Glasgow Secondary Schools

Maths Higher Prelim 2014 Paper II Solutions

Question 1a)

- ●[1] Midpoint M = (4,2)
- ●[2] $m_{CM} = -3$
- ●[3] $y - 2 = -3(x - 4)$

or alternatively $y + 7 = -3(x - 7)$

Question 1b)

- ●[4] $m_{BC} = 2$
- ●[5] $m_{AD} = -\frac{1}{2}$
- ●[6] $y - 1 = -\frac{1}{2}(x + 4)$

Question 1c)

- ●[7] use $y = -3x + 14$ **and** $y = -\frac{1}{2}x - 1$

  (or equivalent)

- ●[8] $x = 6$
- ●[9] $y = -4$

Question 2

●[1]	$\frac{dy}{dx}$	Know to differentiate
●[2]	$10x$	Correct derivative
●[3]	$m = -10$	Evaluate gradient
●[4]	$y-7 = -10(x + 1)$	Correct equation

Question 3

The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$	ic	interpret roots on diagram
$\bullet^2$	ss	know to use y-intercept
$\bullet^3$	pd	process
$\bullet^4$	ss	know to integrate
$\bullet^5$	pd	integrate
$\bullet^6$	ic	express as an equation
$\bullet^7$	ic	interpret constant of integration

Primary Method : Give 1 mark for each ·

$\bullet^1$	$a = 2$ *and* $b = 4$ ***or*** $k(x-2)(x-4)$	
$\bullet^2$	$6 = k(0-2)(0-4)$	
$\bullet^3$	$k = \dfrac{3}{4}$	
$\bullet^4$	$\int \left(\frac{3}{4}(x-2)(x-4)\right) dx$	s/i by $\bullet^5$
$\bullet^5$	*any two terms integrated correctly* ($\frac{3}{12}x^3$ *etc*)	
$\bullet^6$	$y = \frac{1}{4}x^3 - \frac{9}{4}x^2 + 6x + c$	
$\bullet^7$	$c = 6$	

Notes 1

1 For candidates who fail to complete (a) but produce values for k, a and b *ex nihilo*, all 4 marks are available in (b).
A deduction of 1 mark may be made if their choice eases the working.

2 In (b)
For candidates who use $k = 1$, a "fully correct" follow-through solution may be awarded 3 out of the last 4 marks

3 For candidates who retain "k", "a" and "b", $\bullet^4$, $\bullet^5$, $\bullet^6$ and $\bullet^7$ are still available.

Question 4

ans: $a = 6,\ b = 3,\ c = \frac{1}{3}$ 3 marks

- $\bullet^1$ ic : interpret sketch of function
- $\bullet^2$ ic : interpret sketch of function
- $\bullet^3$ ic : interpret sketch of function

- $\bullet^1$ $a = 6$
- $\bullet^2$ $b = 3$
- $\bullet^3$ $c = \frac{1}{3}$

Question 5

- $\bullet^1$ $(1- k)x^2 – 4x + 4 -2k = 0$
- $\bullet^2$ $b^2 – 4ac = 0$ (for equal roots)
- $\bullet^3$ $16 – 4(1- k)(4 – 2k) = 0$
- $\bullet^4$ Solve to give $k = 0$ and $k = 3$
- $\bullet^5$ State $k \neq 0$, so k must be 3.

6

The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$	ss	substitute for $g(x)$ in $f(x)$
$\bullet^2$	ic	complete
$\bullet^3$	ic	sub. and complete for $q(x)$
$\bullet^4$	ss	simplify
$\bullet^5$	pd	differentiate
$\bullet^6$	pd	solve

Primary Method : Give 1 mark for each $\bullet$

$\bullet^1$ $f(x^2-2)$ **s / i by $\bullet^2$**

$\bullet^2$ $3(x^2-2)+1$

$\bullet^3$ $(3x+1)^2-2$

	$\bullet^4$	$\bullet^5$
$\bullet^4$	$3x^2-5$	$9x^2+6x-1$
$\bullet^5$	$6x$	$18x+6$ *or equiv.*

s / i by $\bullet^5$

$\bullet^6$ $x=-\frac{1}{2}$

Notes

1. In (a)
   2 marks are available for finding either $f\left(g(x)\right)$ or $g\left(f(x)\right)$ and 1 mark for finding the other.
2. In (b)
   candidates who start by equating $p(x)$ and $q(x)$ and then differentiate may earn $\bullet^4$ and $\bullet^6$ only.

Common Errors

1

$p(x)$ and $q(x)$ switched round:

X $\bullet^1$ $p(x)=g(3x+1)$

$X\surd$ $\bullet^2$ $p(x)=(3x+1)^2-2$

$X\surd$ $\bullet^3$ $q(x)=\ldots\ldots=3(x^2-2)+1$

2

Candidates who find $f(f(x))$ and $g(g(x))$ can earn no marks in (a) but

$X\surd$ $\bullet^4$ $9x+4$ *and* x^4-4x^2+2

$X\surd$ $\bullet^5$ $9=4x^3-8x$

XX $\bullet^6$ *not available*

3

X $\bullet^4$ $3x^2-1$ *and* $9x^2+6x-1$

$X\surd$ $\bullet^5$ $6x$ *and* $18x+6$

$X\surd$ $\bullet^6$ $x=-\frac{1}{2}$

Alternative for $\bullet^1$ *to* $\bullet^3$:

$\bullet^1$ $f\left(g(x)\right)=3\times g(x)+1$

$\bullet^2$ $f\left(g(x)\right)=3\left(x^2-2\right)+1$

$g\left(f(x)\right)=\left(f(x)\right)^2-2$

$\bullet^3$ $g\left(f(x)\right)=(3x+1)^2-2$

Question 7 (Method 1)

- $\bullet^1$ strategy: split into approp. parts
- $\bullet^2$ $y = 1 \Rightarrow x = \pm 2$
- $\bullet^3$ first rectangular area
- $\bullet^4$ $9 - \frac{1}{4}x^2$ for integrand of shaped area
- $\bullet^5$ $\int_2^6 dx$ for limits of shaped area
- $\bullet^6$ for integrating.....$\left(9x - \frac{1}{12}x^3\right)$
- $\bullet^7$ for evaluating.....$\left(\frac{56}{3}\right)$
- $\bullet^8$ total cross – sectional area $= \frac{208}{3}(m^2)$
- $\bullet^9$ volume $= 4160\ (m^3)$

(Method 2) Please see next page

Alternative marking scheme based on these criteria.

The Primary Method m/s is based on the following generic m/s.
THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME.

- $\bullet^1$ ss : find intersections
- $\bullet^2$ pd : process quadratic to solution
- $\bullet^3$ ss : decide on appropriate areas
- $\bullet^4$ ss : know to integrate
- $\bullet^5$ ic : state limits
- $\bullet^6$ pd : integrate
- $\bullet^7$ pd : evaluate using limits
- $\bullet^8$ pd : evaluate area

Primary Method : Give 1 mark for each •

- $\bullet^1$ $2x - \frac{1}{2}x^2 = 1 \cdot 5$
- $\bullet^2$ $x = 1, x = 3$
- $\bullet^3$ "*split area up*" *stated or implied by* $\bullet^4$
- $\bullet^4$ $\int\left(2x - \frac{1}{2}x^2 - \frac{3}{2}\right)dx$
- $\bullet^5$ $\int_1^3 ..dx$
- $\bullet^6$ $\left[x^2 - \frac{1}{6}x^3 - \frac{3}{2}x\right]_1^3$
- $\bullet^7$ $\left(3^2 - \frac{1}{6}.3^3 - \frac{3}{2}.3\right) - \left(1^2 - \frac{1}{6}.1^3 - \frac{3}{2}.1\right)$
- $\bullet^8$ $\frac{2}{3}$

8 marks

Notes

1 The first two marks may be obtained as follows:

Guess $x = 1$ and check that $y = 1.5$, award $\bullet^1$

Guess $x = 3$ and check that $y = 1.5$, award $\bullet^2$

2 In the Primary method, $\bullet^3$ is clearly not available for subtracting the wrong way round.

$\bullet^8$ will also be lost for statements such as

$-\frac{2}{3} = \frac{2}{3}$

$-\frac{2}{3}$ so ignore the negative

$-\frac{2}{3} = \frac{2}{3}$ squ units

$\bullet^8$ can still be gained for statements such as

$\ldots -\frac{2}{3}$ and so the area $= \frac{2}{3}$

1 Alternative Method

- $\bullet^1$ $2x - \frac{1}{2}x^2 = 1 \cdot 5$
- $\bullet^2$ $x = 1, x = 3$
- $\bullet^3$ $\int\left(2x - \frac{1}{2}x^2\right)dx$
- $\bullet^4$ choose limits, a and b: $0 \le a \le b \le 4$
- $\bullet^5$ $\left[x^2 - \frac{1}{6}x^3\right]$
- $\bullet^6$ evaluate $\left[x^2 - \frac{1}{6}x^3\right]_a^b$ for chosen values of a and b
- $\bullet^7$ state areas to be added/subtracted *st / imp by* $\bullet^8$
- $\bullet^8$ $\frac{2}{3}$

8 marks

- 9 evaluating volume correctly

PLEASE NOTE

- A mark was allocated for recognising that integration was required but if the only integration was $\int \frac{1}{4}x^2 dx$ no mark was awarded for the resultant $\frac{1}{12}x^3$ as this was considered to be simplified integration.
- Point 2 $\cdot^8$ in the above notes was heeded.

Question 8

●[1] $L = aL + 5$ **or** $L = a^2L + 8$ **or** $L = \frac{b}{1-a}$

●[2] $L = \frac{5}{1-a}$ **or** $L = \frac{8}{1-a^2}$

●[3] $\frac{5}{1-a} = \frac{8}{1-a^2}$

●[4] $5a^2 - 8a + 3 = 0$

●[5] $a = 3/5$ **and** $L = 25/2$

Question 9

(a) •[1] strategy: e.g. equate ratios from similar triangles

•[2] $\frac{10}{4} = \frac{10-h}{x}$ or equivalent

•[3] complete proof

•[4] $V = 40x^2 - 10x^3$

(b) •[5] $\frac{dV}{dx} =$

•[6] $80x - 30x^2$

•[7] $\frac{dV}{dx} = 0$ for stationary points

•[8] $0, \frac{8}{3}$

•[9]

x	…	$\frac{8}{3}$	…
$\frac{dV}{dx}$	+	0	−
		max	

•[10] $\frac{16}{3}$ and $\frac{10}{3}$