H

Mathematics
Higher
Paper 2

Time allowed
1 hour 10 minutes
58 marks

NATIONAL
QUALIFICATIONS

Read carefully

1. Calculators may be used in this paper.

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

January 2015

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

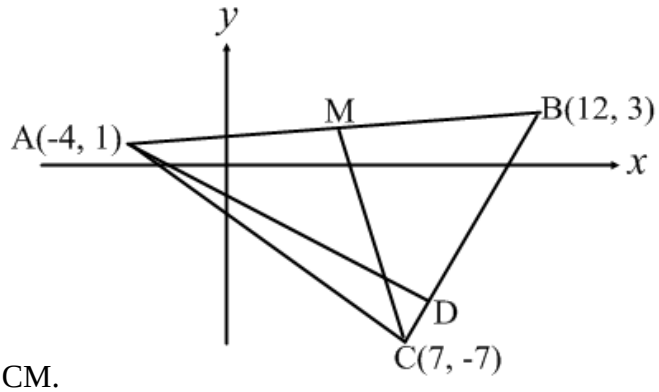
$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL QUESTIONS SHOULD BE ATTEMPTED

1. A triangle ABC has vertices A(-4, 1), B(12, 3) and C(7, -7).



a) Find the equation of the median CM. **3**

b) Find the equation of the altitude AD. **3**

c) Find the coordinates of the point of intersection of CM and AD. **3**

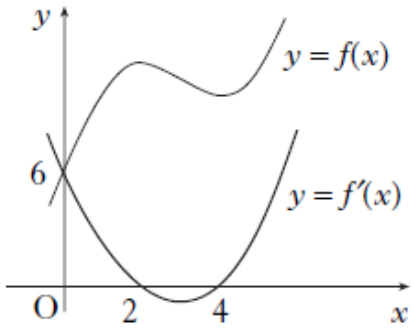
2 The point P(-1, 7) lies on the curve with equation $y = 5x^2 + 2$. Find the equation of the tangent to the curve at P. **4**

3.

The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.

Both graphs pass through the point (0, 6).

The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).



(*a*) Given that $f'(x)$ is of the form $k(x - a)(x - b)$:

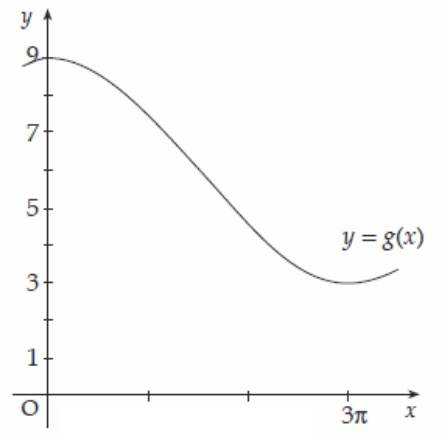
(i) write down the values of a and b;

(ii) find the value of k. **3**

(*b*) Find the equation of the graph of the cubic function $y = f(x)$. **4**

4. The diagram shows an incomplete sketch of the graph of a function g where g is of the form $g(x) = a + b\cos(cx)$.

The graph has a maximum turning point at (0,9) and a minimum turning point at $(3\pi, 3)$.



Find the values of a, b and c. **3**

5 Find the value of k such that the equation $\dfrac{(x-2)^2}{x^2+2} = k$, $k \neq 0$ has equal roots. **5**

6.

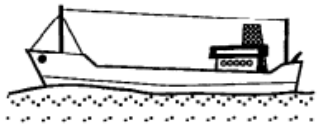
Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

(*a*) (i) Find $p(x)$ where $p(x) = f(g(x))$.

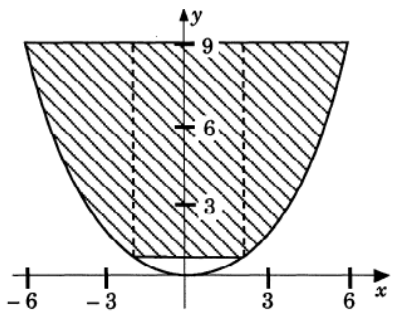
(ii) Find $q(x)$ where $q(x) = g(f(x))$. **3**

(*b*) Solve $p'(x) = q'(x)$. **3**

7. The cargo space of a small bulk carrier is 60m long .



The shaded part of the diagram represents the uniform cross-section of this space. It is shaped like the parabola with equation $y = \frac{1}{4}x^2$, $-6 \le x \le 6$, between the lines $y = 1$ and $y = 9$. Find the area of this cross-section and hence find the volume of cargo that this ship can carry. **9**



(Each unit represents 1m)

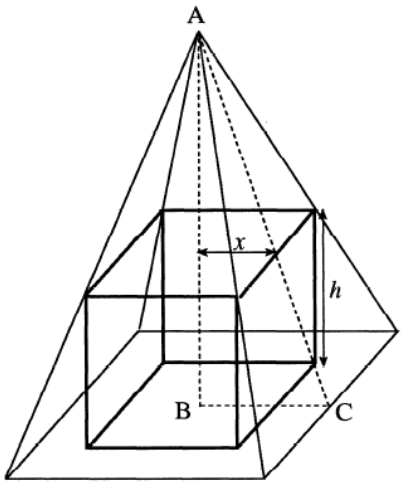
8 Two sequences are generated by the recurrence relations $U_{n+1} = aU_n + 5$ and $V_{n+1} = a^2V_n + 8$.

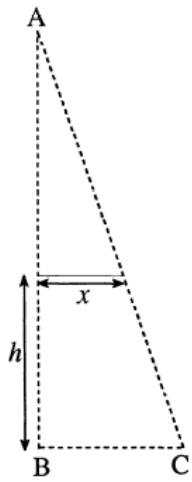
The two sequences approach the same limit as $n \to \infty$.

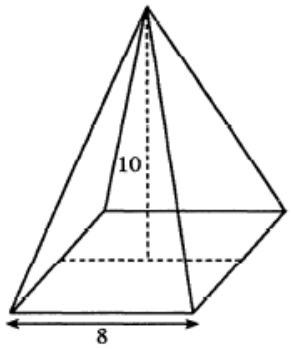
Determine the value of a and evaluate this limit. **5**

9 A cuboid is to be cut out of a right square-based pyramid. The pyramid has a square base of side 8 cm and a vertical height of 10 cm.

(a) The cuboid has a square base of side $2x$ cm and a height of h cm.







If the cuboid is to fit into the pyramid, use the information shown in triangle ABC, or otherwise, to show that

(i) $h = 10 - \frac{5}{2}x.$ **3**

(ii) the volume, V, of the cuboid is given by $V = 40x^2 - 10x^3$ **1**

(b) Hence find the dimensions of the square-based cuboid with the greatest volume which can be cut from the pyramid. **6**

[END OF QUESTION PAPER]