H

Mathematics
Higher
Paper 1

Time allowed
1 hour 30 minutes
70 marks

NATIONAL
QUALIFICATIONS

Read carefully

Calculators may <u>NOT</u> be used in this paper.

Section A – Questions 1 – 20 (40 marks)

Instructions for completion of Section A are given on page three.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks).

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read Carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and Centre Name printed on it.

4 If any of this information is wrong, tell the invigilator immediately.

5 If this information is correct, print your name and seat number in the boxes provided.

6 The answer to each question is either A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below.)

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on the answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

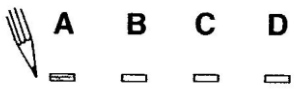
A 8

B 1

C 0

D –4

The correct answer is **A** – 8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Section A

1. The line joining the points (-2,-3) and (6,k) has gradient $\frac{2}{3}$
What is the value of k?

A $\frac{14}{3}$

B $\frac{7}{3}$

C $-\frac{1}{3}$

D -9

2. A sequence is defined by the recurrence relation $u_{n+1} = 0 \cdot 75u_n + 10,$ with $u_0 = 8$.
What is the value of u_2?

A $10 \cdot 75$

B 16

C 19

D 22

3. If $f(x) = 2x^{-3} - 3x^{\frac{1}{2}},\ x > 0,$ find $f'(x)$.

A $-6x^{-4} - 2x^{\frac{3}{2}}$

B $-6x^{-4} - \frac{3}{2}x^{-\frac{1}{2}}$

C $-6x^{-2} - 2x^{\frac{3}{2}}$

D $-6x^{-2} - \frac{3}{2}x^{-\frac{1}{2}}$

4. A tangent to the curve with equation $y = x^2 + 3x + 5$ is drawn at the point (1, 9). What is the gradient of this tangent?

A 2
B 5
C 10
D 21

5. Determine $\int (2x^{\frac{1}{2}} + x^{-2})\,dx$, where $x > 0$.

A $x^{\frac{3}{2}} - 2x^{-3} + c$

B $\frac{4}{3}x^{\frac{3}{2}} - 2x^{-3} + c$

C $x^{\frac{3}{2}} - x^{-1} + c$

D $\frac{4}{3}x^{\frac{3}{2}} - x^{-1} + c$

6. The functions f and g are defined on the set of real numbers by $f(x) = 1 - 2x$ and $g(x) = x^2$
What is the value of $f(g(-2))$?

A –9

B –7

C 9

D 20

7. A straight line passes through the points P(−5, −2) and Q(−2, −1).
What is the equation of the straight line which passes through P and is perpendicular to PQ ?

A $y + 2 = -3(x + 5)$

B $y - 2 = -\frac{3}{7}(x - 5)$

C $y - 1 = -\frac{3}{7}(x - 2)$

D $y - 1 = -\frac{1}{3}(x - 2)$

8. If $\frac{dy}{dx} = 2x + 1$ and $y = 3$ when $x = 1$, express y in terms of x.

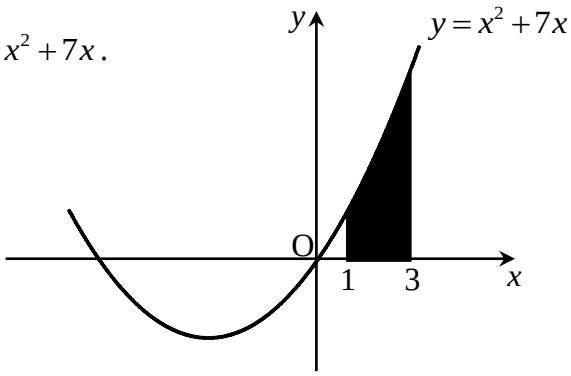
A $y = x^2$

B $y = x^2 + x + 1$

C $y = 2$

D $y = x^2 + 2$

9. The diagram shows the graph of $y = x^2 + 7x$.



Which of the following gives the value of the shaded area?

A $\left[\frac{1}{3}x^3 + \frac{7}{2}x^2\right]_1^3$

B $\left[\frac{1}{3}x^3 + 7\right]_1^3$

C $\left[2x^3 + \frac{7}{2}x^2\right]_1^3$

D $\left[2x^3 + 7\right]_1^3$

10. A fish farm starts with a stock of 5000 fish. Each Friday 30% of the fish are removed for sale and it is then restocked with 400 new fish.
Let u_n represent the number of fish after restocking n times.

What is the recurrence relation that describes the situation after restocking?

A $u_{n+1} = 0.3u_n + 400$ and $u_0 = 5000$

B $u_{n+1} = 0.7u_n + 400$ and $u_0 = 5000$

C $u_{n+1} = 0.3(u_n + 400)$ and $u_0 = 5000$

D $u_{n+1} = 0.7(u_n + 400)$ and $u_0 = 5000$

11. Here are two statements about the roots of the quadratic equation $3x^2 + x - 2 = 0$:

(1) The equation has equal roots.

(2) The equation has rational roots.

Which of the following is true?

A Neither statement is correct.
B Only statement (1) is correct.
C Only statement (2) is correct.
D Both statements are correct.

12.

A sequence is defined by the recurrence relation
$u_{n+1} = 0 \cdot 6u_n + k$ and $u_0 = 3$.

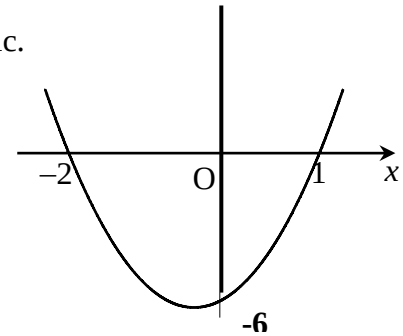
As $n \to \infty$, the limit of this sequence is 5.

What is the value of k?

A 0
B 0·88
C 2
D 8

13. The diagram shows the graph of a quadratic.

What is the equation of this parabola?



A $y = -(x+2)(x-1)$
B $y = -3(x-2)(x+1)$
C $y = (x-2)(x+1)$
D $y = 3(x+2)(x-1)$

14. If $f(x) = x\sqrt{x}$, where $x > 0$, find $f'(x)$.

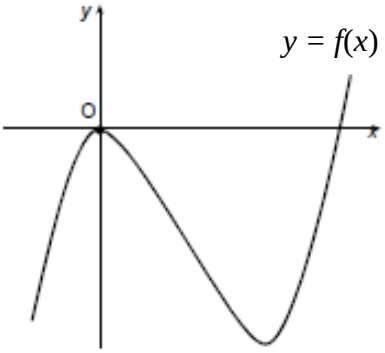
A $1 + \dfrac{1}{2\sqrt{x}}$

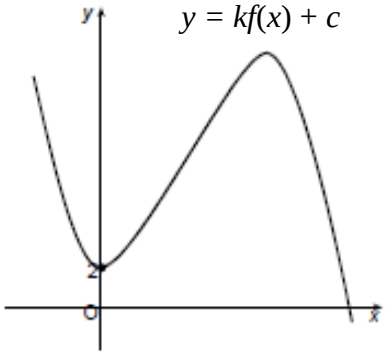
B $\sqrt{x}$

C $\dfrac{3}{2}\sqrt{x}$

D $\dfrac{2}{5}x^{\frac{5}{2}}$

15. The diagram show the sketches of $y = f(x)$ and $y = kf(x) + c$.





What are the values of k and c ?

	k	c
A	−1	2
B	−1	−2
C	1	2
D	1	−2

16. A ball is projected from the ground. The height, h metres, above the ground after t seconds is given by $h(t) = 10t - t^2$.
What is the rate of change of h after 3 seconds?

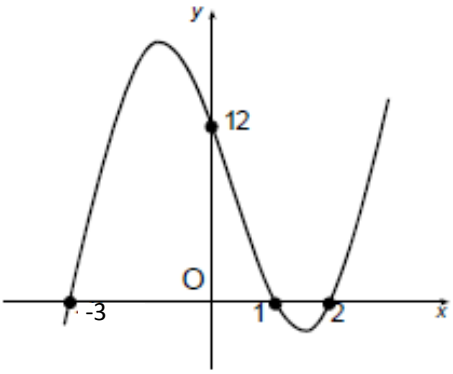
A 4
B 5
C 21
D 36

17. For what values of x is function f, such that $f'(x) = (x-3)(x+2)$, increasing?

A $x = -2,\ x = 3$
B $x \le -2,\ x \ge 3$
C $-2 < x < 3$
D $x < -2,\ x > 3$

18.

The diagram shows part of the graph of a cubic function.



What is the equation of this graph ?

A $y = 2(x-2)(x-1)(x+3)$

B $y = 12(x-2)(x-1)(x+3)$

C $y = -2(x-3)(x+1)(x+2)$

D $y = 12(x-3)(x+1)(x+2)$

19. What is the remainder when $4x^2 + 6x - 2$ is divided by $2x - 1$?

A -4
B -3
C 2
D 8

20. The curve with equation $y = 3x^2 + ax - 5$ has a stationary point at $x = -1$.
What is the value of a?

A -11
B -3
C -2
D 6

END OF SECTION A

SECTION B
ALL questions should be attempted

21. Find the equation of the straight line which is parallel to the line with the equation $2x + 3y = 5$ and which passes through the point (2, -1) **3**

22. (a) (i) Show that $(x - 1)$ is a factor of $f(x) = 2x^3 - x^2 - 4x + 3$
(ii) Hence factorise $f(x)$ fully. **5**

(b) Solve $2x^3 - x^2 - 4x + 3 = 0$ **1**

(c) The line with equation y = 2x -1 is a tangent to the curve with equation $y = 2x^3 - x^2 - 2x + 2$ at the point G.
Find the coordinates of G. **5**

(d) This tangent meets the curve again at the point H.
Write down the coordinates of H. **1**

23. (a) Show that the function $f(x) = 2x^2 + 8x - 3$ can be written in the form

$f(x) = a(x + b)^2 + c$ where a, b and c are constants. **3**

(b) On the same diagram sketch:

i) the graph of $y = f(x)$
ii) the graph of $y = -9 - f(x)$ **4**

(c) Find the range of values of x for which $-9 - f(x)$ is positive. **1**

24. Functions $f(x) = \dfrac{1}{x - 5}$ and $g(x) = 2x - 1$ are defined on suitable domains.

(a) Find an expression for $h(x)$ where $h(x) = f(g(x))$. **2**
(b) Write down any restriction on the domain of h. **1**

25. Two sequences are defined by the recurrence relations

$U_{n+1} = 0.2U_n + p,\ U_0 = 1$ and
$V_{n+1} = 0.6V_n + q,\ V_0 = 1.$

If both sequences have the same limit, express p in terms of q. **3**

END OF QUESTION PAPER