Ex 10E

Q 1, 2, 6, 7 in ij notation

followed by

Q 2-5 in graphical solutions

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(1) (a) $r_A = (-9i + 6j)$ km $\quad v_A = (3i + 12j)$ km/hr

$r_B = (16i + 6j)$ km $\quad v_B = (-9i + 3j)$ km/hr.

$$r_A(t) = (-9i + 6j) + (3i + 12j)t$$

$$r_B(t) = (16i + 6j) + (-9i + 3j)t$$

$$r_{AB} = r_A - r_B$$

$$= \left[(-9 + 3t) - (16 - 9t)\right]\underline{i} + \left[(6 + 12t) - (6 + 3t)\right]\underline{j}$$

$$= (-25 + 12t)\underline{i} + (9t)\underline{j}$$

at $t = 0$

$$r_{AB} = -25i$$

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1 (b) $|{}_A r_B|^2 = (-25+12t)^2 + (9t)^2$

$= 625 - 600t + 144t^2 + 81t^2$

$= 625 - 600t + 225t^2$

$\frac{d}{dt}|{}_A r_B|^2 = -600 + 450t = 0$ at minimum

$t = \frac{600}{450} = \frac{4}{3}$ hours.

$|{}_A r_B|^2 = 625 - 600\left(\frac{4}{3}\right) + 225\left(\frac{4}{3}\right)^2$

$= 225$

$|{}_A r_B| = 15$ km

(c) $t = \frac{4}{3}$ hr

$= 1$ h 20 m

$\Rightarrow$ 1.20 pm

at $t = \frac{4}{3}$

$$r_A = (-9i + 6j) + (3i + 12\underline{j}) \times \frac{4}{3}$$

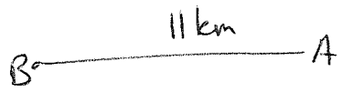
$$= \underline{\underline{(-5i + 22j)\ \text{km}}}$$

$$r_B = (16i + 6\underline{j}) + (-9i + 3j) \times \frac{4}{3}$$

$$= \underline{\underline{(4i + 10j)\ \text{km}}}$$

Ex 10E

(2) (8 am)



$r_A = 11i$ $\qquad$ $r_B = 0$

$v_A = (-4i + 3j)$ $\qquad$ $v_B = (2i + 4j)$

$$r_A(t) = 11i + (-4i + 3j)t$$
$$= (11 - 4t)\underline{i} + 3t\underline{j}$$

$$r_B(t) = 0 + (2i + 4j)t$$
$$= 2t\underline{i} + 4t\underline{j}$$

$$_A r_B(t) = (11 - 4t - 2t)\underline{i} + (3t - 4t)\underline{j}$$
$$= (11 - 6t)\underline{i} + (-t)\underline{j}$$

$$|_A r_B|^2 = (11 - 6t)^2 + (-t)^2$$
$$= 121 - 132t + 36t^2 + t^2$$
$$= 121 - 132t + 37t^2$$

$$\frac{d}{dt}|_A r_B|^2 = -132 + 74t = 0 \text{ at minimum}$$

$$t = \frac{132}{74} = 1.78\text{ hr} = 1\text{ hr } 47\text{ mins}$$

$\Rightarrow$ 9.47 am

$$|_A r_B|^2 = (11 - 6 \times 1.78)^2 + (-1.78)^2 = 3.27$$

$$_A r_B = 1.81\text{ km}$$

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6

$$r_A(t) = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} 7 \\ 3 \end{pmatrix} t$$

$$r_B(t) = \begin{pmatrix} 8 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix} t$$

$$_A r_B = \begin{pmatrix} -3 \\ -6 \end{pmatrix} + \begin{pmatrix} 5 \\ 4 \end{pmatrix} t$$

$$|_A r_B|^2 = (-3+5t)^2 + (-6+4t)^2$$

$$= 9 - 30t + 25t^2 + 36 - 48t + 16t^2$$

$$= 45 - 78t + 41t^2$$

$$\frac{d}{dt}|_A r_B|^2 = -78 + 82t = 0$$

$$t = 0.95h$$

$$= (0.95 \times 60) \text{ mins}$$

$$= 57 \text{ mins}$$

$\Rightarrow$ Time is $\underline{\underline{12.57 \text{ pm}}}$

$$|_A r_B|^2 = (-3+5\times0.95)^2 + (-6+4\times0.95)^2$$

$$= 3.0625 + 4.84$$

$$= 7.9$$

$$|_A r_B| = \underline{\underline{2.81 \text{ km}}}$$

Ex 10 E

7

(a) at 12 noon

$r_A(t) = r_A + v_A t$

$r_A = (3i + j) + (2i + 3j) \times 1$

$= 5i + 4j$

(b) ${}_A r_B = r_A - r_B$

$= (5\underline{i} + 4\underline{j}) - (2\underline{i} - \underline{j})$

$= 3\underline{i} + 5\underline{j}$

$|{}_A r_B| = \sqrt{3^2 + 5^2}$

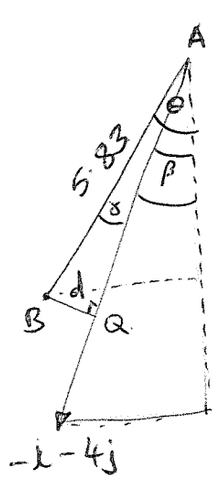
$= \sqrt{34}$

$= 5.83$ km

(c) ${}_A v_B = v_A - v_B$

$= (2i + 3j) - (3i + 7j)$

$= -\underline{i} - 4\underline{j}$



$\theta = \tan^{-1}\left(\frac{3}{5}\right) = 31°$

$\beta = \tan^{-1}\left(\frac{1}{4}\right) = 14°$

$\gamma = 31 - 14 = 17°$

$d = 5.83 \sin 17°$

$= 1.7$ km

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(7) - relative speed $= \sqrt{1^2 + 4^2} = 4.12$ km/hr.

distance moved by A $= 5.83 \cos 17^\circ = 5.57$ km.

$$t = \frac{\text{dist}}{\text{speed}} = \frac{5.57}{4.12} = 1.35 \text{ hr}$$

$$= 1 \text{ hr } (0.35 \times 60) \text{ m}$$

$$= 1 \text{ hr } 21 \text{ m}$$

$$\Rightarrow \underline{\underline{1.21 \text{ pm}}}$$

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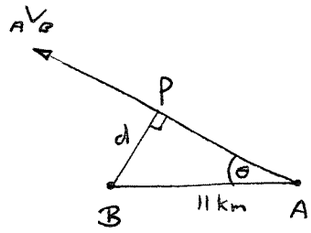
(2)



$$_AV_B = V_A - V_B$$
$$= (-4\underline{i} + 3\underline{j}) - (2\underline{i} + 4\underline{j})$$
$$= -6\underline{i} - \underline{j}$$

$$\theta = \tan^{-1}\frac{1}{6} = 9.5^\circ$$

$$d = 11 \sin 9.5^\circ = \underline{\underline{1.81 \text{ km.}}}$$

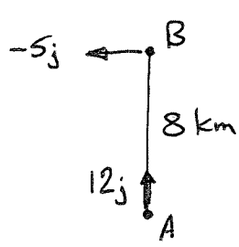
$$|\vec{AP}| = \sqrt{11^2 - 1.81^2} = 10.85 \text{ km.}$$

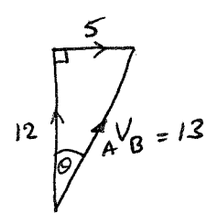
Relative speed $= \sqrt{6^2 + 1^2} = 6.1$ km/hr

$$\text{time} = \frac{d}{s} = \frac{10.85}{6.1} = 1.78 \text{ hr} = 1 \text{ h } 47 \text{ m.}$$

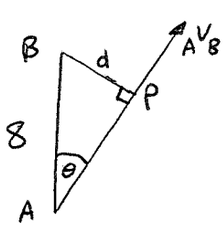
Ships are closest at 8am + 1h 47m = $\underline{\underline{9 \cdot 47 \text{ am}}}$

(3)





$$\tan\theta = \frac{5}{12}$$



$$d = 8 \sin\theta$$
$$= 8 \times \frac{5}{13}$$
$$= \underline{\underline{3.08 \text{ km.}}}$$

Distance travelled by A until closest distance is AP.

$$AP = 8 \cos\theta$$
$$= 8 \times \frac{12}{13}$$
$$= 7.38$$

$$t = \frac{d}{s} = \frac{7.38}{13} = 0.568 \text{ h} = 34 \text{ mins}$$

time = 7·30 pm + 34 m = $\underline{\underline{8 \cdot 04 \text{ pm}}}$

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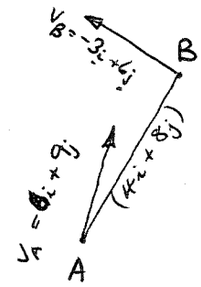
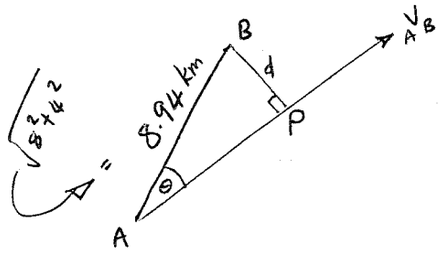
(4)



$$_AV_B = V_A - V_B$$
$$= (6i + 9j) - (-3i + 6j)$$
$$= 9i + 3j$$



$$\theta = \tan^{-1}\left(\frac{8}{4}\right) - \tan^{-1}\left(\frac{3}{9}\right)$$
$$= 63.4 - 18.43$$
$$= 45^\circ$$

Closest distance, $d = 8.94 \times \sin 45^\circ$
$= \underline{\underline{6.32 \text{ km}}}$.

$$t = \frac{d}{s} = \frac{8.94 \cos 45}{\sqrt{9^2+3^2}} = \frac{6.32}{9.48} = 0.666 \text{ h} = 40 \text{ mins}$$

time = 1300 + 0040 = $\underline{\underline{1340}}$

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$$r'_A = r_A + v_A t.$$

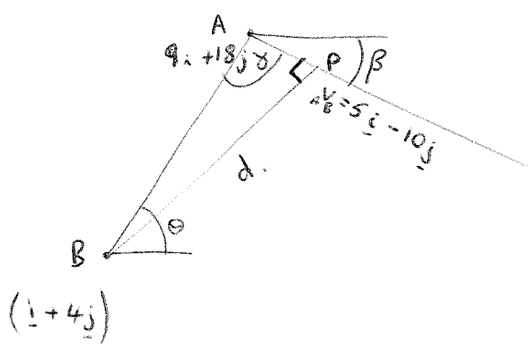
(5) (a) at 1500 hours $r'_A = 20\underline{j} + (9\underline{i} - 2\underline{j}) \times 1$

$$= 9\underline{i} + 18\underline{j}$$

(b) $${}_A v_B = v_A - v_B$$

$$= (9\underline{i} - 2\underline{j}) - (4\underline{i} + 8\underline{j})$$

$$= 5\underline{i} - 10\underline{j}$$



$$|\overrightarrow{AB}| = \sqrt{(9i - i)^2 + (18j - 4j)^2}$$

$$= \sqrt{8^2 + 14^2}$$

$$= 16.1 \text{ km}$$

$$\theta = \tan^{-1}\left(\frac{14}{8}\right) = 60.26$$

$$\beta = \tan^{-1}\left(\frac{10}{5}\right) = 63.4$$

$$\gamma = 180 - \theta - \beta = 56.3°$$

$$d = 16.1 \sin 56.3$$

$$= 13.4 \text{ km}$$

(c) $$t = \frac{\text{dist}}{\text{speed of}} = \frac{\sqrt{16.1^2 - 13.4^2}}{\sqrt{5^2 + 10^2}} = \frac{8.92}{18.2} = 0.798\text{h}$$

$$= 48 \text{ mins.}$$

time $= 1500 + 48 = 1548$