1. *(a)* Using synthetic division

$$\begin{array}{lccccccc} & x^3 & - & 3x^2 & - & 6x & + & 8 \\ x=-2 & 1 & & -3 & & -6 & & 8 \\ & & & -2 & & 10 & & -8 \\ & 1 & & -5 & & 4 & & 0 \end{array}$$

Remainder $= 0; f(-2) = 0$

Since the remainder $= 0, -2$ is a root and $(x+2)$ is a factor

$$x^3 - 3x^2 - 6x + 8 = (x+2)(x^2-5x+4)$$

By factorising the second bracket further . . . $f(x) = (x+2)(x-1)(x-4)$

(b) $f(x)$ meets the x-axis when $y = 0$; $(x+2)(x-1)(x-4) = 0$

$(x+2) = 0$ or $(x-1) = 0$ or $(x-4) = 0$

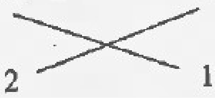
$x = -2, x = 1, x = 4$

coordinates are $(-2, 0)(1, 0)(4, 0)$

$f(x)$ meets the y-axis when $x = 0$; $f(0) = 8$; coordinates are $(0, 8)$

Hence $f(x)$ meets the axes at the points $(-2, 0), (1, 0), (4, 0), (0, 8)$

2. *(a)* $\boldsymbol{a} = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$ $\quad \boldsymbol{b} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}$ $\qquad \overrightarrow{AP} : \overrightarrow{PB} = 2 : 1$



By section formula method

$$P = \frac{1}{3}(\boldsymbol{a} + 2\boldsymbol{b}) = \frac{1}{3}\left[\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} + \begin{pmatrix} -4 \\ 8 \\ 2 \end{pmatrix}\right] = \frac{1}{3}\begin{pmatrix} -3 \\ 6 \\ 6 \end{pmatrix}$$

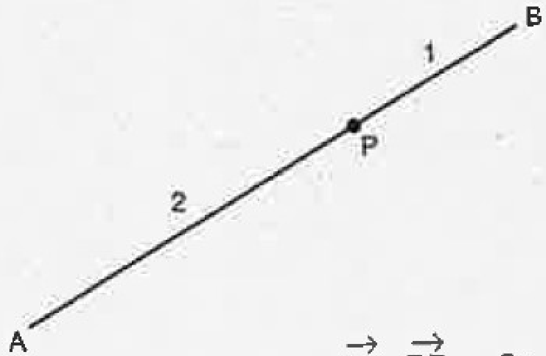
$P = (-1, 2, 2)$

(b) $\overrightarrow{BP} = -\overrightarrow{PB}$

hence $\overrightarrow{AP} : \overrightarrow{BP}$

$= \overrightarrow{AP} : -\overrightarrow{PB}$

$= 2 : -1$



$\overrightarrow{AP} : \overrightarrow{BP} = 2 : -1$

* For Alternative Method see *

Paper A Question 11 and Paper C Question 7

3. $L(2, 4), M(-1, -2), N(3, 7)$

$m_{MN} = \dfrac{7-(-2)}{3-(-1)} = \dfrac{9}{4}$

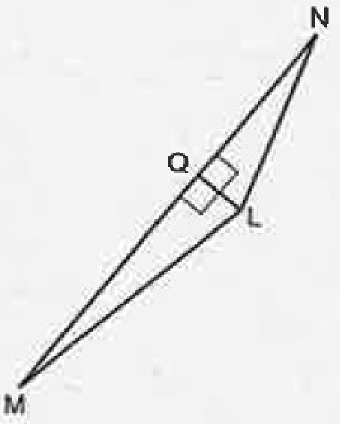
$m_{MN} \cdot m_{LQ} = -1$ for LQ $\perp$ to MN

Hence $m_{LQ} = -\dfrac{4}{9}$ through point L(2, 4)

$y - 4 = -\dfrac{4}{9}(x-2)$

$9y - 36 = -4x + 8$

$9y + 4x = 44$ = equation of LQ.



4. *(a)* $x^2 + y^2 - 6x + 8y + 9 = 0, C(-g, -f), r = \sqrt{g^2 + f^2 - 9}$

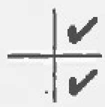
$2g = -6, -g = 3,$

$2f = 8, -f = -4$

Centre $= (3, -4)$, radius $= \sqrt{3^2 + 4^2 - 9} = \sqrt{25 - 9} = \sqrt{16}$

Radius $= 4$

(b) After reflection in x-axis



$(3, -4) \rightarrow (3, 4)$

Radius unchanged.

Equation is $x^2 + y^2 - 6x - 8y + 9 = 0$

* For Alternative Method see *
Paper B Question 3*(b)*

5. *(a)* $f(x) = 4x^3 + mx$

$f'(x) = 12x^2 + m = m \tan = 0$ {m tan is the gradient of the tangent to the curve.}

$x = \pm \dfrac{\sqrt{3}}{2} \quad 12\left(\dfrac{\sqrt{3}}{2}\right)^2 + m = 0$ {m tan = 0 for stationary values.}

$12\left(\dfrac{3}{4}\right) + m = 0$

$9 + m = 0, \Rightarrow m = -9$

(b) Hence $f(x) = 4x^3 - 9x$

and $f(-2) = 4(-2)^3 - 9(-2)$

$\Rightarrow 4(-8) + 18$

$\Rightarrow -32 + 18$

$f(-2) = -14$

6. $(2\sqrt{3} - 5\sqrt{2})^2 = (2\sqrt{3}) - 5\sqrt{2})(2\sqrt{3} - 5\sqrt{2})$

$= 2\sqrt{3}(2\sqrt{3} - 5\sqrt{2}) - 5\sqrt{2}(2\sqrt{3} - 5\sqrt{2})$

$= 12 - 10\sqrt{6} - 10\sqrt{6} + 50$

$= 62 - 20\sqrt{6}$

7. $f(x) = \dfrac{x^3 + 2x^2 - 3x - 1}{3x^2}$

$$f(x) = \frac{x^3}{3x^2} + \frac{2x^2}{3x^2} - \frac{3x}{3x^2} - \frac{1}{3x^2}$$

$$f(x) = \frac{x}{3} + \frac{2}{3} - \frac{1}{x} - \frac{1}{3x^2}$$

$$f(x) = \frac{x}{3} + \frac{2}{3} - x^{-1} - \frac{x^{-2}}{3}$$

We can now differentiate

$$f'(x) = \frac{1}{3} + x^{-2} + \frac{2x^{-3}}{3}$$

$$f'(x) = \frac{1}{3} + \frac{1}{x^2} + \frac{2}{3x^3}$$

8. $f(x) = x^3 - 5x^2 - x + d$

If $f(-1) = 0$, then $(x + 1)$ is a factor.

$$\begin{array}{r|rrrl} & 1 & -5 & -1 & d \\ -1 & & -1 & 6 & -5 \\ \hline & 1 & -6 & 5 & -5 + d, \; d - 5 = 0, \; d = 5 \end{array}$$

Hence $f(x) = (x + 1)(x^2 - 6x + 5)$

$f(x) = (x + 1)(x - 1)(x - 5)$ fully factorised.

9. *(a)* $u_{r+1} = mu_r + c, \; u_0 = 3, \; u_1 = 2, \; u_2 = 4$

$u_1 = mu_0 + c \Rightarrow 2 = 3m + c$ ①

$u_2 = mu_1 + c \Rightarrow 4 = 2m + c$ ②

② − ① $\quad 2 = -m$

Substitute $m = -2$ in ①, $\quad 2 = 3(-2) + c$

$\Rightarrow 2 = -6 + c$

$c = 8$

$m = -2 \quad c = 8$

$u_{r+1} = -2u_r + 8$

(b) $u_3 = -2u_2 + 8$

$= -2(4) + 8 = 0$

u_{-1} is found by using $u_0 = -2u_{-1} + 8, u_0 = 3$

$3 = -2u_{-1} + 8$

$-5 = -2u_{-1}$

$\frac{5}{2} = u^{-1}$

Hence $u_3 = 0, \; u_{-1} = \frac{5}{2}$

(c) $u_r = u_{r+1} \Rightarrow u_r = -2u_r + 8$

$\Rightarrow 3u_r = 8$

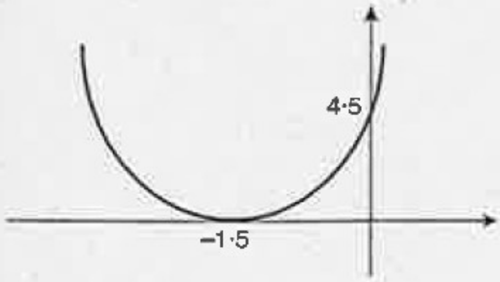
$u_r = \frac{8}{3}$

Check $-2\left(\frac{8}{3}\right) + 8 = \frac{-16}{3} + \frac{24}{3} = \frac{8}{3}$

$(u_r, u_{r+1}), \left(\frac{8}{3}, \frac{8}{3}\right)$

10. *(a)* $f(x) = 2x^2 + 6x + p$ in form $ax^2 + bx + c$
$a = 2, b = 6, c = p$, and using the discriminant $b^2 - 4ac = 0$ if $f(x)$ has equal roots

$$\begin{aligned} b^2 - 4ac &= (6)^2 - 4(2)(p) = 0 \\ 36 - 8p &= 0 \\ 8p &= 36 \\ p &= \frac{36}{8}\ ;\ p = 4{\cdot}5 \end{aligned}$$



substitute $p = 4{\cdot}5$ into $f(x)$
$f(x) = 2x^2 + 6x + 4{\cdot}5$,
$a = 2, b = 6, c = 4{\cdot}5$,
and using the discriminant $b^2 - 4ac = 0$
if $f(x)$ has equal roots

$$\begin{aligned} b^2 - 4ac &= (6)^2 - 4(2)(4{\cdot}5) = 0 \\ 36 - 36 &= 0 \end{aligned}$$

root is $\frac{-b}{2a}$; $= \frac{-6}{4}$; $= \frac{-3}{2}$ or $-1{\cdot}5$

(b) coordinates of the root $= (-1{\cdot}5, 0)$

(c) y-intercept $= (0, 9)$, x-intercept $= (-1{\cdot}5, 0)$

11. $\int_{\pi/3}^{\pi/2} \sin x\, dx = [-\cos x]_{\pi/3}^{\pi/2}$

$$\begin{aligned} F(\pi/2) - F(\pi/3) &= (-\cos \pi/2) - (-\cos \pi/3) \\ &= 0 - (-0{\cdot}5) \\ &= 0{\cdot}5 \end{aligned}$$

$(\pi/2)$ radians $= 90°$ $(\pi/3)$ radians $= 60°$

12.

$$\begin{aligned} 4 \sin 2x - 2 &= 0 \\ 4 \sin 2x &= 2 \\ \sin 2x &= \frac{2}{4} \\ &= 0{\cdot}5 \\ \sin^{-1}(0{\cdot}5) &= 30 \text{ in quadrant 1 and } 180 - 30 \text{ in quadrant 2} \\ 2x &= 30° \text{ or } 150° \\ x &= 15° \text{ or } 75° \end{aligned}$$

Period of $\sin 2x = \frac{360}{2} = 180°$ (i.e., 2 cycles in 360°) add 180 to each of 15, and 75.

Hence x has 4 possible values, 15°, 75°, 195° and 255°.