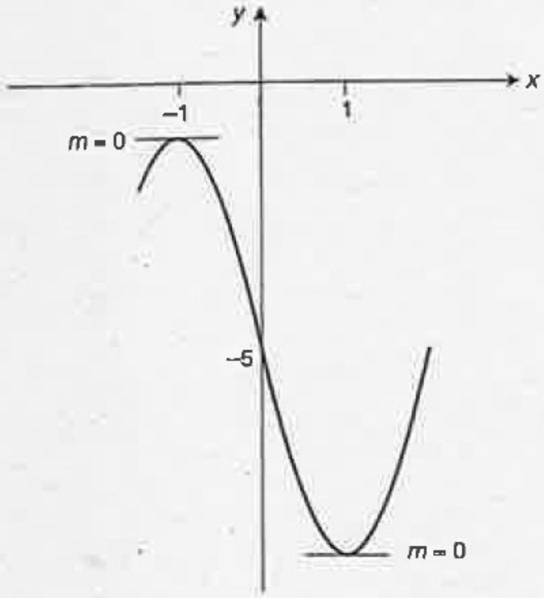
1. $f(x) = x^3 - 3x - 5$

$f'(x) = 3x^2 - 3 \;\; = m \text{ tan } = 0$ for S.V.

$3(x^2 - 1) = 0$

$x = \pm 1$



x	→	-1	→	1	→
$(x^2 - 1)$	+	0	−	0	+
$f'(x)$	↗	→	↘	→	↗

shape

$f'(x)$ decreasing $-1 < x < 1$

Alternative Method:

Using second derivative

$f''(x) = 6x$

$f''(-1)$ −ve ↘ max T.P. $f(-1)$

$f''(1)$ +ve ↗ min T.P. $f(1)$

$f(x)$ decreasing for $\{-1 < x < 1\}$

2. $3xy = -2$

$y = \dfrac{-2}{3x}$ $\quad x = 1, y = -\dfrac{2}{3}, x = -1, y = \dfrac{2}{3}$

$P\left(1, -\dfrac{2}{3}\right)$, $Q\left(-1, \dfrac{2}{3}\right)$

$$m_{PQ} = \frac{\frac{4}{3}}{-2} = -\frac{4}{6} = -\frac{2}{3}$$

gradient of PQ $= -\dfrac{2}{3}$

3. $\cos 3x + \cos x$

$= \cos(2x + x) + \cos(2x - x)$

$= \cos 2x \cos x - \sin 2x \sin x$

$\quad + \cos 2x \cos x + \sin 2x \sin x$

$= 2 \cos 2x \cos x$

Hence $\cos 3x + \cos x = 2 \cos 2x \cos x$.

4. $2x^2 + x + 2$

$$= 2\left(x^2 + \frac{1}{2}x + 1\right)$$

$$= 2\left(x + \frac{1}{4}\right)^2 \quad + 2\left(-\frac{1}{16} + 1\right)$$

$$= 2\left(x + \frac{1}{4}\right)^2 \quad + 2\left(\frac{15}{16}\right)$$

$$= 2\left(x + \frac{1}{4}\right)^2 \quad + \frac{15}{8}$$

minimum value $= \frac{15}{8}$ when $x = -\frac{1}{4}$, $\left(-\frac{1}{4}, \frac{15}{8}\right)$ min. T.P.

5. $\frac{x - 2y}{3} = \frac{y - 2x}{2} \Rightarrow 2(x - 2y) = 3(y - 2x)$

$$2x - 4y = 3y - 6x$$

$$8x = 7y$$

$$x = \frac{7}{8}y$$

To find value of $\frac{7x - 2y}{3x + y}$

Substitude $x = \frac{7}{8}y$

$$= \frac{7\left(\frac{7}{8}\right)y - 2y}{3\left(\frac{7}{8}\right)y + y}$$

$$= \frac{\left(\frac{49}{8} - 2\right)y}{\left(\frac{21}{8} + 1\right)y} = \frac{\frac{49 - 16}{8}}{\frac{21 + 8}{8}} = \frac{33}{29}$$

Hence $\frac{7x - 2y}{3x + y} = \frac{33}{29}$

6. $f(x) = x^2 - 3, \; g(x) = 2 - x$

$$f(g(x)) = f(2 - x) = (2 - x)^2 - 3$$

$$= 4 - 4x + x^2 - 3$$

$$f(g(x)) = x^2 - 4x + 1$$

$$f(g(2)) = 2^2 - 4(2) + 1$$

$$= 4 - 8 + 1$$

$$= -3$$

<u>Alternative Method</u>:
$g(2) = 2 - 2 = 0$
$f(0) = 0^2 - 3 = -3$

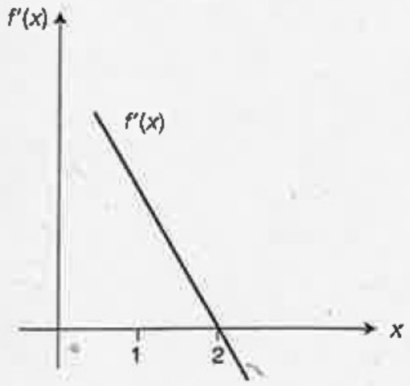
7.

x	< 2	2	> 2
$f'(x)$	$+$	0	$-$ve
Plot	above	on	below

in relation to x-axis
intercepts x-axis at $(2, 0)$

$\frown$ shape $- x^2$ $\qquad f'(x) \Rightarrow m$ is $-$ve $\searrow$

$f(x)$ quadratic, $f'(x)$ linear



8. $$\int_1^4 \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx = \int_1^4 \left(x^{1/2} + \frac{1}{2}(x)^{-1/2}\right) dx$$

$$= \left[\frac{2x^{3/2}}{3} + x^{1/2}\right]_1^4$$

$$= \left(\frac{16}{3} + 2\right) - \left(\frac{2}{3} + 1\right)$$

$$= \frac{14}{3} + 1 = \frac{17}{3} \text{ or } 5\frac{2}{3}$$

9. $f(x) = 2\cos 3x$

$f'(x) = -6\sin 3x, \quad f'\left(\frac{\pi}{2}\right) = -6\sin\left(\frac{3\pi}{2}\right)$

$= -6 \times -1$

$= 6$

10. General equation of a circle is $x^2 + y^2 - 2gx + 2fy + c = 0$

Centre $= (-g, -f)$

$x^2 + y^2 - 6x + 8y = 0, \qquad 2g = -6 \Rightarrow -g = 3$

$\qquad 2f = 6 \Rightarrow -f = -4$

$(-g, -f) \Rightarrow$ centre $= (3, -4)$

$C_1 \rightarrow C_2$ under reflection in x-axis

$\Rightarrow (3, -4) \rightarrow (3, 4)$

$C_2 = (3, 4) \qquad -g = 3 \Rightarrow 2g = -6$

$\qquad -f = 4 \Rightarrow 2f = -8$

Equation of circle under reflection in x-axis $= x^2 + y^2 - 6x - 8y = 0$

11. $y = 2_{\log 5}(2x + 5)$

At B, $x = 0$

$y = 2_{\log 5}(0 + 5)$

$y = 2_{\log 5}(5)$

$y = 2 \times 1 = 2$

coordinates of B(0, 2)

$y = 2_{\log 5}(2x + 5)$

At C, $y = 4$

$y = 2_{\log 5}(2x + 5) = 4$

$2_{\log 5}(2x + 5) = 2$

$5^2 = 2x + 5$

$2x + 5 = 25$

$2x = 20;\ x = 10$

coordinates of C(10, 4)

$y = 2_{\log 5}(2x + 5)$

At A, $y = 0$

$2_{\log 5}(2x + 5) = 0$

$_{\log 5}(2x + 5)^2 = 0$

$5^0 = (2x + 5)^2$

$(2x + 5)^2 = 1$

$2x + 5 = 1$

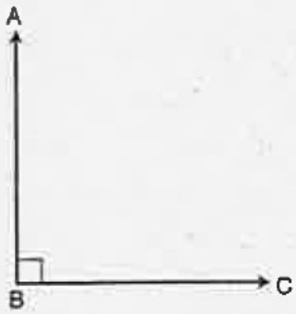
$2x = -4;\ x = -2$

coordinates of A(−2, 0)

12. $\cos ABC = \dfrac{BA \cdot BC}{|BA||BC|}$

If AB is perpendicular to BC

then BA . BC = 0



$BA = \boldsymbol{a} - \boldsymbol{b}$

$BC = \boldsymbol{c} - \boldsymbol{b}$

$$\begin{matrix} \boldsymbol{a} & - & \boldsymbol{b} \end{matrix}$$

$$\begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\begin{matrix} \boldsymbol{c} & - & \boldsymbol{b} \end{matrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$$

$BA \cdot BC = (1 \times -1) + (-1 \times 1) + (-1 \times -2) = -1 + -1 + 2 = 0$

Since $BA \cdot BC = 0$, BA is perpendicular to BC, and angle $ABC = 90°$.