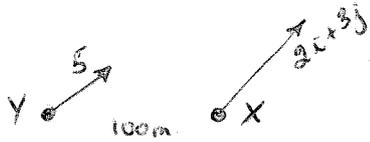
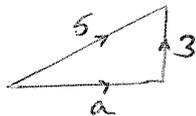


①



$$\begin{aligned} {}_Y v_x &= v_y - v_x \\ &= (a\mathbf{i} + b\mathbf{j}) - (2\mathbf{i} + 3\mathbf{j}) \\ &= (a-2)\mathbf{i} + (b-3)\mathbf{j} \end{aligned}$$

For collision ${}_Y v_x$ has no vertical component $\therefore b-3=0$
 $\underline{\underline{b=3}}$



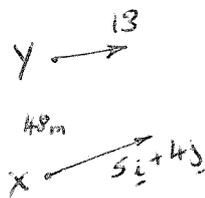
$$a = \sqrt{5^2 - 3^2} = 4$$

b. $\underline{\underline{v_y = 4\mathbf{i} + 3\mathbf{j}}}$

and ${}_Y v_x = (a-2)\mathbf{i} = (4-2)\mathbf{i} = \underline{\underline{2\mathbf{i}}}$

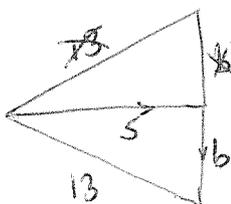
$$\text{time} = \frac{\text{dist}}{\text{velocity}} = \frac{100}{2} = \underline{\underline{50 \text{ secs}}}$$

②



$$\begin{aligned} {}_Y v_x &= v_y - v_x \\ &= (a\mathbf{i} + b\mathbf{j}) - (5\mathbf{i} + 4\mathbf{j}) \\ &= (a-5)\mathbf{i} + (b-4)\mathbf{j} \end{aligned}$$

Since ${}_Y v_x$ has no horizontal component for collision
 $a-5=0$
 $a=5$



$$b = \sqrt{13^2 - 5^2} = 12$$

$$\underline{\underline{v_y = 5\mathbf{i} + 12\mathbf{j}}}$$

$$\text{time} = \frac{\text{dist}}{{}_Y v_x} = \frac{-48}{(-12-4)} = \underline{\underline{3 \text{ secs}}}$$

$$\textcircled{3} \quad r_B - r_A = (6\hat{i} + 4\hat{j}) - (\hat{i} + 7\hat{j}) = 5\hat{i} - 3\hat{j}$$

$$\begin{aligned} v_{A/B} &= v_A - v_B = (6\hat{i} + 2\hat{j}) - (-4\hat{i} + 8\hat{j}) = 10\hat{i} - 6\hat{j} \\ &= 2(5\hat{i} - 3\hat{j}) \end{aligned}$$

Since the velocity of A relative to B is in the same direction as $A \rightarrow B$ then a collision will occur.

$$|A \rightarrow B| = \sqrt{5^2 + (-3)^2} = 4 \text{ km.}$$

$$|v_{A/B}| = \sqrt{10^2 + 6^2} = 8 \text{ km/h}$$

$$\text{time} = \frac{\text{dist}}{\text{speed}} = \frac{4}{8} = 0.5 \text{ h} = \underline{\underline{30 \text{ mins}}} \text{ after 12 noon}$$

$\text{is } \underline{\underline{12:30 \text{ pm}}}$

At point of collision P,

$$\begin{aligned} r_P &= r_A + v_A t \\ &= (\hat{i} + 7\hat{j}) + (6\hat{i} + 2\hat{j}) \times 0.5 \\ &= \underline{\underline{4\hat{i} + 8\hat{j}}} \end{aligned}$$

④

$$r_B - r_A = (7\hat{i} + 7\hat{j}) - (5\hat{i} + 2\hat{j}) = 2\hat{i} + 5\hat{j}$$

$${}_A v_B = v_A - v_B = (15\hat{i} + 10\hat{j}) - (9\hat{i} - 5\hat{j}) = 6\hat{i} + 15\hat{j} = 3(2\hat{i} + 5\hat{j})$$

Since ${}_A v_B$ has same direction as $|r_B - r_A|$ they will collide.

$$\text{Dist, } |A \rightarrow B| = \sqrt{2^2 + 5^2} = 5.38 \text{ km.}$$

$$\text{Speed} = |{}_A v_B| = \sqrt{6^2 + 15^2} = 16.16 \text{ km/h.}$$

$$\text{Time} = \frac{\text{dist}}{\text{speed}} = \frac{5.38}{16.16} = 0.333 \text{ h} = \underline{\underline{20 \text{ mins}}}$$

$$\text{Collision} = 12 \text{ noon} + 20 \text{ mins} = \underline{\underline{12.20 \text{ pm.}}}$$

$$\begin{aligned} \text{Collision at } P, = r_P &= r_A + v_A t \\ &= (5\hat{i} + 2\hat{j}) + (15\hat{i} + 10\hat{j}) \times \frac{1}{3} \\ &= \underline{\underline{10\hat{i} + 5\frac{1}{3}\hat{j}}} \end{aligned}$$

⑤ (see next page).

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(5) If a collision occurs then $r_A' = r_B'$ at same time t after 12

$$(-6\mathbf{i} + 12\mathbf{j}) + (16\mathbf{i} - 4\mathbf{j})(t + 0.5) = (12\mathbf{i} - 15\mathbf{j}) + (8\mathbf{i} + 16\mathbf{j})t$$

$$-6\mathbf{i} + 12\mathbf{j} + (16t)\mathbf{i} - (4t)\mathbf{j} + 8\mathbf{i} - 2\mathbf{j} = 12\mathbf{i} - 15\mathbf{j} + 8t\mathbf{i} + 16t\mathbf{j}$$

$$(2 + 16t)\mathbf{i} + (10 - 4t)\mathbf{j} = (12 + 8t)\mathbf{i} + (-15 + 16t)\mathbf{j}$$

$$\Rightarrow 2 + 16t = 12 + 8t \quad \text{and} \quad 10 - 4t = -15 + 16t$$

$$2 + 8t = 12$$

$$8t = 10$$

$$t = \frac{10}{8} = \frac{5}{4} \text{ h.}$$

$$25 - 4t = 16t$$

$$25 = 20t$$

$$\frac{25}{20} = t$$

$$t = \frac{5}{4} \text{ h.}$$

$\frac{5}{4}$

Collision occurs at $r_B' = r_B + v_B t$

$$= (12\mathbf{i} - 15\mathbf{j}) + (8\mathbf{i} + 16\mathbf{j}) \frac{5}{4}$$

$$= (12 + 10)\mathbf{i} + (-15 + 20)\mathbf{j}$$

$$= \underline{\underline{22\mathbf{i} + 5\mathbf{j}}}$$

Collision occurs at 12.00 + t

$$= 12 + \frac{5}{4} \text{ h}$$

$$= 12 + 1 \frac{1}{4}$$

$$= \underline{\underline{1.15 \text{ pm}}}$$

⑥ $r(t) = r_0 + vt$

$$r_A(t) = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 15 \\ 10 \end{pmatrix} t$$

$$r_B(t) = \begin{pmatrix} 7 \\ 7 \end{pmatrix} + \begin{pmatrix} 9 \\ -5 \end{pmatrix} t$$

If a collision takes place A & B will be at the same place at the same time.

x-components: $5 + 15t = 7 + 9t$

$$6t = 2$$

$$t = \frac{1}{3} \text{ hr}$$

y-components: $2 + 10t = 7 - 5t$

$$15t = 5$$

$$t = \frac{1}{3} \text{ hr.}$$

Collision at $12.00 + \frac{1}{3}(60) \text{ m}$

= 12 20 pm.

$$r_A\left(\frac{1}{3}\right) = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 15 \\ 10 \end{pmatrix} \frac{1}{3} = \underline{\underline{\begin{pmatrix} 10 \\ 5\frac{1}{3} \end{pmatrix}}}$$

(7a) If they collide then they must be at the same place at the same time for both horizontal & vertical components. i.e.

$$\begin{aligned} r_A' &= r_B' \\ r_A + v_A t &= r_B + v_B t \\ (5\mathbf{i} + \mathbf{j}) + (9\mathbf{i} + 18\mathbf{j})t &= (12\mathbf{i} + 5\mathbf{j}) + (-12\mathbf{i} + 6\mathbf{j})t \\ (5 + 9t)\mathbf{i} + (1 + 18t)\mathbf{j} &= (12 - 12t)\mathbf{i} + (5 + 6t)\mathbf{j} \end{aligned}$$

do.

$$\begin{aligned} 5 + 9t &= 12 - 12t & \text{and} & & 1 + 18t &= 5 + 6t \\ 21t &= 7 & & & 12t &= 4 \\ t &= \frac{1}{3} \text{ hour} & & & t &= \frac{1}{3} \text{ hour} \end{aligned}$$

do collide at $2 \text{ pm} + \frac{1}{3} \text{ h} = \underline{\underline{2:20 \text{ pm}}}$

$$r_A' = (5\mathbf{i} + \mathbf{j}) + (9\mathbf{i} + 18\mathbf{j})\frac{1}{3} = \underline{\underline{8\mathbf{i} + 7\mathbf{j}}}$$

$$\begin{aligned} (b) \quad r_C' &= r_C + v_C t \\ &= (13\mathbf{i} - 3\mathbf{j}) + (9\mathbf{i} + 12\mathbf{j})\frac{1}{3} \\ &= \underline{\underline{16\mathbf{i} + \mathbf{j}}} \end{aligned}$$

$$\text{Distance from collision} = \sqrt{(16-8)^2 + (1-7)^2} = 10 \text{ km.}$$

$$(c) \quad \text{Speed of } C = \sqrt{9^2 + 12^2} = 15$$

$$\text{time} = \frac{\text{dist}}{\text{speed}} = \frac{10}{15} = \frac{2}{3} \text{ h} = \underline{\underline{40 \text{ mins}}} \text{ after } 2:20 \text{ pm.}$$

$$\Rightarrow \underline{\underline{3:00 \text{ pm}}}$$