

Sequences and Recurrence Relations 1

1.
 - (a) If £300 is invested at an interest rate of 4.5% per annum, find the total value of the investment after 5 years .
 - (b) How many years, at the same rate of interest, would it take to double the investment ?

2. A recurrence relationship is defined as $U_{n+1} = aU_n + b$.
 - (a) List the four terms U_1 to U_4 of this sequence when $U_0 = 30$, $a = 0.8$ and $b = 4$.
 - (b) Describe why this particular sequence has a limit.
 - (c) Calculate the limit of this sequence.

3. A sequence is defined by the recurrence relation $U_{n+1} = kU_n + c$, where k and c are constants.
 - (a) Given that $U_0 = 60$, $U_1 = 10$ and $U_2 = -15$, form a system of equations and solve it to find k and c .
 - (b) Find U_4 .

4. An unstable atomic particle decays in mass by half every two hours. At the end of each 2 hour period it is bombarded by atomic matter which allows it to recover a mass of 12 a.mu.
 - (a) By considering a suitable recurrence relationship and taking the original mass of the particle to be 40 a.m.u. , calculate the number of hours it will take to decay to a value which is below 32% of its original mass.
 - (b) If the mass of this particle falls below 12.3 a.m.u. it becomes highly unstable resulting in an explosion.
Should the scientists allow this experiment to continue over a long period of time ?
Explain your answer.

5. An antibiotic in the body dissipates at the rate of 8% during each 10 minute period from the initial administration.
 - (a) If an initial dose of 100 units is given what amount of antibiotic will still be present in a patients bloodstream after 1 hour ?
 - (b) A doctor prescribes this drug to a patient. It is known that in order to be effective a level of at least 100 units must be maintained over the long term, however the drug becomes toxic if the level exceeds 300 units.
The doctor prescribes 100 units every hour. Is this treatment satisfactory?
Your answer must be accompanied by the appropriate working and justifications.