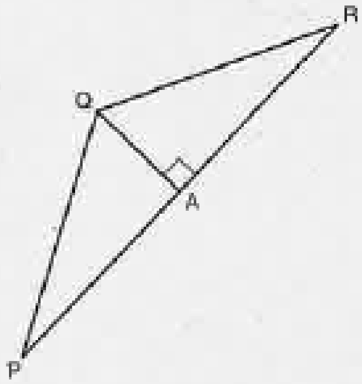
WORKED EXAMPLE – TEST PAPER B

1. P(2, –1), Q(3, 2), R(6, 5)



gradient of PR $= \dfrac{5-(-1)}{6-2}$

$$\frac{6}{4} = \frac{3}{2} = m$$

If line QA is perpendicular to line PR
then $m_{QA} \cdot m_{PR} = -1$

$m_{PR} = \dfrac{3}{2}$ hence $m_{QA} = -\dfrac{2}{3}$

Point on QA = Q(3, 2) gradient of QA $= -\dfrac{2}{3}$

$$y - b = m(x - a)$$
$$y - 2 = -\frac{2}{3}(x - 3)$$
$$3y - 6 = -2x + 6$$

$3y + 2x = 12$ is the equation of the altitude QA.

2. P(2, a, –3), Q(1, a, a)

$\overrightarrow{OP} = p = \begin{pmatrix} 2 \\ a \\ -3 \end{pmatrix}$, $\overrightarrow{OQ} = q = \begin{pmatrix} 1 \\ a \\ a \end{pmatrix}$

If $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$, then $p \cdot q = 0$

$$\Rightarrow \begin{pmatrix} 2 \\ a \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ a \\ a \end{pmatrix} = 0$$

$\Rightarrow \quad 2 + a^2 - 3a = 0$
$\Rightarrow \quad a^2 - 3a + 2 = 0$
$(a-1)(a-2) = 0$
$a = 2$ or $a = 1$

Check with $a = 2$; $\begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 2 + 4 - 6 = 0$

Check with $a = 1$; $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 2 + 1 - 3 = 0$

3. *(a)* $x^2 + y^2 - 4x + 2y + 1 = 0$

General equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$

where centre $= (-g, -f)$
radius $= \sqrt{g^2 + f^2 - c}$

$x^2 + y^2 - 4x + 2y + 1 = 0$

$2g = -4$, $2f = 2$

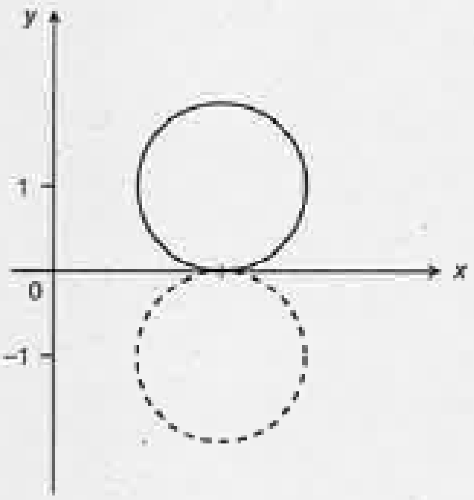
$-g = 2$, $-f = -1$, $c = 1$

radius $= \sqrt{2^2 + 1^2 - 1} = \sqrt{4} = 2$

Hence centre = (2, –1), radius = 2

(*b*) Reflected in the x-axis
centre = (2, 1)
radius unchanged
New equation is

$$x^2 + y^2 - 4x - 2y + 1 = 0$$



4. $f(x) = x^3 + mx$

$f'(x) = 3x^2 + m$

$3x^2 + m$ = gradient of tangent

gradient = 0 for stationary value

$\Rightarrow 3x^2 + m = 0$

$x = \pm 1 \Rightarrow 3(-1)^2 + m = 0$

$3 + m = 0$

$\Rightarrow m = -3$

$f(x) = x^3 - 3x$

5. $(\sqrt{3} + 2\sqrt{2})^2$.

$= (\sqrt{3} + 2\sqrt{2})(\sqrt{3} + 2\sqrt{2})$

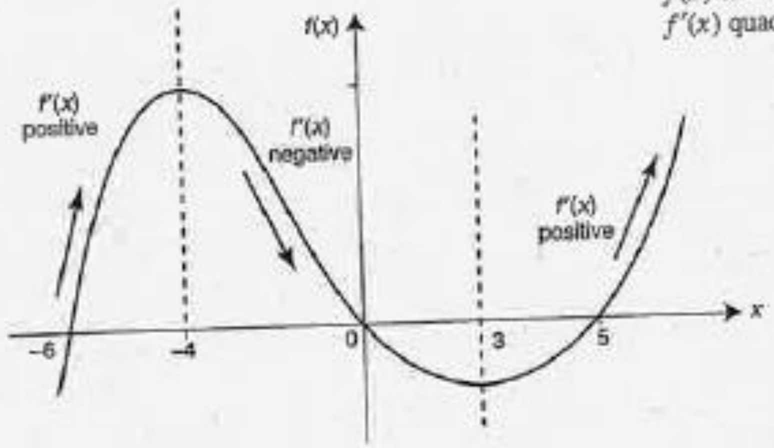
$= \sqrt{3}(\sqrt{3} + 2\sqrt{2}) + 2\sqrt{2}(\sqrt{3} + 2\sqrt{2})$

$= 3 + 2\sqrt{6} + 2\sqrt{6} + 8$

$= 11 + 4\sqrt{6}$

6.

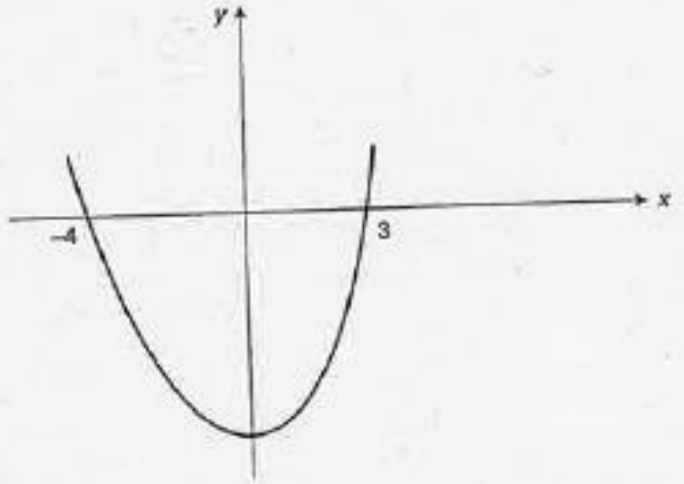
$f(x)$ is cubic
$f'(x)$ quadratic (assumed)



	$x<-4$	$x=-4$	$-4<x<3$	$x=3$	$x>3$
$f'(x)$	+ve	0	–ve	0	+ve
Plot $f'(x)$ in relation to the x-axis	above	on	below	on	above

sketch of $f'(x$

points $(-4, 0)(3, 0$



7. $f(x) = 3x^3 - 16x^2 + px + 10$

If $f(x)$ is divisible by $(x-1)$, then by synthetic division remainder $= 0$.

	x^3	x^2	x^1	x^0	
$\div (x-1)$	3	-16	p	10	
1		3	-13	$-13+p$	
	3	-13	$-13+p$	$-3+p$	(Remainder)
			$\Rightarrow$	$-3+p=0$	
				$p=3$	

$3x^3 - 16x^2 + 3x + 10 \div (x-1)$

$= 3x^2 - 13x - 10$ (since $-13 + p = -13 + 3 = -10$

factorised $= (3x+2)(x-5)$

fully factorised $f(x) = (x-1)(x-5)(3x+2)$

8. $f'(x) = 3x - 2$

$f(x) = \frac{3x^2}{2} - 2x + c$

$f(2) = \frac{3(2)^2}{2} - 2(2) + c$

$= 6 - 4 + c$

$= 2 + c$

given $f(2) = 7$

$\Rightarrow \quad 2 + c = 7$

$\Rightarrow \quad c = 5$

Hence $f(x) = \frac{3x^2}{2} - 2x + 5$

9. $f(x) = 2x^2$

$g(x) = 3x - 1$

$f(g(x)) = f(3x - 1)$

$= 2(3x - 1)^2$

$= 2(9x^2 - 6x + 1)$

$f(g(x)) = 18x^2 - 12x + 2$

10. *(a)* $3x^2 - 4x + 2$

$a = 3, b = -4, c = 2$

$b^2 - 4ac < 0$ for no real roots

$(-4)^2 - 4(3)(2)$

$16 - 24 = -8$

since $\quad -8 \in R$

function has no real roots.

(b) $3x^2 - 4x + 2$

$3\left(x^2 - \frac{4}{3}x + \frac{2}{3}\right)$

$3\left[\left(x - \frac{2}{3}\right)^2 - \frac{4}{9} + \frac{2}{3}\right]$

$3\left[\left(x - \frac{2}{3}\right)^2 + \frac{2}{9}\right]$

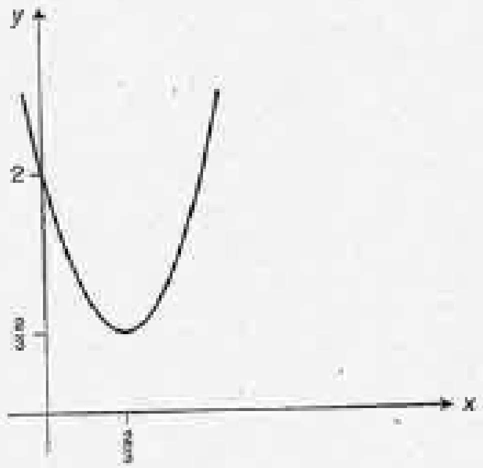
$= 3\left(x - \frac{2}{3}\right)^2 + \frac{2}{3}$

minimum value $= \frac{2}{3}$ when $x = \frac{2}{3}$

minimum turning point $= \left(\frac{2}{3}, \frac{2}{3}\right)$

cuts y-axis at $(0, 2)$

(c) Sketch of $f(x)$



11. (a) $u_{r+1} = mu_r + c,\ u_0 = 1,\ u_1 = -3,\ u_2 = 21$

$u_1 = mu_0 + c \Rightarrow -3 = m(1) + c$ ①

$u_2 = mu_1 + c \Rightarrow 21 = m(-3) + c$ ②

① − ② $-24 = 4m, \quad \Rightarrow \quad m = -6$

Substitute $m = -6$ in ①

$-3 = -6(1) + c$

$3 = c$

$m = -6 \quad c = 3$

$u_{r+1} = -6u_r + 3$

$u_2 = -6u_1 + 3$

check $-6(-3) + 3$

$= 18 + 3 = 21$

(b) $u_3 = -6u_2 + c$

$= -6(21) + 3$

$= -126 + 3 = -123$

$u_3 = -123$

To find u_{-1} use $u_0 = -6u_{-1} + 3$

$u_0 = 1 \Rightarrow 1 = -6u_{-1} + 3$

$\Rightarrow -2 = -6u_{-1}$

$\Rightarrow \dfrac{1}{3} = u_{-1}$

(c) To find $u_r = u_{r+1}$

$u_{r+1} = -6u_r + 3$

$\Rightarrow \quad u_r = -6u_r + 3$

$\Rightarrow \quad 7u_r = 3$

$\Rightarrow \quad u_r = \dfrac{3}{7}$

Check $u_r = \dfrac{3}{7},\ u_{r+1} = -6\left(\dfrac{3}{7}\right) + 3$

$= \dfrac{-18}{7} + 3$

$= \dfrac{-18}{7} + \dfrac{21}{7} = \dfrac{3}{7}$

Gives (u_r, u_{r+1}), $\left(\dfrac{3}{7}, \dfrac{3}{7}\right)$, $u_r = u_{r+1}$

12. Using Pythagoras' Theorem, third side is $\sqrt{21}$

$\sin x = \frac{2}{5} \quad \cos x = \frac{\sqrt{21}}{5}$

$\sin 2x = 2 \sin x \cos x = 2\left(\frac{2}{5}\right) \cdot \left(\frac{\sqrt{21}}{5}\right) = \left(\frac{4\sqrt{21}}{25}\right)$

$\cos 2x = \cos^2 x - \sin^2 x = \left(\frac{\sqrt{21}}{5}\right)^2 - \left(\frac{2}{5}\right)^2$

$\frac{21}{25} - \frac{4}{25} = \frac{17}{25}$