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(1) $|\overrightarrow{OA}| = \sqrt{7^2 + 24^2}$

$= \underline{\underline{25\,m}}$

(2) (a) $|\overrightarrow{OB}| = \sqrt{8^2 + (-15)^2} = \underline{\underline{17\,m}}$

(b) $|\overrightarrow{OC}| = \sqrt{5^2 + (-12)^2} = \underline{\underline{13\,m}}$

(c) $\overrightarrow{BC} = \underline{c} - \underline{b}$

$= (5\underline{i} - 12\underline{j}) - (8\underline{i} - 15\underline{j})$

$= \underline{\underline{(-3\underline{i} + 3\underline{j})\,m}}$

(d) $|\overrightarrow{BC}| = \sqrt{(-3)^2 + 3^2} = \sqrt{18} = \sqrt{9}\sqrt{2} = \underline{\underline{3\sqrt{2}}}\,m$

(3) (a) $|\overrightarrow{OA}| = \sqrt{3^2 + (-4)^2 + 5^2}$

$= \sqrt{50}$

$= \sqrt{25}\sqrt{2}$

$= \underline{\underline{5\sqrt{2}}}\,m$

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3 (b) $|\overrightarrow{OB}| = \sqrt{8^2 + 8^2 + (-3)^2} = \underline{\underline{\sqrt{137}\ m}}$

(c) $|\overrightarrow{OC}| = \sqrt{4^2 + 3^2} = \underline{\underline{5\ m}}$

(d) $\overrightarrow{AB} = \underline{b} - \underline{a}$

$= (8\underline{i} + 8\underline{j} - 3\underline{k}) - (3\underline{i} - 4\underline{j} + 5\underline{k})$

$= \underline{\underline{(5\underline{i} + 12\underline{j} - 8\underline{k})\ m}}$

(e) $|\overrightarrow{BC}| = \underline{c} - \underline{b}$

$= (4\underline{i} + 3\underline{k}) - (8\underline{i} + 8\underline{j} - 3\underline{k})$

$= \underline{\underline{(-4\underline{i} - 8\underline{j} + 6\underline{k})\ m}}$

(f) $|\overrightarrow{CB}| = -|\overrightarrow{BC}| = \underline{\underline{(4\underline{i} + 8\underline{j} - 6\underline{k})\ m}}$

(g) $|\overrightarrow{AB}| = \sqrt{5^2 + 12^2 + (-8)^2} = \underline{\underline{\sqrt{233}\ m}}$

(h) $|\overrightarrow{BC}| = \sqrt{(-4)^2 + (-8)^2 + 6^2} = \sqrt{116} = \underline{\underline{2\sqrt{29}\ m}}$

(4) $\sqrt{6^2+(-8)^2} = \underline{\underline{10 \text{ m/s}}}$

(5) $\sqrt{7^2+(-24)^2} = \underline{\underline{25 \text{ m/s}}}$

(6) $\sqrt{(-4)^2+1^2} = \underline{\underline{\sqrt{17} \text{ m/s}}}$

(7) $\sqrt{4^2+(-10)^2+1^2} = \underline{\underline{\sqrt{117} \text{ m/s}}} = \sqrt{9}\sqrt{13} = \underline{\underline{3\sqrt{13}}} \text{ m}$

(8) $\sqrt{3^2+(-1)^2+(-7)^2} = \underline{\underline{\sqrt{59} \text{ m/s}}}$

(9) A: $\sqrt{5^2+2^2} = \sqrt{29}$

B: $\sqrt{(-4)^2+4^2} = \sqrt{32}$

<u>B is faster.</u>

(10) $\sqrt{2^2+a^2} = 5.2$

$2^2+a^2 = 27.04$

$a^2 = 23.04$

$\underline{\underline{a = \pm 4.8 \text{ m/s}}}$

(11) $\sqrt{b^2 + (b+7)^2} = 17$

$b^2 + (b^2 + 14b + 49) = 289$

$2b^2 + 14b - 240 = 0$

$b^2 + 7b - 120 = 0$

$(b+15)(b-8) = 0$

$\underline{\underline{b = -15 \text{ or } 8}}$

(12) $S(t) = S_0 + vt$

$= (5\underline{i} + 3\underline{j}) + (2\underline{i} + 4\underline{j})t$

$= (5 + 2t)\underline{i} + (3 + 4t)\underline{j}$

(a) $S(1) = (5 + 2\times1)\underline{i} + (3 + 4\times1)\underline{j}$

$= \underline{\underline{7\underline{i} + 7\underline{j}}}$

(b) $S(2) = (5 + 2\times2)\underline{i} + (3 + 4\times2)\underline{j}$

$= \underline{\underline{9\underline{i} + 11\underline{j}}}$

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(13) $s(t) = (5\underline{i} + 4\underline{j}) + (2\underline{i} - \underline{j})t$

$= (5+2t)\underline{i} + (4-t)\underline{j}$

(a) $s(3) = (5+2\times3)\underline{i} + (4-3)\underline{j}$

$= \underline{\underline{(11\underline{i} + \underline{j})\,m}}$

(b) $s(5) = (5+2\times5)\underline{i} + (4-5)\underline{j}$

$= \underline{\underline{(15\underline{i} - \underline{j})\,m}}$

(14) $s(t) = (7\underline{i} + 5\underline{j}) + (a\underline{i} + b\underline{j})t$

$s(t) = (7+at)\underline{i} + (5+bt)\underline{j}$

$s(3) = (7+3a)\underline{i} + (5+3b)\underline{j} = 10\underline{i} - \underline{j}$

So $7+3a = 10$

$3a = 3$

$\underline{\underline{a = 1}}$

$5+3b = -1$

$3b = -6$

$\underline{\underline{b = -2}}$

(15) $s(t) = s_0 + vt$

$= (\underline{i} + 6\underline{j}) + (5\underline{i} - 12\underline{j}) \times 3$

$= \underline{\underline{(16\underline{i} - 30\underline{j})}}\ m$

speed $= |\underline{v}| = \sqrt{5^2 + (-12)^2} = \underline{\underline{13\ m/s}}$

(16) (a) $s(t) = (4\underline{i} + 3\underline{j} + 9\underline{k}) + (3\underline{i} - 2\underline{j} - 5\underline{k})t$

$= \underline{\underline{(4+3t)\underline{i} + (3-2t)\underline{j} + (9-5t)\underline{k}}}$

(b) $s(5) = (4 + 3\times5)\underline{i} + (3 - 2\times5)\underline{j} + (9 - 5\times5)\underline{k}$

$= \underline{\underline{19\underline{i} - 7\underline{j} - 16\underline{k}}}$

Distance $= \sqrt{19^2 + (-7)^2 + (-16)^2} = \sqrt{666} = \underline{\underline{3\sqrt{74}}}\ m$

17 (a) $s(t) = s_0 + vt$

$$s(t) = (a\underline{i} + b\underline{j} + c\underline{k}) + (3\underline{i} + \underline{j} + 4\underline{k})t$$

$$= (a+3t)\underline{i} + (b+t)\underline{j} + (c+4t)\underline{k}$$

$$s(2) = (a+6)\underline{i} + (b+2)\underline{j} + (c+8)\underline{k} = (7i + \underline{j} + 11\underline{k})$$

Comparing coefficients gives;

$a+6=7$ $\quad$ $b+2=1$ $\quad$ $c+8=11$

$\underline{\underline{a=1}}$ $\quad$ $\underline{\underline{b=-1}}$ $\quad$ $\underline{\underline{c=3}}$

(b) $$s(3) = (1+3\times3)\underline{i} + (-1+3)\underline{j} + (3+4\times3)\underline{k}$$

$$= (10\underline{i} - 2\underline{j} + 15\underline{k})\text{ m}$$

$$\text{distance} = \sqrt{10^2 + (-2)^2 + 15^2}$$

$$= \underline{\underline{\sqrt{329}\text{ m}}}$$

(18) $s = s_0 + v_1 t + v_2 t$

$= (7\underline{i} - 6\underline{j} + 3\underline{k}) + (4\underline{i} - 6\underline{k}) \times 2 + (a\underline{i} + b\underline{j} + c\underline{k}) \times 3$

$= (15 + 3a)\underline{i} + (-6 + 3b)\underline{j} + (-9 + 3c)\underline{k}$

arrived back at origin so;

$15 + 3a = 0$ $\quad$ $-6 + 3b = 0$ $\quad$ $-9 + 3c = 0$

$\underline{\underline{a = -5}}$ $\quad$ $\underline{\underline{b = 2}}$ $\quad$ $\underline{\underline{c = 3}}$

(19) see next page.

(19) For A

$$s(t) = (2\underline{i} + 3\underline{j} - 4\underline{k}) + (-\underline{i} + 3\underline{j} + 5\underline{k})t$$

$$s(4) = (2\underline{i} + 3\underline{j} - 4\underline{k}) + (-\underline{i} + 3\underline{j} + 5\underline{k}) \times 4$$

$$= -2\underline{i} + 15\underline{j} + 16\underline{k}$$

For B

$$s(t) = (8\underline{i} + 6\underline{k}) + (\underline{v}t)$$

$$s(5) = 8\underline{i} + 6\underline{k} + 5(a\underline{i} + b\underline{j} + c\underline{k}) = -2\underline{i} + 15\underline{j} + 16\underline{k}$$

Comparing coefficients gives;

$8 + 5a = -2$

$5a = -10$

$\underline{\underline{a = -2}}$

$5b = 15k$

$\underline{\underline{b = 3}}$

$6 + 5c = 16$

$5c = 10$

$\underline{\underline{c = 2}}$

So $\underline{\underline{\underline{v} = (-2\underline{i} + 3\underline{j} + 2\underline{k})}}$ m/s