

# Homework Sheet 3

1	The graph has an equation of the form $y = k(x-a)(x-b)$ . What is the equation of the graph?		$y = k(x-a)(x-b)$ (0,12) $12 = k(-1)(4)$ $k = 3$ $y = 3(x-1)(x-4)$
2	For what values of $x$ is $6 + x - x^2 < 0$ ?		$-1(x^2 - x - 6) < 0$ $(x-3)(x+2) < 0$ $x < -2$ or $x > 3$
3	Express $\log_a 25 + \log_a 4 - \log_a 20$ as the logarithm of a single number.		$\log_a \left(\frac{25 \times 4}{20}\right) = \log_a 5$
4	Solve $\cos 2x - 3\cos x + 2 = 0$ for $0 \leq x < 360$ .		$\cos 2x = 2\cos^2 x - 1$ $2\cos^2 x - 1 - 3\cos x + 2 = 0$ $2\cos^2 x - 3\cos x + 1 = 0$ $(2\cos x - 1)(\cos x - 1) = 0$ $\cos x = \frac{1}{2}$ or $\cos x = 1$ $60^\circ$ or $300^\circ$ or $0^\circ$ or $360^\circ$
5	The diagram shows two right-angled triangles with sides and angles given. What is the value of $\sin(p+q)$ ?		$\sin(p+q) = \sin p \cos q + \cos p \sin q$ $\sin p = \frac{2}{\sqrt{5}}$ $\cos p = \frac{1}{\sqrt{5}}$ $\sin q = \frac{4}{3\sqrt{5}}$ $\cos q = \frac{1}{3\sqrt{5}}$ $\sin(p+q) = \frac{2}{\sqrt{5}} \cdot \frac{1}{3\sqrt{5}} + \frac{1}{\sqrt{5}} \cdot \frac{4}{3\sqrt{5}} = \frac{2}{15} + \frac{4}{15} = \frac{6}{15} = \frac{2}{5}$
6	What is the derivative of $(x^3 + 4)^2$ ?		$y = u^2$ $\frac{dy}{dx} = 2u \cdot \frac{du}{dx} = 2(x^3 + 4) \cdot 3x^2 = 6x^2(x^3 + 4)$
7	The point $P(5, 12)$ lies on the curve with equation $y = x^2 - 4x + 7$ . Find the equation of the tangent to the curve.		$\frac{dy}{dx} = 2x - 4$ at $x=5$ , $y-12 = 6(x-5)$ $y-12 = 6x - 30$ $6x - y - 18 = 0$
8	Find $\int 4 \sin(2x+3) dx$ .		$= -\frac{4}{2} \cos(2x+3) + C$ $= -2 \cos(2x+3) + C$
9	Find $\int_{-2}^2 (x+1)^2 dx$ .		$\left[ \frac{1}{3}(x+1)^3 \right]_{-2}^2$ $\left( \frac{1}{3}(3)^3 \right) - \left( \frac{1}{3}(-1)^3 \right) = 9 + \frac{1}{3} = 9\frac{1}{3}$ units
10	Write $2\sin x + 3\cos x$ in the form $k \sin(x-a)$ , for $k > 0$ and $0 \leq a \leq 360$ .		$k \sin(x-a) = k \sin x \cos a - k \cos x \sin a$ $k \cos a = 2$ $k \sin a = -3$ $\tan a = -\frac{3}{2}$ $k = \sqrt{2^2 + (-3)^2} = \sqrt{13}$ $a = 146.3^\circ$ $= -\sqrt{13}$

$\frac{\sin A}{\sin C} = \frac{\sqrt{13} \sin(x-146.3^\circ)}{\sqrt{13} \sin(x-146.3^\circ)}$

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11	Functions $f$ and $g$ are defined on a suitable domain by $f(x) = \cos x$ and $g(x) = x + \frac{\pi}{6}$ . What is the value of $f\left(g\left(\frac{\pi}{3}\right)\right)$ ?	$f(g(x)) = \cos\left(x + \frac{\pi}{6}\right)$ $= \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ Let $x = \frac{\pi}{6}$
12	The diagram shows the graph of $y = f(x)$ . Sketch $y = f(x+2) - 1$ .	
13	Given that $u = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ and $v = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$ , find $3u - 2v$ in component form.	$3\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \\ -8 \end{pmatrix} = \begin{pmatrix} 8 \\ -4 \\ -5 \end{pmatrix}$
14	The vectors $xi + 5j + 7k$ and $-3i + 2j - k$ are perpendicular. What is the value of $x$ ?	$Q \cdot b = 0$ if perp. $a \cdot b = -3x + 10 - 7 = 0$ $-3x + 3 = 0$ $-3x = -3$ $x = 1$
15	Prove that $2\cos^2 A + 3\sin^2 A - 2 = \sin^2 A$ .	$\cos^2 A = 1 - \sin^2 A$ $2(1 - \sin^2 A) + 3\sin^2 A - 2$ $2 - 2\sin^2 A + 3\sin^2 A - 2 = \sin^2 A$
16	A line makes an angle of $30^\circ$ with the positive direction of the $x$ -axis as shown. What is the gradient of the line?	
17	Find the equation of the perpendicular bisector of the line joining $P(3, -3)$ to $Q(-1, 9)$ .	$M = \left(\frac{3-1}{2}, \frac{-3+9}{2}\right) = \left(1, 3\right)$ $m_{PQ} = \frac{9-(-3)}{-1-3} = \frac{12}{-4} = -3$ $m_{\perp} = \frac{1}{3}$ $y-3 = \frac{1}{3}(x-1)$ $3y-9 = x-1$ $x-3y+8=0$
18	Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$	$q = 4$ $p = 2$ $C = -38$ $C(-4, -2)$ $r = \sqrt{4^2 + 2^2 + 38}$ $= \sqrt{58}$
19	A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 3$ and $u_0 = 1$ . What is the value of $u_3$ ?	$u_1 = 2 \times 1 + 3 = 5$ $u_2 = 2 \times 5 + 3 = 13$ $u_3 = 2 \times 13 + 3 = 29$
20	Calculate the shaded area enclosed by the line $y = 2x - 3$ and the curve $y = x^2 - 5x - 3$ .	$\int_0^3 (2x-3) - (x^2-5x-3) dx$ $\int_0^3 (-x^2 + 7x) dx$ $\left[ -\frac{1}{3}x^3 + \frac{7}{2}x^2 \right]_0^3$ $= \left[ -\frac{1}{3}(27) + \frac{7}{2}(9) \right] = \frac{343}{6} = 57\frac{1}{6}$ sq units