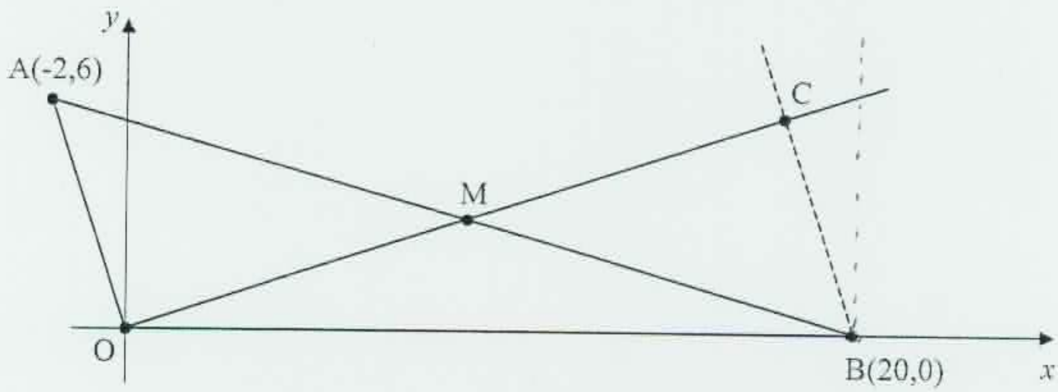
**1.** The diagram shows triangle OAB with M being the mid-point of AB.
The coordinates of A and B are (-2,6) and (20,0) respectively.




(a) Establish the coordinates of M. **1**

(b) Hence find the equation of the median OM. **2**

(c) A line through B, perpendicular to OM meets OM produced at C.



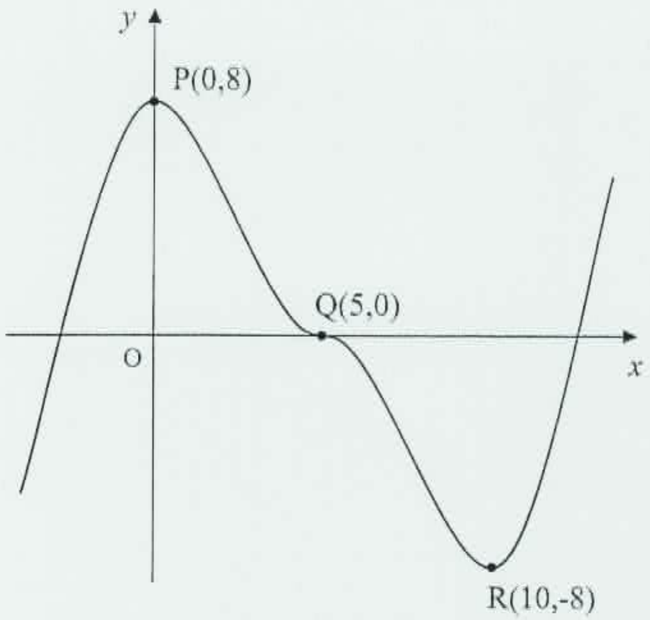

(i) Find the equation of the line BC and hence establish the coordinates of C. **4**

(ii) What can you say about triangles OAM and BMC? Explain your answer. **2**

**2.** A curve has as its equation $y = \dfrac{x^2 - 4x}{\sqrt{x}}$, where $x \in R$ and $x > 0$.

Find the gradient of the tangent to this curve at the point where $x = 4$. **6**

3. The diagram shows part of the graph of $y = f(x)$.




The function has stationary points at P(0,8), Q(5,0) and R(10,-8) as shown.

Sketch a possible graph for $y = f'(x)$, where $f'(x)$ is the derivative of $f(x)$. **4**

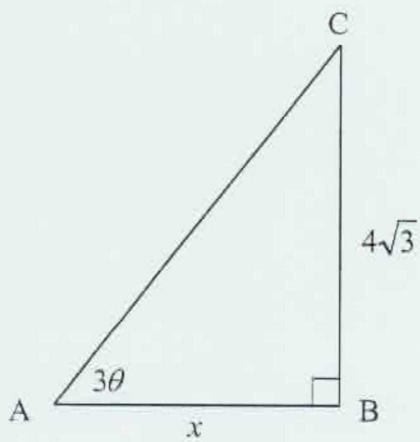
4. Two functions, defined on suitable domains, are given as

$$g(x) = x^2 - 3x \quad \text{and} \quad h(x) = 2x + 1.$$

Show that the composite function $g\left(h(x)\right)$ can be written in the form $a(ax+b)(x-b)$, where $a$ and $b$ are constants, and state the value(s) of $a$ and $b$. **4**

5. Consider the triangle opposite.

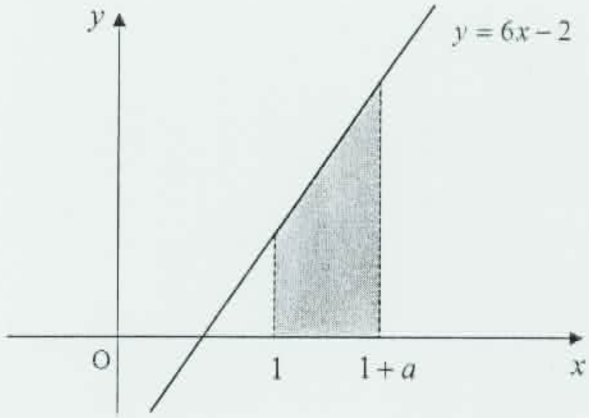
AB is $x$ units long, BC $= 4\sqrt{3}$ units long and angle BAC $= 3\theta$ radians.




(a) Given that the exact area of the triangle is $8\sqrt{3}$ units$^2$, **show clearly** that $x = 4$. **3**

(b) Hence find the value of $\theta$, **in radians**, given that $3\theta$ is acute. **3**

**6.** The diagram below, which is not to scale, shows part of the graph of the line with equation $y = 6x - 2$. Also shown are ordinates at $x = 1$ and at $x = 1 + a$.




Find $a$ given that the shaded part of the diagram has an area of 4 square units. **7**

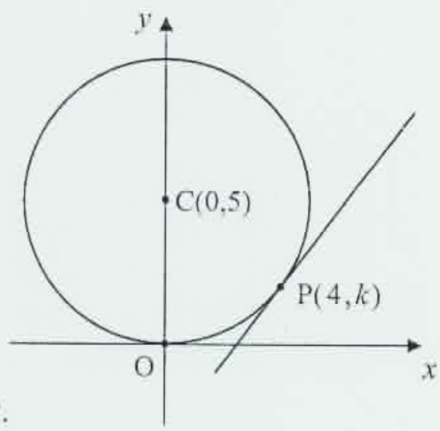
**7.** Two sequences are defined by the following recurrence relationships

$U_{n+1} = 0 \cdot 6U_n + 20$ and $U_{n+1} = 0 \cdot 9U_n + b$ , where $b$ is a constant.

(a) Explain why both sequences have a limit as $n \to \infty$. **1**

(b) Find the value of $b$ if both these sequences have the same limit. **4**

**8.** A circle passes through the origin and has the point C(0,5) as its centre.




(a) Establish the equation of this circle giving your answer in **expanded form.** **4**

(b) The point P(4, $k$) lies on the circumference of this circle as shown. Find **algebraically** the value of $k$. **5**

(c) Find the equation of the tangent to the circle at P. **3**

**9.** A curve has as its equation $y = (p+1)x^3 - 3px^2 + 4x + 1$, where $p$ **is a positive integer**.

(a) Find $\dfrac{dy}{dx}$. **2**

(b) Hence establish the value of $p$ given that this curve has only **one stationary point**. **5**

[ END OF QUESTION PAPER ]

# Belmont Academy

*Prelim Examination 2005*
*(Assessing Units 1 & 2)*

# MATHEMATICS
## Higher Grade - Paper II

**Time allowed - 1 hour 30 minutes**

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Read Carefully

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## FORMULAE LIST

**Circle:**

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

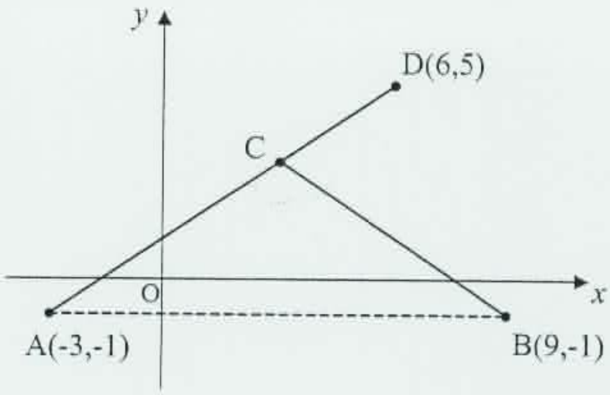
**Trigonometric formulae:**

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

**All questions should be attempted**

1. The diagram shows a line joining the points A(-3,-1) and D(6,5).

B has coordinates (9,-1) and C is a point on AD.




(a) Find the equation of the line AD. **2**

(b) Hence establish the coordinates of C given that triangle ABC is isosceles. **3**

(c) Use gradient theory to calculate the size of angle BCD, giving your answer correct to the nearest degree. **3**

2. A lead shot is discharged from a gun at a clay pigeon.

The height, $h$ feet, of the shot after $t$ seconds is given by the function

$$h(t) = 288t - 48t^2 .$$

(a) What is the maximum height the shot can reach ? **4**

(b) For the shot to actually break the clay pigeon it must strike the pigeon at a speed greater than <u>or</u> equal to 48 feet per second.

The speed, $s$ , of the shot after $t$ seconds can be found from $s = h'(t)$, where $0 < t \leq 3$ .

Will the shot break the clay pigeon after a flight of $2 \cdot 7$ seconds ? Explain. **2**

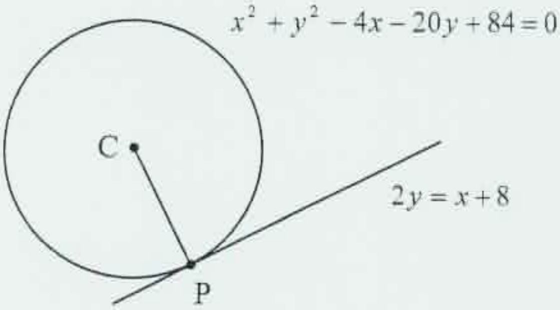
(c) Calculate the maximum **height** the shot can reach **and** still break the clay pigeon. **3**

3. Solve algebraically the equation

$$9\sin x^\circ + 4 = 2\cos 2x^\circ \qquad \text{where } 0 \leq x < 360$$ **6**

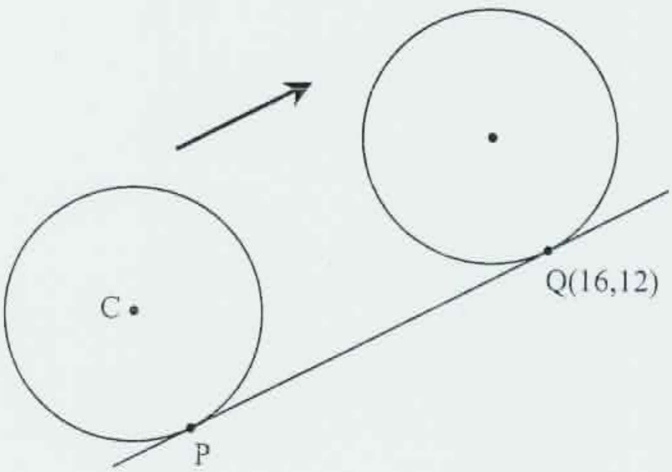
**4.** A circle, centre C, has as its equation $x^2 + y^2 - 4x - 20y + 84 = 0$.

It touches the line with equation $2y = x + 8$ at point P, as shown.




(a) Find **algebraically** the coordinates of P. **4**

(b) The circle is rolled up the line until Q(16,12) becomes the new point of tangency.




Establish the equation of the circle in this new position. **5**

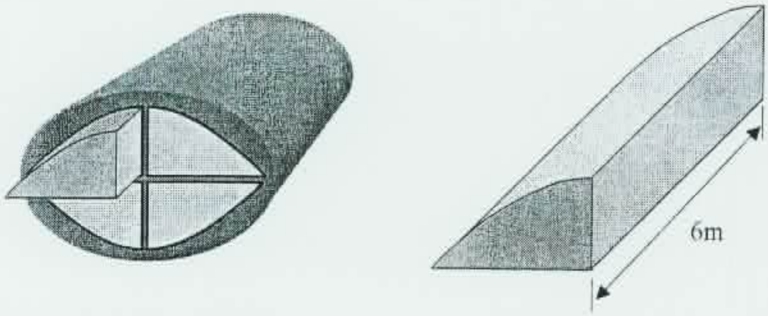
**5.** A sequence is defined by the recurrence relation $U_{n+1} = aU_n + b$, where $a$ and $b$ are constants.

(*a*) Given that $U_0 = a - 2$ and $b = 1$, show clearly that $U_1 = a^2 - 2a + 1$. **2**

(*b*) Hence find an expression for $U_2$ in terms of $a$. **2**

(c) Given now that $U_2 = 37$, form an equation and solve it to find $a$.
Explain why there is only one possible answer for $a$. **4**

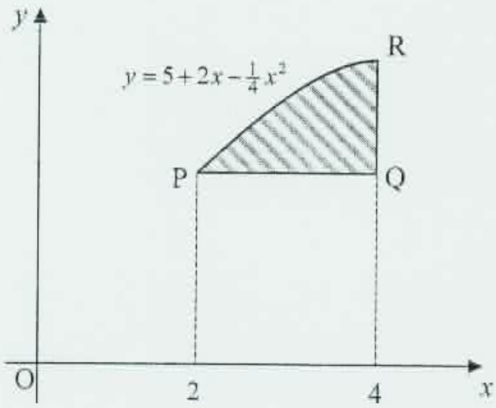
6. A titanium rod from a nuclear reactor is a solid prism which slots into an elliptical chamber along with three other identical rods. It has a cross-sectional shape made up of two straight lines and a curved edge.




Each rod has a depth of 6 metres.

The cross section of a rod is shown geometrically in the coordinate diagram below where the **units are in metres**. The diagram is not drawn to scale.

The curved section is part of the graph of the curve with equation $y = 5 + 2x - \frac{1}{4}x^2$.
PQ is horizontal and QR is vertical.




(a) Calculate the shaded area in square metres. **7**

(b) Hence calculate the **total volume** of titanium contained in **all four rods**. **2**

7. The angle $\theta$ is such that $\tan\theta = \frac{2}{\sqrt{2}}$ where $0 < \theta < \frac{\pi}{2}$.

(a) Find the exact values of $\sin\theta$ and $\cos\theta$. **3**

(b) Hence show clearly that the exact value of $\sin(\theta + \frac{\pi}{3})$ can be expressed as

$$\sin(\theta + \tfrac{\pi}{3}) = \tfrac{1}{6}(\sqrt{6} + 3).$$ **5**

**8.** Three functions are defined on suitable domains as

$$f(x) = x - 1\ ,\ \ g(x) = 3x^2 - 3\ \text{ and }\ h(x) = x^3 - 6x\ .$$

(a) Given that $y = g(f(x)) - h(x)$, find a formula for $y$ in its simplest form. 3

(b) Hence find the coordinates of the maximum turning point of the graph of $y = g(f(x)) - h(x)$, **justifying your answer**. 4

**9.** An equation is given as $ax(x-1) = c(x-1)$, where $a \neq 0,\ c \neq 0$, and $a$ and $c$ are constants.

(a) Show clearly that this equation can be written in the form

$$ax^2 - (a+c)x + c = 0\ .$$ 2

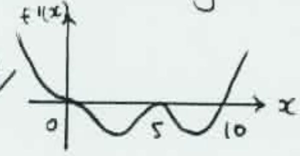
(b) What condition needs to be met for this quadratic equation to have equal roots? 4

[ END OF QUESTION PAPER ]

# ANSWERS

## 2005 PAPER 1

1/(a) $M(9,3)$ (b) $x - 3y = 0$ (c) $y + 3x - 60 = 0$ (d) Congruent

2/ $m = 2$ 3/ 

4/ $a = 2,\ b = 1$ 5/(a) $x = 4$

(b) $\theta = \frac{\pi}{9}$ 6/ $a = \frac{2}{3}$ 7/(a) $-1 < 0.6 < 1,\ -1 < 0.9 < 1$

(b) $b = 5$ 8/(a) $x^2 + y^2 - 10y = 0$ (b) $k = 2$ (c) $4x - 3y - 10 = 0$

9/(a) $(3p+3)x^2 - 6px + 4$ (b) $p = 2$

## 2005 Paper 2

1/(a) $3y = 2x + 3$ (b) $C(3,3)$ (c) $67°$ 2/(a) 432 feet

(b) No. $h'(2.7) = 28.8$ (c) 420 feet 3/ $194.5°,\ 345.5°$

4/(a) $P(4,6)$ (b) $(x-14)^2 + (y-16)^2 = 20$ 5/(b) $u_2 = a^3 - 2a^2 + a + 1$

(c) $a = 4$, $b^2 - 4ac < 0$ for $a^2 + 2a + 9$ 6/(a) $1\frac{1}{3}$ (b) $32m^3$

7/(a) $\sin\theta = \frac{2}{\sqrt{6}}$, $\cos\theta = \frac{1}{\sqrt{3}}$ (b) $\frac{1}{6}(\sqrt{6} + 3)$ 8/(a) $y = 3x^2 - x^3$

(b) $(2, 4)$ 9/(b) $a = c$