

Belmont Academy

Department of Mathematics



Higher
Mathematics

Prelim
Paper 1

Time : 1 Hour 10 minutes

JANUARY 2004

Read Carefully

1. Calculators may **NOT** be used in this paper.
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. Question paper must not be removed from examination room.

Higher Prelim

Formulae List

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle with centre $(-g, -f)$ and radius $r = \sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle, centre (a, b) and radius r .

Trigonometric Formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

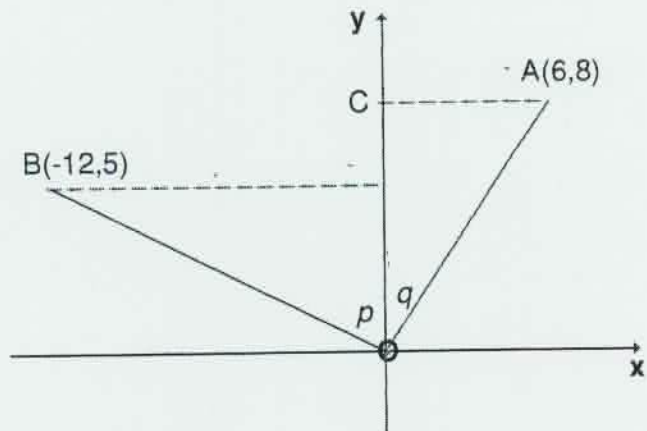
$$\sin 2A = 2\sin A \cos A$$

All questions should be attempted.

1. On the coordinate diagram shown ,
A is the point (6,8) and B is the
point (-12,5).
Angle AOC = q and angle COB = p .

Find the exact value of :

- (i) $\cos (p + q)$
(ii) $\sin 2q$



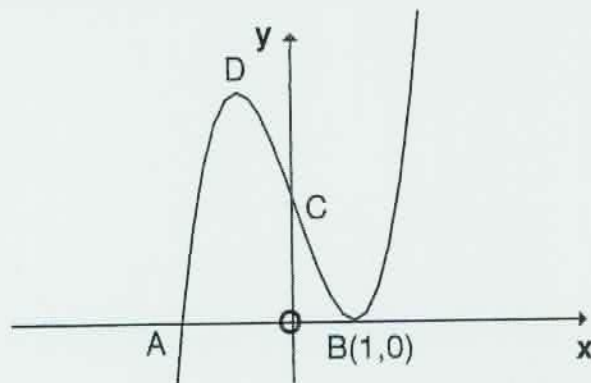
[4]

[2]

2. The three points P(-6,1) Q(2,-7) and R(k,-3) are collinear.
Find the value of k.

[3]

3. The graph below is a sketch of the function $f(x) = x^3 - 3x + 2$.



- (a) The graph crosses the y-axis at C and touches the x - axis at B(1,0).

(i) Write down the coordinates of C.

[1]

(ii) Find the coordinates of A.

[5]

- (b) Find the coordinates of the turning point at D.

[4]

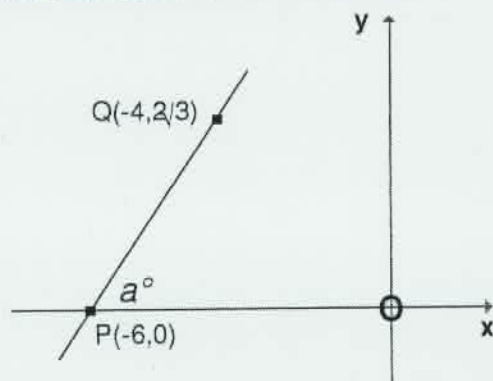
- (c) Hence sketch the graph of $y = f(-x)$.

[2]

On the same diagram , sketch the graph of $y = 2f(-x)$.

[2]

4. Find the size of the angle a° that the line joining the points $P(-6,0)$ and $Q(-4,2\sqrt{3})$ makes with the positive direction of the x -axis.



[4]

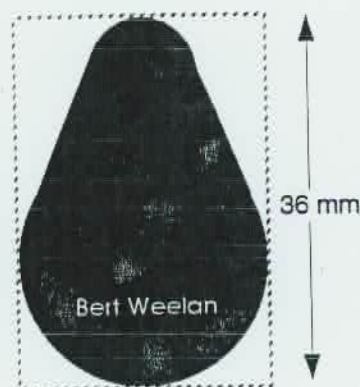
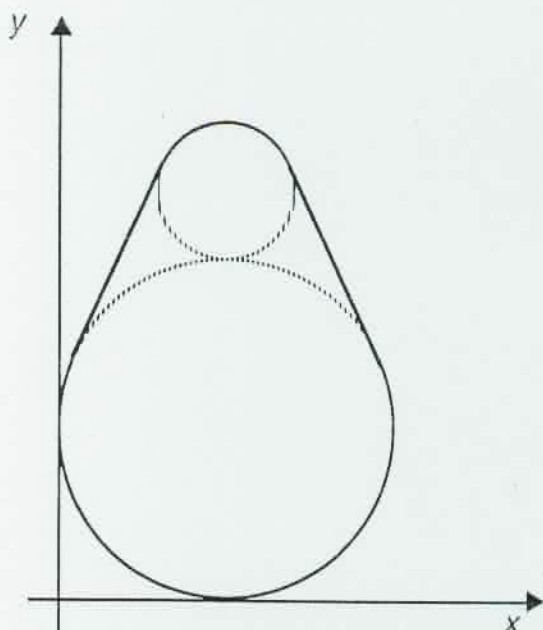
5. The functions f and g are defined on suitable domains by $f(x) = x - 5$ and $g(x) = \frac{4}{x}$.

Find an expression for the function $h(x) = g(f(x))$ and state any restriction on the domain of h .

[2]

6. A new Bert Weelan guitar pickum is designed based on two touching circles.

When it was designed, the planner drew it as shown below, with the larger circle just touching the axes.



The equation of the larger circle is $x^2 + y^2 - 24x - 24y + 144 = 0$.

Find the equation of the smaller circle.

[5]

7. Solve : $2\sin 3x - 1 = 0$ for $0 \leq x \leq \pi$

[4]

8. A curve is such that its derivative is defined as $\frac{dy}{dx} = x - \frac{4}{x^2}$.

If the curve passes through the point $(-4, \frac{5}{2})$, find the equation of the above curve.

[5]

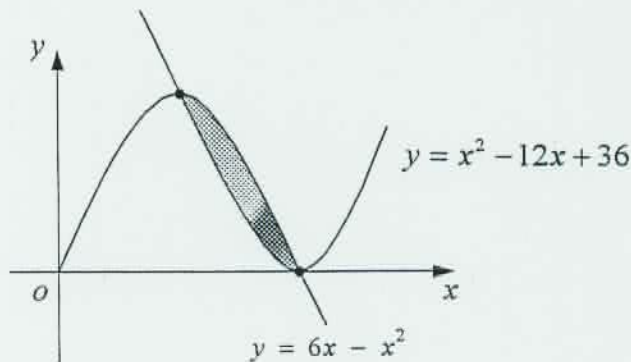
9. (a) Express the function $f(x) = (2x - 1)(2x + 9)$ in the form $a(x + b)^2 + c$, where a , b and c are constants.

[4]

(b) Hence find the coordinates of the turning point of the function f .

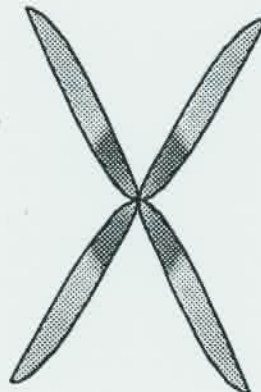
[1]

10. The diagram below shows parts of the graphs of two quadratic curves with equations $y = 6x - x^2$ and $y = x^2 - 12x + 36$.



The design below has been created by reflecting the area enclosed between the two curves in the horizontal and vertical axes.

Calculate the area of this design.



[6]

11. Find the values of k given that the equation

$$(8k + 4)x^2 + 4kx + k = 8x + 2 \text{ has real roots.}$$

[6]

END OF QUESTION PAPER

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Higher
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Prelim
Paper 2

Time : 1 hour 30 minutes

JANUARY 2004

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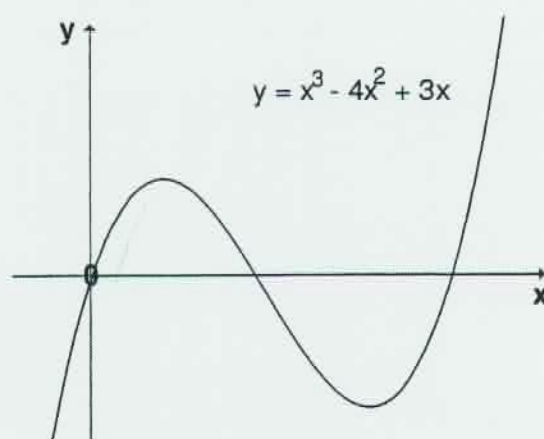
$$= 1 - 2\sin^2 A$$

$$\sin 2A = 2\sin A \cos A$$

All questions should be attempted.

1. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + 3x$.

- (a) Find the equation of the tangent to this curve at the point where $x = 1$. [5]
- (b) The tangent at the point $(3,0)$ has equation $y = 6x - 18$. Find the coordinates of the point where this tangent meets the curve again. [6]

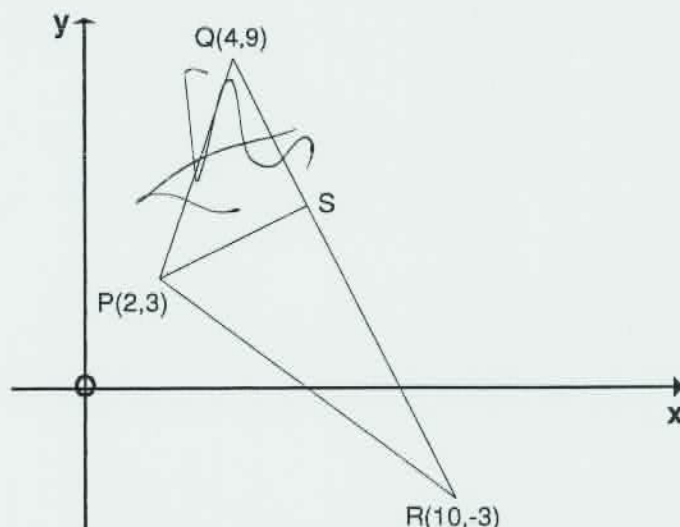


2. A woman borrowed £6000 to buy a car and agreed to make monthly payments of £150. Interest is calculated annually at 12% on the outstanding balance due at the start of each year.

- (a) Establish a recurrence relation to describe the amount, A_{n+1} , owing at the end of each year in relation to the amount, A_n , owed at the end of the previous year. [2]

- (b) During which year will she succeed in paying off the loan? [3]

3. P, Q and R are the points with coordinates $(2,3)$, $(4,9)$ and $(10,-3)$ respectively.



- (a) Find the equation of PS, the altitude of triangle PQR. [3]

- (b) Determine the equation of the circle which passes through the points P, S and R. [4]

- (c) Show that the line $x = 1$ is a tangent to this circle. [3]

4. Two functions are defined on suitable domains as $f(x) = \frac{3x + p}{2}$ and $g(x) = p^2 + x$.

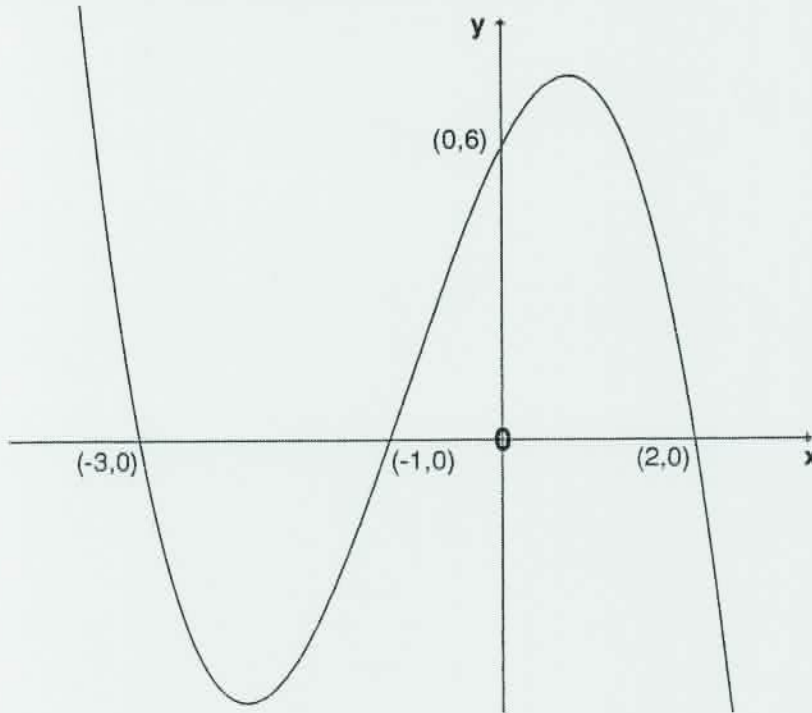
(a) Find expressions for $f(g(x))$ and $g(f(x))$.

[4]

(b) Find the value of the constant p when $f(g(x)) - g(f(x)) = 18$ and $p > 0$.

[3]

5. The diagram shows the graph of a cubic function.



Find the equation of the graph of this function $y = f(x)$.

[4]

6. Two sequences are defined by the recurrence relations :

$$u_{n+1} = au_n + 3, \quad u_0 = 1 \quad \text{and}$$

$$v_{n+1} = bv_n + 2, \quad v_0 = 1$$

(a) Write down the restriction on a and b if these sequences have a limit.

[1]

(b) If both sequences have the same limit, express a in terms of b .

[3]

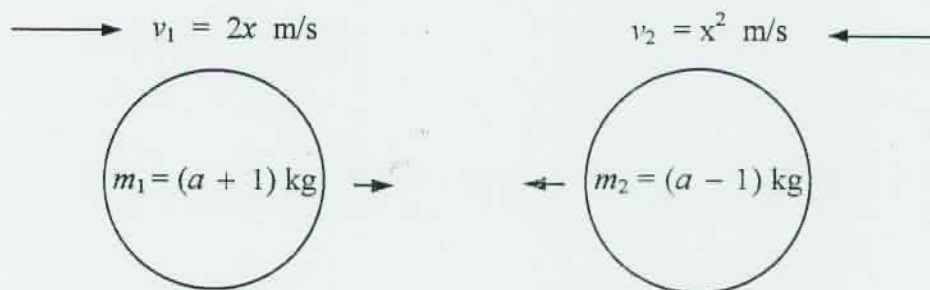
7. The function $f(x) = \frac{4 + kx - 2x^2}{x^3}$, $x \neq 0$, has a stationary point at $x = -2$.

Find the value of k , where k is a constant.

[6]

8. The two objects shown below collide on a horizontal surface.

The momentum (M) of each object before the collision can be found using the formula $M = mv$, where m is the mass of the object, in *kilograms*, and v is its velocity in *metres per second*.



- (a) Given that the difference between the two momenta before the impact is 9, with the object on the left having a greater momentum, show that the following equation can be constructed :

$$(a - 1)x^2 - (2a + 2)x + 9 = 0 . \quad [3]$$

- (b) Given that $a > 2$, find the value of a for which the equation $(a - 1)x^2 - (2a + 2)x + 9 = 0$ has **equal** roots.

[4]

- (c) Calculate the velocity of each object **before** impact.

[3]

9. Solve the equation $4\cos 2x^\circ - 2\cos x^\circ = -1$ in the interval $0^\circ \leq x < 360^\circ$.

[5]

10. A manufacturer designs a cylindrical can to hold 500 ml of oil.

The radius of the can is r centimetres and the height of the can is h centimetres.



The cost of the can depends on its surface area.

(a) Show that the surface area, S , of the can is given by :

$$S(r) = \frac{1000}{r} + 2\pi r^2, \text{ where } r \text{ is the radius of the can.}$$

[2]

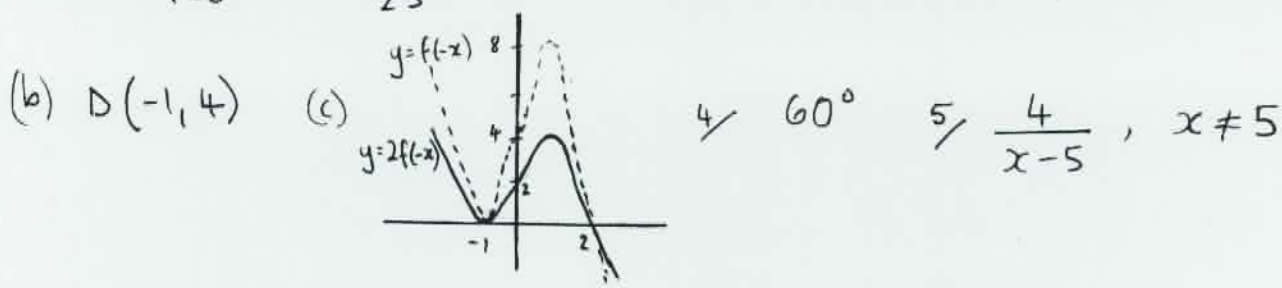
(b) Find the value of r which makes the cost a minimum.

[6]

ANSWERS

2004 PAPER 1

1/ (i) $-\frac{32}{120}$ (ii) $\frac{24}{25}$ 2/ $k = -2$ 3/ (a) (i) $C(0, 2)$ (ii) $A(-2, 0)$



6/ $(x-30)^2 + (y-12)^2 = 36$ 7/ $\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$

8/ $y = \frac{x^2}{2} - \frac{4}{x} - \frac{9}{2}$ 9/ (a) $4(x+2)^2 - 25$ (b) $(-2, -25)$

10/ 36 units^2 11/ $-3 \leq k \leq 2$

2004 PAPER 2

1/ (a) $y = -2x + 2$ (b) $(-2, 30)$ 2/ (a) $A_{n+1} = 1.12A_n - 1800$

(b) Year 5 3/ (a) $2y = x + 4$ (b) $(x-6)^2 + y^2 = 25$

4/ (a) $f(g(x)) = \frac{3p^2 + 3x + p}{2}$ $g(f(x)) = p^2 + \frac{3x+p}{2}$ (b) $p = 6$

5/ $y = -(x+3)(x+1)(x-2)$ 6/ (a) $-1 < a < 1, -1 < b < 1$ (b) $a = \frac{3b-1}{2}$

7/ $k = 1$ 8/ (b) $a = 5$ (c) $V_1 = 3 \text{ ms}^{-1}, V_2 = 2\frac{1}{4} \text{ ms}^{-1}$

9/ $41.4^\circ, 120^\circ, 240^\circ, 318.6^\circ$ 10/ (b) $r = 4.3 \text{ cm}$