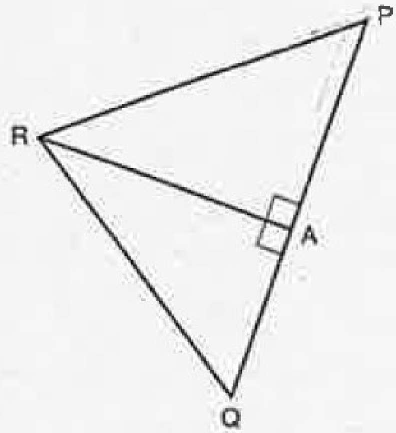
1. (a) $a = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$ $b = \begin{pmatrix} -2 \\ 2 \\ -2 \end{pmatrix}$

$$P = \frac{1}{5}(3a + 2b) = \frac{1}{5}\left[\begin{pmatrix} 9 \\ 3 \\ 9 \end{pmatrix} + \begin{pmatrix} -4 \\ 4 \\ -4 \end{pmatrix}\right] = \frac{1}{5}\begin{pmatrix} 5 \\ 7 \\ 5 \end{pmatrix}$$

$$P = \left(1, \frac{7}{5}, 1\right)$$

2. P(2, 7), Q(0, −1), R(−5, 4)

$$m_{PQ} = \frac{7 - (-1)}{2 - 0} = \frac{8}{2} = 4$$



$$m_{PQ} \cdot m_{AR} = -1 \Rightarrow m_{AR} = -\frac{1}{4}$$ through R(−5, 4)

$$y - 4 = -\frac{1}{4}(x + 5)$$

$$4y - 16 = -x - 5$$

$4y + x = 11$ equation of altitude AR

3. Let P, Q, R represent the given points P(1, –1), Q(a, 2), R(b, 1).

$$\boldsymbol{p} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \boldsymbol{q} = \begin{pmatrix} a \\ 2 \end{pmatrix}, \boldsymbol{r} = \begin{pmatrix} b \\ 1 \end{pmatrix}$$

$$\overrightarrow{PQ} = \boldsymbol{q} - \boldsymbol{p} = \begin{pmatrix} a \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} a-1 \\ 3 \end{pmatrix}$$

$$\overrightarrow{QR} = \boldsymbol{r} - \boldsymbol{q} = \begin{pmatrix} b \\ 1 \end{pmatrix} - \begin{pmatrix} a \\ 2 \end{pmatrix} = \begin{pmatrix} b-a \\ -1 \end{pmatrix}$$

If collinear then vectors are parallel.

Hence $\begin{pmatrix} a-1 \\ 3 \end{pmatrix} = \begin{pmatrix} b-a \\ -1 \end{pmatrix}$

where $k = -3$

Hence $-3 \begin{pmatrix} b- \\ -1 \end{pmatrix} = \begin{pmatrix} a-1 \\ 3 \end{pmatrix}$

$$\Rightarrow -3b + 3a = a - 1$$
$$\Rightarrow -3b + 2a = -1$$
$$\Rightarrow 3b - 2a = 1$$

4. $4x^3 + mx = f(x)$

$f'(x) = 0$ for stationary value(s)

$$f'(x) = 12x^2 + m = 0$$
$$12x^2 = -m$$
$$x^2 = \frac{-m}{12}$$
$$x = \pm\frac{3}{2}, \left(-\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2$$

$$x = \frac{3}{2}, \quad \frac{9}{4} = \frac{-m}{12} \Rightarrow \frac{108}{4} = -m, \quad 27 = -m, \quad m = -27$$

Hence equation is $4x^3 - 27x$.

5. $$(2\sqrt{3} + 3\sqrt{2})^2 = (2\sqrt{3} + 3\sqrt{2})(2\sqrt{3} + 3\sqrt{2})$$
$$= 2\sqrt{3}(2\sqrt{3} + 3\sqrt{2}) + 3\sqrt{2}(2\sqrt{3} + 3\sqrt{2})$$
$$= 12 + 6\sqrt{6} + 6\sqrt{6} + 18$$
$$= 30 + 12\sqrt{6}$$

6. $\int_{-3}^{0} (\text{upper} - \text{lower})\,dx + \int_{0}^{5} (\text{upper} - \text{lower})\,dx$

$= \int_{-3}^{0} (f_2(x) - f_1(x))\,dx + \int_{0}^{5} (f_1(x) - f_2(x))\,dx$

7. *(a)* $u_{r+1} = Ku_r + t \quad u_0 = 0, u_1 = 2, u_2 = -4$

$$
\begin{aligned}
u_1 &= Ku_0 + t \Rightarrow 2 = t \\
u_2 &= Ku_1 + t \Rightarrow -4 = K(2) + t, t = 2 \\
-4 &= 2K + 2 \\
-6 &= 2K \\
t = 2, \quad K &= -3
\end{aligned}
$$

$u_{r+1} = Ku_r + t \Rightarrow u_{r+1} = -3u_r + 2$

(b)

$$
\begin{aligned}
u_4 &= -3u_3 + 2 \\
u_3 &= -3u_2 + 2 \\
&= -3(-4) + 2 \\
u_3 &= 14
\end{aligned}
$$

Hence $u_4 = -3(14) + 2 = -42 + 2 = -40$

To find u_{-1} use $u_0 = -3u_{-1} + 2$

$$
\begin{aligned}
0 &= -3u_{-1} + 2 \\
-2 &= -3u_{-1} \\
\frac{2}{3} &= u_{-1}. \\
u_4 &= -40 \\
u_{-1} &= \frac{2}{3}
\end{aligned}
$$

8. $4x^2 + 4x + 5$

$$
\begin{aligned}
4\left(x^2 + x + \frac{5}{4}\right) &= 4\left(x + \frac{1}{2}\right)^2 + 4\left(-\frac{1}{4} + \frac{5}{4}\right) \\
&= 4\left(x + \frac{1}{2}\right)^2 + 4
\end{aligned}
$$

Minimum value $= 4$ when $x = -\frac{1}{2}$

9. $y = 3x^2 - 2x + 1$

$\frac{dy}{dx} = 6x - 2 = m \tan$

parallel to $y = x - 3$

$\Rightarrow m = 1$

$\Rightarrow 6x - 2 = 1$

$6x = 3$

$x = \frac{1}{2}, f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 1$

$\frac{3}{4} - 1 + 1 = \frac{3}{4}$

Point $\left(\frac{1}{2}, \frac{3}{4}\right)$ $m = 1$

$y - \frac{3}{4} = 1\left(x - \frac{1}{2}\right)$

$4y - 3 = 4\left(x - \frac{1}{2}\right)$

$4y - 3 = 4x - 2$

$4y - 4x = 1$ or $4y = 4x + 1$ (equation of the tangent)

10. *(a)* $f(x) = 2x^2 - bx + 3$ for equal roots $b^2 - 4ac = 0$

$a = 2, \quad b = -b, \quad c = 3$

$b^2 - 4ac = (-b)^2 - 4(2)(3)$

$b^2 - 24 = 0$

$b^2 = 24$

$b = \pm\sqrt{24}$

$b = 2\sqrt{6}$ or $-2\sqrt{6}$

(b) For real roots $b^2 - 4ac \geq 0$, $4ac = 24$

$b \leq -2\sqrt{6}$ or $b \geq 2\sqrt{6}$

Since $(-b)^2 > 24$ for these values

then $b^2 - 4ac > 0$

(c) For no real roots

$b^2 - 4ac < 0$

$\Rightarrow b^2 - 24 < 0$

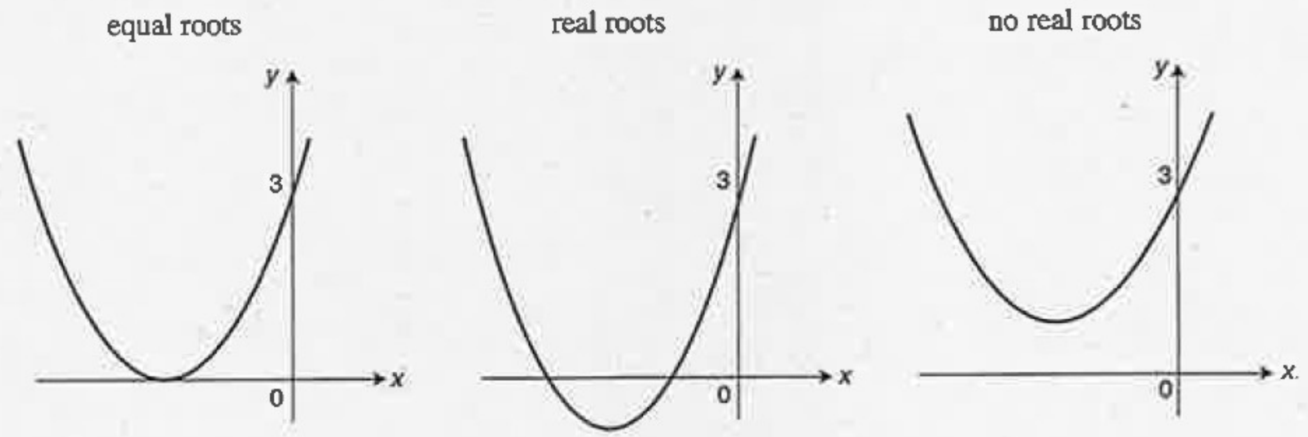
$b^2 < 24$

$-2\sqrt{6} < b < 2\sqrt{6}$

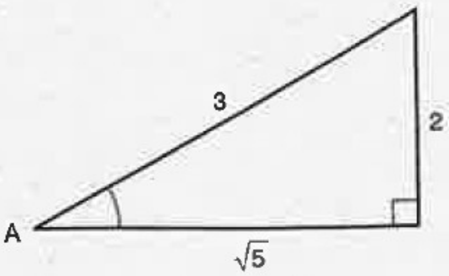
(d) $f(x)$ cuts y-axis at $(0, 3)$

equal roots $b = \pm 2\sqrt{6}$

$\frac{-b \pm \sqrt{0}}{2a} = \frac{-b}{2a} = \frac{-2\sqrt{6}}{4} = \frac{-\sqrt{6}}{2}$ or $\frac{\sqrt{6}}{2}$



11. Sin A = $\frac{2}{3}$. Find the exact value of tan 2A.



$\sqrt{9-4} = \sqrt{5}$

opposite $= 2$
hypotenuse $= 3$
third side $= \sqrt{5}$

sin A $= \frac{2}{3}$ and cos A $= \frac{\sqrt{5}}{3}$

$$\tan 2A = \frac{\sin 2A}{\cos 2A}$$

$$\sin 2A = 2 \sin A \cos A \longrightarrow = 2 \cdot \left(\frac{2}{3}\right) \cdot \left(\frac{\sqrt{5}}{3}\right) \longrightarrow \frac{4\sqrt{5}}{9}$$

$$\cos 2A = \cos^2 A - \sin^2 A \longrightarrow = \left(\frac{\sqrt{5}}{3}\right)^2 - \left(\frac{2}{3}\right)^2 \longrightarrow \frac{1}{9}$$

$$\frac{\sin 2A}{\cos 2A} = \frac{\frac{4\sqrt{5}}{9}}{\frac{1}{9}} = 4\sqrt{5} = \tan 2A$$

12. *(a)* $x^2 + y^2 - 10x + 2y + 1 = 0;$

centre $= (-g, -f);$

$2g = -10,\ 2f = 2,$ centre $= (5, -1)$

(b) $C = (5, -1);\ Q = (1, 2);\ m_{CP} = \frac{2-(-1)}{1-5} = \frac{3}{-4}$ $m_{tangent} \times m_{CP} = -1$

$\frac{3}{-4} \times \frac{4}{3} = -1$ hence $m_{tangent} = \frac{4}{3}$ through $P(1, 2) = (a, b)$

$y - b = m(x - a)$

$y - 2 = \frac{4}{3}(x - 1)$

$3y - 6 = 4(x - 1)$

$3y - 6 = 4x - 4$

$3y = 4x + 2$ is the equation of the tangent to the circle

(c) $3y = 4x + 2$; when $x = 0, y = \frac{2}{3}$; when $y = 0, 4x + 2 = 0, x = -\frac{1}{2}$

Coordinates are $\left(0, \frac{2}{3}\right)$ and $\left(\frac{-1}{2}, 0\right)$