1. AB : BC

2 : 3 hence 3AB = 2BC

$$3(b - a) = 2(c - b)$$
$$3b - 3a = 2c - 2b$$
$$5b = 3a + 2c$$

$$3\begin{bmatrix}4\\-1\\5\end{bmatrix} + 2\begin{bmatrix}-1\\4\\10\end{bmatrix} = \begin{bmatrix}12 - 2\\-3 + 8\\15 + 20\end{bmatrix} = \begin{bmatrix}10\\5\\35\end{bmatrix} = 5b \qquad b = \begin{bmatrix}2\\1\\7\end{bmatrix}$$

The coordinates of B = (2, 1, 7)

2. $\int_a^0 (f_1(x) - f_2(x))dx + \int_0^b (f_2(x) - f_1(x))dx = $ (upper – lower)

$x < 0$ line is upper, line $= f_1(x)$

$x > 0$ curve is upper, curve $= f_2(x)$

3. $f : x \to \sin 3x \qquad 0 \le x < 360$

period $= \dfrac{360}{3} = 120$

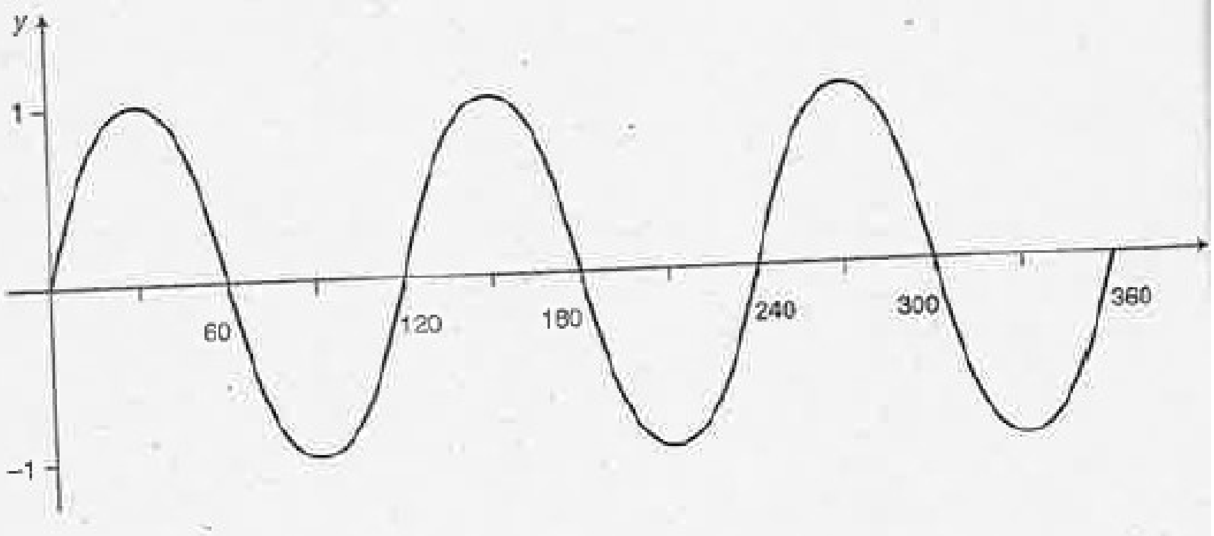
curve repeats 3 times in interval 0 to 360.

cuts x-axis when $\sin 3x = 0$

$\Rightarrow 3x = 0, 180, 360, 540$, etc.

$x = \{0, 60, 120, 180, 240, 300\}$

$\sin 3x$ meets the x-axis 6 times.



4. $x^2 + y^2 - 8x + 6y + 21 = 0$

(radius remains the same)

$2g = -8, -g = 4$

$2f = 6, -f = -3$

centre $(-g, -f)$

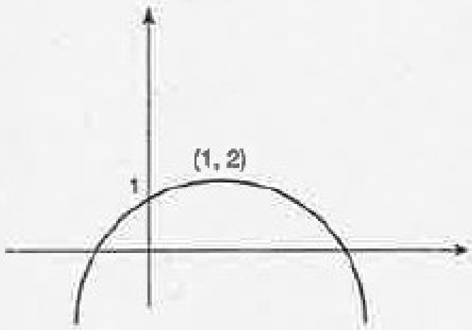
centre (4, –3) reflected in y-axis centre = (–4, –3)

equation is $x^2 + y^2 + 8x + 6y + 21 = 0$

5. *(a)* $1 + 2x - x^2 = -x^2 + 2x + 1 = -1(x^2 - 2x - 1) = -1[(x^2 - 2x + 1) - 1 - 1]$

$$= -1[(x - 1)^2 - 2]$$
$$= -1(x - 1)^2 + 2$$
$$= 2 - (x - 1)^2$$

Since the least value of any square number is 0,
then $(x - 1)^2$ has minimum value 0.
Hence the maximum value of the function is $2 - 0$
when $x = 1$.
Maximum value of the function is $2 - (1 - 1)^2$
and the coordinates of the maximum turning point (1, 2).



(b) y-intersection (0, 1), maximum turning point (1, 2).

(c) Since the maximum turning point (1, 2) lies above the x-axis, the curve crosses the x-axis in two places and the equation has 2 real roots.

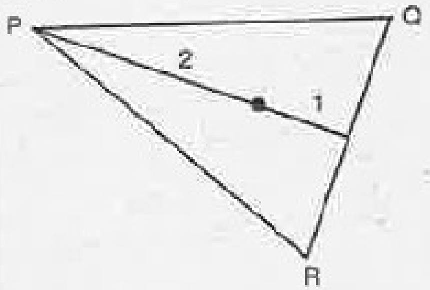
6. $f(x) = 2 \sin 3x,$

$$f'(x) = 6 \cos 3x,$$

$$f'\left(\frac{\pi}{4}\right) = 6 \cos 3\left(\frac{\pi}{4}\right)$$

$$= 6 \times \frac{-1}{\sqrt{2}} = \frac{-6}{\sqrt{2}} = -3\sqrt{2}$$

7. *(a)* P(1, 2), Q(6, 3), R(5, −2)



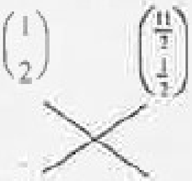
Let mid-point of QR = L

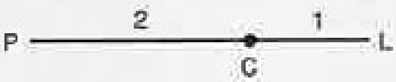
$$L = \left(\frac{6+5}{2}, \frac{3-2}{2}\right)$$

$$L = \left(\frac{11}{2}, \frac{1}{2}\right)$$

Let C = centroid

PC : CL = 2 : 1





$$C = \frac{1}{3}\left[\begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}11\\1\end{pmatrix}\right], \quad = \frac{1}{3}\begin{pmatrix}12\\3\end{pmatrix} = (4, 1)$$

<u>Alternative Method</u>

$$\overrightarrow{OC} = \frac{1}{3}(p + q + r) = \left(\frac{1+6+5}{2} \quad \frac{2+3+(-2)}{3}\right) = (4, 1) = C$$

(b) P(1, 2), Q(6, 3), R(5, −2)

$$M = \frac{1}{2}(p + r) = \frac{1}{2}\begin{pmatrix}6\\0\end{pmatrix} = (3, 0)$$

$$N = \frac{1}{2}(q + p) = \frac{1}{2}\begin{pmatrix}7\\5\end{pmatrix} = \left(\frac{7}{2}, \frac{5}{2}\right)$$

$$\overrightarrow{QC} = c - q \quad c\begin{pmatrix}4\\1\end{pmatrix}, q\begin{pmatrix}6\\3\end{pmatrix}$$

$$= \begin{pmatrix}-2\\-2\end{pmatrix}$$

$$\overrightarrow{QM} = m - q, m\begin{pmatrix}3\\0\end{pmatrix}, q\begin{pmatrix}6\\3\end{pmatrix}$$

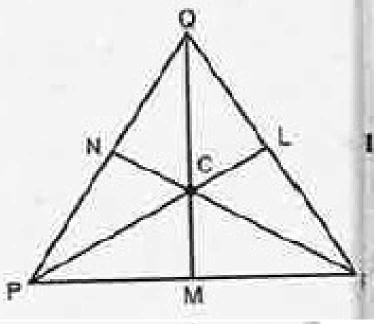
$$= \begin{pmatrix}-3\\-3\end{pmatrix}$$

$$\overrightarrow{QC} : \overrightarrow{QM}$$
$$= 2 : 3$$

$$\overrightarrow{RC} = c - r, c\begin{pmatrix}4\\1\end{pmatrix}, r\begin{pmatrix}5\\-2\end{pmatrix} \qquad \overrightarrow{CN} = n - c, n = \begin{pmatrix}\frac{7}{2}\\\frac{5}{2}\end{pmatrix}, \quad c = \begin{pmatrix}4\\1\end{pmatrix}$$

$$c - r = \begin{pmatrix}4 - 5\\1 - (-2)\end{pmatrix} \qquad n - c = \begin{pmatrix}\frac{7}{2} - \frac{8}{2}\\\frac{5}{2} - \frac{2}{2}\end{pmatrix}$$

$$= \begin{pmatrix}-1\\3\end{pmatrix}, \qquad = \begin{pmatrix}-\frac{1}{2}\\\frac{3}{2}\end{pmatrix}$$



Ratio $\overrightarrow{RC} : \overrightarrow{CN} = 2 : 1$

8. $f(x) = x^3 - 3x^2 - 4x + 12$

	1	−3	−4	+12
−2		−2	10	−12
	1	−5	+6	0

Hence $f(x) = (x + 2)(x^2 - 5x + 6)$
$= (x + 2)(x - 3)(x - 2)$

Try factors of −12
$\{\pm1, \pm2, \pm3, \pm4, \pm6\}$
If $f(x) \div (x + 2)$ has remainder zero then $(x + 2)$ is a facto

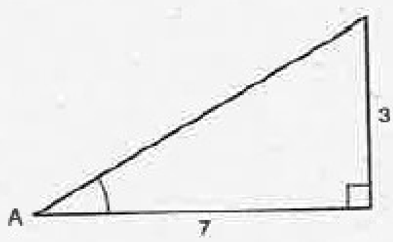
9. By Pythagoras' theorem the 3rd side of the triangle $= \sqrt{3^2 + 7^2} = \sqrt{58}$

$$\cos A = \frac{7}{\sqrt{58}} \qquad \cos 2A = 2\cos^2 A - 1$$

$$= 2\left(\frac{7}{\sqrt{58}}\right)^2 - 1$$

$$= 2\left(\frac{49}{58}\right) - 1$$

$$= \frac{98}{58} - \frac{58}{58} = \frac{40}{58} = \frac{20}{29}$$



$$\cos 2A = \frac{20}{29}$$

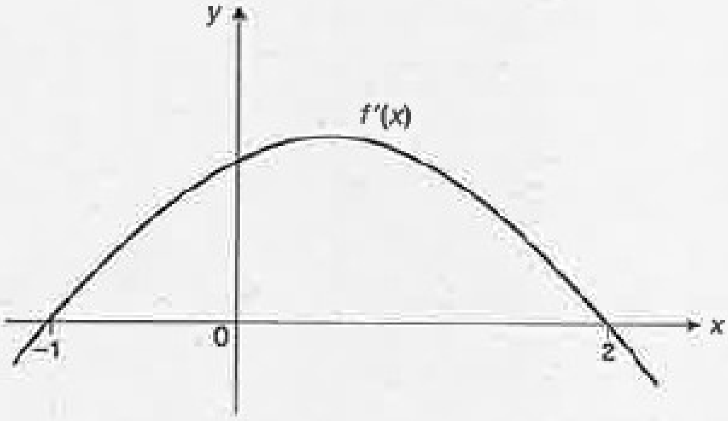
10. $f(x) = (2x^2 - x)^5$

By chain rule method

$f'(x) = 5(2x^2 - x)^4(4x - 1)$

$f'(x) = 5(4x - 1)(2x^2 - x)^4$

11.



x	<-1	-1	>-1	2	>2
$f'(x)$	−ve	0	+	0	−ve

$f(x)$ has general shape of ax^3, a negative

$f'(x)$ has general shape of $-3x^2$ 

Plot $(-1, 0)(2, 0)$

12. $C = 3\cos\left(x + \frac{\pi}{3}\right)$ maximum value $=$ 3 when $x + \frac{\pi}{3} = 0$ or 2π

minimum value $= -3$ when $x + \frac{\pi}{3} = \pi$

min, $x + \frac{\pi}{3} = \pi, x = \pi - \frac{\pi}{3}, x = \frac{2\pi}{3}$

$x = \frac{5\pi}{3} \quad y = 3.$

max, $x = -\frac{\pi}{3}$ or $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$x = \frac{2\pi}{3} \quad y = -3$