Differentiation 1

1. Differentiate each of the following with respect to x :

(a) $3x^2 + 6x$ (b) $x^4 - 2x^{-1} + 4$ (c) $(2x+3)^2$

2. Differentiate each of the following functions with respect to the relevant variable :

(a) $f(x) = x^3(x - x^2)$ (b) $g(x) = 3x^2 + \frac{1}{x^3}$ (c) $h(x) = \frac{1}{x} + \frac{1}{3x^2}$

(d) $f(t) = t^{1/2}(t^2 + t^{3/2})$ (e) $g(p) = \frac{1}{p}(p^{-1/3} - p^{2/3})$ (f) $h(u) = \sqrt{u} + \frac{1}{2\sqrt{u}}$

(g) $g(x) = \frac{x^5 + 2x^2}{x^4}$ (h) $f(v) = \frac{5v+2}{\sqrt{v}}$ (i) $h(t) = \frac{1}{\sqrt{t}}\left(\frac{t^2 + t^{3/2}}{t}\right)$

3. Calculate the rate of change of :

(a) $f(x) = x^3 - 2x^2$ at $x = 2$ (b) $h(t) = (t-1)(2t+3)$ at $t = -1$

4. Find the equation of the tangent to the curve with equation $y = 3x^2 - 2x$ at the point where $x = -1$.

5. A function f is given by $f(x) = 3x^2 - 2x^3$. Determine the interval on which f is increasing.

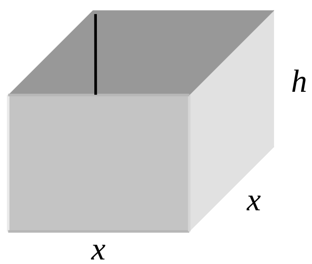
6. A curve has as its equation $y = x(2-x)^2$.

(a) Find the stationary points of the curve and determine the nature of each.

(b) Write down the coordinates of the points where the curve meets the coordinate axes.

(c) Sketch the curve.

7. An open cistern with a square base and vertical sides is to have a capacity of 4000 cubic feet.



(*a*) Taking the length of the square base to be x feet , find an expression for the height h in terms of x .

(*b*) Hence show that the surface area, A square feet, of the cistern can be written in the form

$$A(x) = x^2 + \frac{16000}{x}$$

(*c*) Find the dimensions of the cistern so that the cost of cladding it in lead sheet will be a minimum.