1. $(5+2\sqrt{3})^2 = (5+2\sqrt{3})(5+2\sqrt{3})$

$= 5(5+2\sqrt{3}) + 2\sqrt{3}(5+2\sqrt{3})$

$= 25 + 10\sqrt{3} + 10\sqrt{3} + 12$

$= 37 + 20\sqrt{3}$

2. $\cos 4x$ $\qquad 0 \le x \le 180$

period $= \dfrac{360}{4} = 90$

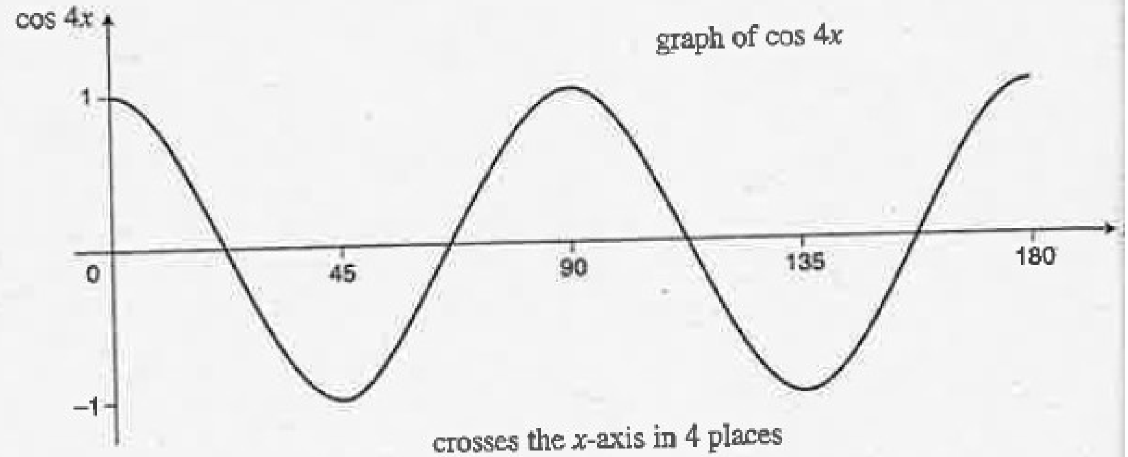
repeats twice in 180°

$\cos 4x = 0 \Rightarrow 4x = 90, 270$

$4x = 90, x = 22\frac{1}{2}, 112\frac{1}{2}$

$4x = 270, x = 67\frac{1}{2}, 157\frac{1}{2}$

cuts at $x = \{22\frac{1}{2}, 67\frac{1}{2}, 112\frac{1}{2}, 157\frac{1}{2}\}$



crosses the x-axis in 4 places

3. A(2, 1), B(a, 5), C(b, −7) in component form.

$\mathbf{a} = \begin{pmatrix}2\\1\end{pmatrix}, \mathbf{b} = \begin{pmatrix}a\\5\end{pmatrix}, \mathbf{c} = \begin{pmatrix}b\\-7\end{pmatrix}$

If points are collinear, B is a shared point and $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$.

$\Rightarrow \mathbf{b} - \mathbf{a} = k(\mathbf{c} - \mathbf{b})$

$\mathbf{b} - \mathbf{a} = \begin{pmatrix}a-2\\4\end{pmatrix}, \mathbf{c} - \mathbf{b} = \begin{pmatrix}b-a\\-12\end{pmatrix}$

If collinear, ratio is $4 : -12 = 1 : -3$

$\Rightarrow -3(a-2) = b - a$

$-3a + 6 = b - a$

$\Rightarrow 6 = 2a + b$

<u>Alternative method</u>

A(2, 1), B(a, 5), C(b, −7)

$m_{AB} = \dfrac{5-1}{a-2}, \ m_{BC} = \dfrac{5-(-7)}{a-b}$

$\dfrac{4}{a-2} = \dfrac{12}{a-b}$ if parallel

$4a - 4b = 12a - 24$

$\Rightarrow 24 = 8a + 4b \Rightarrow 6 = 2a + b$ as given.

4. General equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Centre $= (-g, -f)$, $\quad r = \sqrt{g^2 + f^2 - c}$

Given equation $= x^2 + y^2 - 8x + 6y + 21 = 0$

$2g = -8, \Rightarrow -g = 4, \; 2f = 6 \Rightarrow -f = -3$

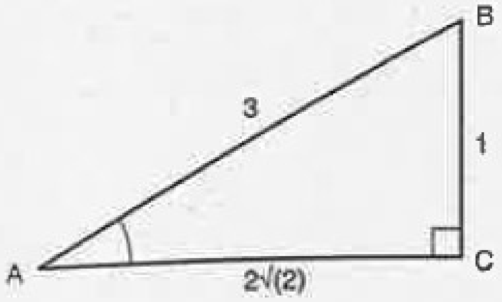
Radius $= \sqrt{4^2 + 3^2 - 21} = \sqrt{4} = 2$

Centre $= (4, -3)$ reflected in $y = -x$ becomes $(3, -4)$, $r = 2$.

Hence equation of circle after reflection in $y = -x$:

$$x^2 + y^2 - 6x + 8y + 21 = 0$$

5. By Pythagoras' Theorem $\qquad AB = \sqrt{(2\sqrt{2})^2 + 1^2} = 3$



$$\cos A = \frac{2\sqrt{2}}{3}$$

$$\sin A = \frac{1}{3}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\sin 2A = 2\left(\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right) = \frac{4\sqrt{2}}{9}$$

$$\cos 2A = 2\cos^2 A - 1 = 2\left(\frac{2\sqrt{2}}{3}\right)^2 - 1$$

$$= 2\left(\frac{8}{9}\right) - 1$$

$$\cos 2A = \frac{16}{9} - \frac{9}{9} = \frac{7}{9}$$

$$\cos 2A = \frac{7}{9}, \quad \sin 2A = \frac{4\sqrt{2}}{9}$$

6. (a) $u_{n+1} = 4u_n - 5$

$u_{n+2} = 4u_{n+1} - 5$

$= 4(4u_n - 5) - 5$

$u_{n+2} = 16u_n - 25$

(b) $u_{n+1} = 4u_n - 5$

$u_{n+3} = 4u_{n+2} - 5$

$= 4(16u_n - 25) - 5$

$u_{n+3} = 64u_n - 105$

$u_{n+3} = 87$

$64u_n - 105 = 87$

$64u_n = 192$

$u_n = 3$

(c) Find u_{n-1}

$u_n = 4u_{n-1} - 5$

$3 = 4u_{n-1} - 5$

$4u_{n-1} = 8; \quad u_{n-1} = 2$

Find u_{n+4}

$u_{n+4} = 4u_{n+3} - 5$

$= 4(87) - 5$

$u_{n+4} = 343$

7. $f(x) = (3x^2 - 2x)^4$

By chain rule method:

$f'(x) = 4(3x^2 - 2x)^3(6x - 2)$

$= 4(6x - 2)(3x^2 - 2x)^3$

$f'(x) = 4(6x - 2)(3x^2 - 2x)^3$

$f'(-1) = 4(6(-1) - 2)(3(-1)^2 - 2(-1))^3$

$f'(-1) = 4(-6 - 2)(3 + 2)^3$

$= 4(-8)(5)^3$

$f'(-1) = -32 \times 125$

$= -4000$

8. $D = 2\cos\left(x - \frac{\pi}{2}\right)$ minimum value $= -2$ when $x - \frac{\pi}{2} = \pi$

$$\Rightarrow \quad x = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$$

maximum value $= 2$ when $\left(x - \frac{\pi}{2}\right) = 0$ or 2π,

$$x = \frac{\pi}{2} \text{ or } \frac{5\pi}{2} \text{ (too large)}$$

$$(x, y) = \left(\frac{\pi}{2}, 2\right), \left(\frac{3\pi}{2}, -2\right)$$

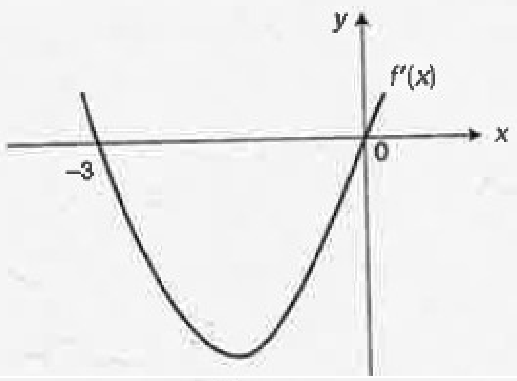
9. $f(x) = ax^3$, a +ve

$f'(x) = 3ax^2 \Rightarrow$ shape 

x	<-3	-3	>-3	0	>0
$f'(x)$	+	0	−	0	+
Plot	above	on	below	on	above

Points in relation to x-axis.

Plot $(-3, 0)(0, 0)$



10. (a) $a \begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix}$ $b \begin{pmatrix} -2 \\ 2 \\ -3 \end{pmatrix}$

1 3

$P = \frac{1}{4}(3a + b) = \frac{1}{4}\left[\begin{pmatrix} 6 \\ -6 \\ 15 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \\ -3 \end{pmatrix}\right] = \frac{1}{4}\begin{pmatrix} 4 \\ -4 \\ 12 \end{pmatrix}$

$P = (1, -1, 3)$

(b) $\overrightarrow{AB} = b - a = \begin{pmatrix} -4 \\ 4 \\ -8 \end{pmatrix}$

$\overrightarrow{PB} = b - p = \begin{pmatrix} -3 \\ 3 \\ -6 \end{pmatrix}$

Ratio $\overrightarrow{AB} : \overrightarrow{PB} = 4 : 3$

11. $f'(x) = 3x^2 - 4x + 5$ $f(x)$ passes through $(-2, 6)$

$f(x) = \int (3x^2 - 4x + 5)dx = x^3 - 2x^2 + 5x + c$ from the given point, when $x = -2, f(x) = 6$,

therefore, $(-2)^3 - 2(-2)^2 + 5(-2) + c = 6;\ c = 32$

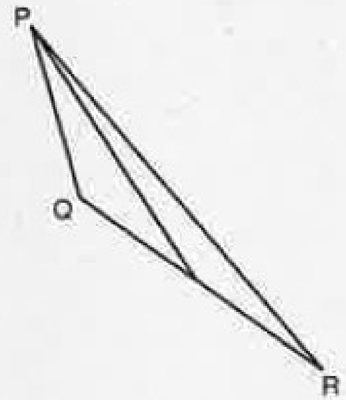
Hence, $f(x) = x^3 - 2x^2 + 5x + 32$

and $f(3) = (3)^3 - 2(3)^2 + 5(3) + 32; = 27 - 18 + 15 + 32 = 56$

$f(3) = 56$

12. Let given points be P, Q, R respectively.

(2, 5)	(4, 1)	(8, −3)
P	Q	R

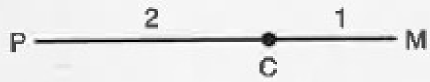


Mid point QR $= \left(\frac{4+8}{2}, \frac{1+(-3)}{2}\right)$

$= (6, -1)$

$M = (6, -1)$

Let C = centroid PC : CM = 2 : 1



$p\begin{pmatrix}2\\5\end{pmatrix}$ $m\begin{pmatrix}6\\-1\end{pmatrix}$

2   1

$C = \frac{1}{3}\left[\begin{pmatrix}12\\-2\end{pmatrix} + \begin{pmatrix}2\\5\end{pmatrix}\right] = \frac{1}{3}\begin{pmatrix}14\\3\end{pmatrix}$ $C = \left(\frac{14}{3}, 1\right)$

<u>Alternative Method</u>

$\overrightarrow{OC} = \frac{1}{3}(p + q + r)$

$= \frac{1}{3}\left[\begin{pmatrix}2\\5\end{pmatrix} + \begin{pmatrix}4\\1\end{pmatrix} + \begin{pmatrix}8\\-3\end{pmatrix}\right] = \frac{1}{3}\begin{pmatrix}14\\3\end{pmatrix}$

$C = \left(\frac{14}{3}, 1\right)$