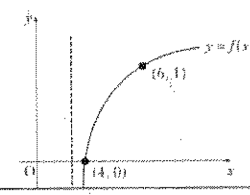
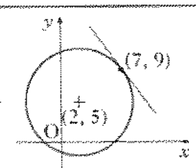
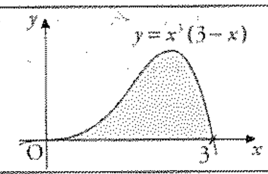
Homework Sheet 2

1	Show that $(x-1)$ is a factor of x^3-3x+2. Hence or otherwise factorise x^3-3x+2 fully.	$1 \mid 1\ 0\ -3\ 2$; $1\ 1\ -2$; $1\ 1\ -2\ 0$ $\quad (x-1)(x^2+x-2)$ $=(x-1)(x+2)(x-1)$ $=(x-1)^2(x+2)$
2	$2x^2+4x+7$ is expressed in the form $2(x+p)^2+q$. What is the value of q.	$2(x^2+2x)+7$ $2[(x+1)^2-1]+7$ $2(x+1)^2-2+7$ $2(x+1)^2+5$
3	If $log_4 12 - log_4 x = log_4 6$, what is the value of x?	$\log_4\left(\frac{12}{x}\right)=\log_4 6$ $\frac{12}{x}=6 \quad 12=6x \quad x=2$
4	Solve $2cosx=\sqrt{3}$ for x, where $0\le x<2\pi$.	$\cos x=\frac{\sqrt{3}}{2}$ $x=\frac{\pi}{6}$ or $\frac{11\pi}{6}$ S A T C $\theta=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}$
5	If the exact value of $cosx$ is $\frac{1}{\sqrt{5}}$, find the exact value of $cos2x$.	$\cos 2x=2\cos^2x-1$ $=2\left(\frac{1}{\sqrt{5}}\right)^2-1$ $=2\left(\frac{1}{5}\right)-1=-\frac{3}{5}$ $\sin x=\frac{2}{\sqrt{5}}$ $\quad \tan x=\frac{1}{2}$
6	Given that $f(x)=(4-3x^2)^{\frac{-1}{2}}$ on a suitable domain, find $f'(x)$.	$f(x)=(4-3x^2)^{-\frac{1}{2}}$ $f'(x)=-\frac{1}{2}(4-3x^2)^{-\frac{3}{2}}(-6x)$ $=\frac{3x}{\sqrt{(4-3x^2)^3}}$ $4-3x^2\neq 0$ $-3x^2\neq -4$ $x^2\neq\frac{4}{3}$ $\quad x\neq\pm\sqrt{\frac{4}{3}}$
7	Find the coordinates of the stationary points on the curve $f(x)=x^3-3x+2$ and determine their nature.	$f(x)=x^3-3x+2$ $f'(x)=3x^2-3=0$ $3x^2=3$ $x^2=1$ $x=\pm 1$ $f(-1)=-1+3+2=4$ $f(1)=1-3+2=0$ $(-1,4)\ (1,0)$ $f''(x)=6x$ $f''(-1)=-6$ Max $f''(1)=6$ Min
8	Find $\int\left(4x^{\frac{1}{2}}+x^{-3}\right)dx$, where $x>0$.	$\int 4x^{\frac{1}{2}}+x^{-3}\,dx$ $\frac{8}{3}x^{\frac{3}{2}}-\frac{1}{2}x^{-2}+C$
9	The graph of $y=f(x)$ passes through the point $\left(\frac{\pi}{9},1\right)$. If $f'(x)=\sin(3x)$ express y in terms of x.	$f'(x)=\sin(3x)$ $f(x)=\int\sin(3x)$ $f(x)=-\frac{1}{3}\cos 3x+C$ $1=-\frac{1}{3}\cos\frac{\pi}{3}+C$ $1=-\frac{1}{3}\times\frac{1}{2}+C$ $\quad C=\frac{7}{6}$ $f(x)=-\frac{1}{3}\cos 3x+\frac{7}{6}$
10	Write $sinx-cosx$ in the form $ksin(x-a)$ stating the values of k and a where $k>0$ and $0\le a\le 2\pi$.	$k\sin(x-a)=k\sin x\cos a-k\cos x\sin a$ $k\sin a=1$ $\quad \tan a=1$ $\quad k=\sqrt{2}$ $k\cos a=1$ $\quad a=\frac{\pi}{4}$ $\sin x-\cos x=\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$

Homework Sheet 2 Mark:

11	Functions f and g are given by $f(x)=3x+1$ and $g(x)=x^2-2$. Find $f(g(x))$ and $g(f(x))$.	$f(g(x))=3(x^2-2)+1=3x^2-6+1=3x^2-5$ $g(f(x))=(3x+1)^2-2=9x^2+6x+1-2=9x^2+6x-1$
12	The diagram shows the graph of $y=f(x)$ where f is a logarithmic function. What are the values of a and b for $f(x)=log_a(x-b)$? 	$b=3$ $\quad \therefore a=3$. $1=\log_a(6-3)$
13	The vectors $\boldsymbol{u}=\begin{pmatrix}k\\-1\\1\end{pmatrix}$ and $\boldsymbol{v}=\begin{pmatrix}0\\4\\k\end{pmatrix}$ are perpendicular. What is the value of k?	$u\cdot v=0$ if perpendicular $(k\cdot 0)+(-1\times 4)+(1\times k)=0$ $-4+k=0$ $\quad \therefore k=4$
14	D, E and F have coordinates $(10,-8,-15)$, $(1,-2,-3)$ and $(-2,0,1)$ respectively. Show that D, E and F are collinear and find the ratio in which E divides DF.	$\vec{DE}=e-d=\begin{pmatrix}1\\-2\\-3\end{pmatrix}-\begin{pmatrix}10\\-8\\-15\end{pmatrix}=\begin{pmatrix}-9\\6\\12\end{pmatrix}$ $\vec{EF}=f-e=\begin{pmatrix}-2\\0\\1\end{pmatrix}-\begin{pmatrix}1\\-2\\-3\end{pmatrix}=\begin{pmatrix}-3\\2\\4\end{pmatrix}$ as $\vec{DE}=3\vec{EF}$ and have common point E, collinear
15	Prove that $\frac{cos^3x}{1-sin^2x}=cosx$.	$\frac{\cos^2x\cdot\cos x}{1-\sin^2x}=\frac{(1-\sin^2x)\cos x}{1-\sin^2x}$ $=\cos x$
16	The line L passes through the point $(-2,-1)$ and is parallel to the line with equation $5x+3y-6=0$. What is the equation of L?	$3y=-5x+6$ $y=-\frac{5}{3}x+2$ $m=-\frac{5}{3}$ $y+1=-\frac{5}{3}(x+2)$ $3y+3=-5x-10$ $5x+3y+13=0$
17	Triangle PQR has vertices at $P(-3,-2)$, $Q(-1,4)$ and $R(3,6)$. PS is a median. What is the gradient of PS?	Q R P $S=\left(\left(\frac{-1+3}{2}\right),\left(\frac{4+6}{2}\right)\right)=(1,5)$ $M_{PS}=\frac{5+2}{1+3}=\frac{7}{4}$
18	The diagram shows a circle, centre $(2,5)$ and a tangent drawn at the point $(7,9)$. What is the equation of this tangent? 	~~$(x-2)^2+(y-5)^2$~~ $M_r=\frac{9-5}{7-2}=\frac{4}{5}$ $M_T=-\frac{5}{4}$ $y-9=-\frac{5}{4}(x-7)$ $4y-36=-5x+35$ $5x+4y-71=0$
19	A sequence is generated by the recurrence relation $u_{n+1}=0.4u_n-240$. What is the limit of this sequence as $\rightarrow\infty$?	$L=\frac{b}{1-a}=\frac{-240}{1-0.4}=\frac{-240}{\frac{6}{10}}=-\frac{240\times 10}{6}=-400$.
20	Calculate the shaded area enclosed by the curve $y=x^3(3-x)$ and the x-axis between $x=0$ and $x=3$. 	$y=x^3(3-x)$ $y=3x^3-x^4$ $\int_0^3 3x^3-x^4$ $=\left[\frac{3}{4}x^4-\frac{1}{5}x^5\right]_0^3$ $=\frac{3}{4}(3)^4-\frac{1}{5}(3)^5=60.75-48.6$ $=12.15$ sq un