Algebra - Solutions

① 2010 Paper 1 Q3

$t = \frac{7s + 4}{2}$ • rearrange to get $s = \ldots$ • get rid of fraction..

$2t = 7s + 4$ ✓

$7s + 4 = 2t$

$7s = 2t - 4$ ✓

$s = \frac{2t - 4}{7}$ ✓ (3KU)

---

② 2010 Paper 1 Q4

$f(x) = x^2 - 4x$ $g(x) = 2x + 7$

(a) $f(x) = g(x)$

$x^2 - 4x = 2x + 7$ ✓

$x^2 - 6x - 7 = 0$ ✓ (2RE)

(b) $x^2 - 6x - 7 = 0$ quadratic equation ⇒ factorise

$(x - 7)(x + 1) = 0$ ✓

$x - 7 = 0$ $x + 1 = 0$ (2RE)

$x = 7$ $x = -1$ ✓

---

③ 2010 Paper 2 Q2

$x(x-1)^2 = x(x^2 - 2x + 1)$ ✓

$= x^3 - 2x^2 + x$ ✓

Remember: $(x-1)^2 = (x-1)(x-1)$

(2KU)

---

④ 2009 Paper 1 Q3

$f(x) = x^2 + 3$

(a) $f(-4) = (-4)^2 + 3$

$= 16 + 3$

$= 19$ ✓ (2KU)

(b) $f(t) = t^2 + 3$ $f(t) = 52$

so $t^2 + 3 = 52$ ✓

$t^2 = 49$ ✓

$t = \pm 7$ (2RE)

(5) 2009 Paper 1 Q.4

(a) $x^2 - 4y^2 = \underline{(x-2y)(x+2y)}$ ✓ (difference of 2 squares)

(b) $(2x-1)(x+4) = 2x^2 + 8x - x - 4$

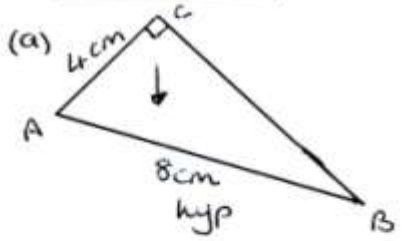
$= \underline{2x^2 + 7x - 4}$ ✓

(c) $x^{\frac{1}{2}}(3x + x^{-2}) = \underline{3x^{\frac{3}{2}} + x^{-\frac{3}{2}}}$ ✓ ✓ (4ku)

---

(6) 2009 Paper 1 Q.5

(a)



Right-angled △ ⇒ Pythagoras.

$BC^2 = 8^2 - 4^2$ ✓

$BC^2 = 64 - 16$

$BC^2 = 48$ ✓

$BC = \sqrt{48}$

$BC = \sqrt{16 \times 3}$

$BC = \underline{\underline{4\sqrt{3}}}$ ✓ (3ku)

---

(7) 2009 Paper 1 Q.9

$f = \frac{kd^2}{20}$

- rearrange to get $d = \ldots$
- get rid of fraction ....

$20f = kd^2$ ✓

$kd^2 = 20f$

$d^2 = \frac{20f}{k}$ ✓ (3ku)

$d = \underline{\underline{\sqrt{\frac{20f}{k}}}}$ ✓

---

(8) 2008 Paper 1 Q.2

$5x^2 - 45 = 5(x^2 - 9)$ ✓

- common factor
- difference of 2 squares.

$= \underline{\underline{5(x-3)(x+3)}}$ ✓

(2ku)

(9) 2008 Paper 1 Q.3

$W = BH^2$ • rearrange to get H = ...

$BH^2 = W$

$H^2 = \frac{W}{B}$ ✓

$H = \sqrt{\frac{W}{B}}$ ✓ (2KU)

---

(10) 2008 Paper 1 Q.5

$\frac{1}{p} + \frac{2}{(p+5)}$

$= \frac{(p+5) + 2p}{p(p+5)}$ ✓

$= \frac{3p+5}{p(p+5)}$ ✓ (2KU)

---

(11) 2008 Paper 1 Q.6

(a) distance = S x T

$= 2(x+8)$ ✓ (2KU)

(b) distance = $\frac{1}{2}x$ ✓ (30mins = ½hr)

(c) total distance = 46

$2(x+8) + \frac{1}{2}x = 46$ ✓

$2\frac{1}{2}x + 16 = 46$

$2\frac{1}{2}x = 30$

$\frac{5}{2}x = 30$ ✓

$x = 30 \times \frac{2}{5}$ (3RE)

$x = 12$

running speed = x = 12 km/hr. ✓

(12) 2008 Paper 1 Q.9

$m^3 \times \sqrt{m} = m^3 \times m^{\frac{1}{2}}$ ✓

$= \underline{m^{\frac{7}{2}}}$ ✓ (2ku)

---

(13) 2008 Paper 1 Q.11

Right-angled △ => Pythagoras.

$AC^2 = (\sqrt{50})^2 - (\sqrt{32})^2$ ✓

$AC^2 = 50 - 32$

$AC^2 = 18$ ✓

$AC = \sqrt{18}$

$AC = \sqrt{9 \times 2}$

$\underline{AC = 3\sqrt{2}}$ ✓ (3ku)

---

(14) 2008 Paper 1 Q.13

add x to numerator + denominator:

$\frac{(17+x)}{(24+x)} = \frac{2}{3}$ ✓

- brackets around sums
- as we have fraction = fraction we can cross-multiply.

$3(17+x) = 2(24+x)$ ✓

$51 + 3x = 48 + 2x$

$\underline{x = -3}$ ✓ (3RE)

---

(15) 2007 Paper 1 Q.4

$P = \frac{2(m-4)}{3}$

- rearrange to get $m = \ldots$
- get rid of fraction...

$3p = 2m - 8$ ✓

$2m - 8 = 3p$

$2m = 3p + 8$ ✓ (3ku)

$\underline{m = \frac{3p+8}{2}}$ ✓ (or $m = \frac{3p}{2} + 4$)

(16) 2007 Paper 1 Q.5

$(2x+3)^2 - 3(x^2-6)$ ✓

$= 4x^2 + 12x + 9 - 3x^2 + 18$ ✓

$= \underline{x^2 + 12x + 27}$ ✓ (3KU)

• remember: $(2x+3)^2 = (2x+3)(2x+3)$

---

(17) 2008 Paper 2 Q.10

(a) $25 \times 4 + (640 - 200) \times 0{\cdot}12 = \underline{£152{\cdot}80}$ ✓ (1KU)

↑ 4 days; ↑ 200 miles free; ↑ 12p per additional miles.

(b) $C = 25d + (m - 200) \times 0{\cdot}12$

$\underline{C = 25d + 0{\cdot}12m - 24}$ ✓ (3RE)

---

(18) 2007 Paper 1 Q.7

$a^{\frac{1}{2}}(a^{\frac{1}{2}} - 2) = a^1 - 2a^{\frac{1}{2}}$ ✓ (x bases, + powers)

$= \underline{a - 2\sqrt{a}}$ ✓ (2KU)

---

(19) 2007 Paper 2 Q.4

• solve ⇒ get $x < \ldots\ldots$

$\frac{x}{4} - \frac{1}{2} < 5$ (× everything by 4)

$x - 2 < 20$ ✓

$\underline{x < 22}$ ✓ (2KU)

---

(20) 2006 Paper 1 Q.3

$f(x) = 4 - x^2$

$f(-3) = 4 - (-3)^2$ ✓ (2KU)

$= 4 - 9$ ✓

$= \underline{-5}$

(21) 2006 Paper 1 Q.6

(a) $4x^2 - y^2 = (2x-y)(2x+y)$ ✓ • difference of 2 squares.

(b) $\frac{4x^2-y^2}{6x+3y} = \frac{(2x-y)(2x+y)}{3(2x+y)}$ ✓ • hence means using previous answer.

$= \frac{(2x-y)}{3}$ ✓ (3KU)

---

(22) 2006 Paper 1 Q.6

$x - 2(x+1) = 8$ • solve means get $x = \ldots$

$x - 2x - 2 = 8$ ✓

$-x = 10$ ✓

$x = -10$ ✓ (3KU)

---

(23) 2006 Paper 1 Q.8

(a) Area of $\triangle = \frac{1}{2}bh$ (base + height are perpendicular)

$= \frac{1}{2} \times 20 \times 15$

$= 150\ m^2$ ✓ (1KU)

(b) Area of $\triangle = \frac{1}{2}bh$

$150 = \frac{1}{2} \times 25 \times BD$ ✓

$150 = 12.5 \times BD$

$BD = 150 \div 12.5$ ✓

$BD = 12$ (3RE)

height $BD = 12\ cm$. ✓

(24) 2006 Paper 1 Q.11

(a) $3x$ ✓ (1ku)

(b) i) $20 + (15-6) \times 2 = £38$ ✓ (1ku)

ii) $20 + (15-6) \times 2 = 20 + 2x - 12$ ✓
$= \underline{8 + 2x}$. ✓ (2RE)

(c) monthly card < sessions

$8 + 2x < 3x$ ✓

$-x < -8$

$x > 8$ ✓ (3RE)

minimum n° of sessions = 9. ✓

(Guess + check would be ok as long as all working is shown)

---

(25) 2006 Paper 2 Q.4

(a) $(x+4)(3x-1) = 3x^2 - x + 12x - 4$
$= \underline{3x^2 + 11x - 4}$ ✓ (1ku)

(b) $m^{\frac{1}{2}}(2 + m^2) = 2m^{\frac{1}{2}} + m^{5\frac{1}{2}}$ ✓
$= \underline{2\sqrt{m} + m^{5\frac{1}{2}}}$ ✓ (2ku)

(c) $2\sqrt{20} - 3\sqrt{5} = 2\sqrt{4 \times 5} - 3\sqrt{5}$
$= 4\sqrt{5} - 3\sqrt{5}$ ✓
$= \sqrt{5}$ ✓ (2ku)

---

(26) 2005 Paper 1 Q.6

$\frac{2}{x} + 1 = 6$

$\frac{2}{x} = 5$ ✓

$2 = 5x$ ✓ (3ku)

$\underline{\underline{x = 2/5}}$ ✓

(27) 2005 Paper 1 Q9

a) £30 of petrol . £0·75 per litre . 5ℓ/hour used .

- 3hours => 3×5 = 15ℓ of petrol used .
- £30 => 30 ÷ 0·75 = 40ℓ to start with ✓

  ($30 \div \frac{3}{4} = 30 \times \frac{4}{3} = 40$)

  (2KU)
- 40ℓ − 15ℓ = 25ℓ left . ✓

b) £20 of petrol, $\frac{c}{100}$ per litre . Kℓ/hour used . t hours

$R = \left(20 \div \frac{c}{100}\right) - k \cdot t$ ✓

$R = 20 \times \frac{100}{c} - kt$

(3RE)

$R = \frac{2000}{c} - kt$ ✓

---

(28) 2005 Paper 1 Q.11

$f(x) = 4\sqrt{x} + \sqrt{2}$

(a) $f(72) = 4\sqrt{72} + \sqrt{2}$ ✓

$= 4\sqrt{36 \times 2} + \sqrt{2}$

$= 4\sqrt{36}\sqrt{2} + \sqrt{2}$ ✓

$= 4 \times 6\sqrt{2} + \sqrt{2}$ (3KU)

$= 24\sqrt{2} + \sqrt{2}$

$= 25\sqrt{2}$ ✓

(b) $f(t) = 3\sqrt{2}$

$4\sqrt{t} + \sqrt{2} = 3\sqrt{2}$ ✓

$4\sqrt{t} = 2\sqrt{2}$

(3RE)

$\sqrt{t} = \frac{2\sqrt{2}}{4}$ ✓

$\sqrt{t} = \frac{\sqrt{2}}{2}$ => $\sqrt{t} = \frac{2}{4}$ => $t = \frac{1}{2}$ ✓

(29) 2004 Paper 1 Q3

$A = 2x^2 - y^2$

$x = 3$
$y = (-4)$

$A = 2(3)^2 - (-4)^2$ ✓

$A = 18 - 16$ (2KU)

$A = 2$ ✓

---

(30) 2004 Paper 1 Q4

$\frac{3}{m} + \frac{4}{(m+1)}$

$= \frac{3(m+1)+4(m)}{m(m+1)}$ ✓ ✓ (3KU)

$= \frac{7m+3}{m(m+1)}$ ✓

---

(31) 2004 Paper 1 Q.11

(a) $2\sqrt{75} = 2\sqrt{25 \times 3}$

$= 2\sqrt{25}\sqrt{3}$ ✓

$= 2 \times 5\sqrt{3}$ (2KU)

$= 10\sqrt{3}$ ✓

(b) $2^0 + 3^{-1}$

$= 1$ ✓ $+ 1/3$ ✓

$= 1\frac{1}{3}$. (2KU)

---

(32) 2004 Paper 1 Q12

Circumference = length of wire

$\pi d = 10$ ✓

$d = \frac{10}{\pi}$ ✓

$r = \frac{1}{2}$ of $\frac{10}{\pi}$

$r = \frac{5}{\pi}$ ✓ (4RE)

Area $= \pi r^2$ ✓

$= \pi \times (\frac{5}{\pi})^2 = \pi \times \frac{25}{\pi^2} = \frac{25}{\pi}$.

(33) 2003 Paper 1 Q.3

$3(2x-4)-4(3x+1)$

$= 6x-12-12x-4$ ✓✓ (3ku)

$= \underline{-6x-16}$ ✓

---

(34) 2003 Paper 1 Q4

$f(x) = 7-4x$

(a) $f(-2) = 7-4\times(-2)$

$f(-2) = 7+8$

$\underline{f(-2) = 15}$ ✓ (1ku)

(b) $f(t) = 9$

$7-4t = 9$ ✓ (2ku)

$-4t = 2$

$\underline{t = -\frac{1}{2}}$ ✓ $(-\frac{2}{4})$

---

(35) 2003 Paper 1 Q5

Factorise ⇒ put brackets back.

$2x^2-7x-15 = \underline{(2x+3)(x-5)}$ ✓ ✓

(2ku)

---

(36) 2003 Paper 1 Q.12

(a) $8^{\frac{2}{3}} = \sqrt[3]{8^2}$ ✓ $= 2^2 = \underline{4}$ ✓ (2ku)

(b) $\frac{\sqrt{24}}{\sqrt{2}} = \sqrt{\frac{24}{2}} = \sqrt{12} = \sqrt{4\times3} = \underline{2\sqrt{3}}$ ✓ (2ku)

---

(37) 2003 Paper 2 Q.5

$d = \frac{n(n-3)}{2}$

$d=20, \quad 20 = \frac{n(n-3)}{2}$ ✓

$40 = n(n-3)$

$40 = n^2-3n$

$0 = n^2-3n-40$ ✓

$n^2-3n-40=0$

$(n+5)(n-8) = 0$ ✓

$n+5=0 \quad n-8=0$

$n=-5 \quad n=8$

The polygon has 8 sides. ✓

• quadratic eqn - factorise or use formula to solve. (4RE)

(38) 2003 Paper 2 Q.11

(a) $d = x$ $\quad t = D/S$

$S = 75 \text{ km/hr}$ $\quad t = \frac{x}{75}$ ✓ (1KU)

$t = ?$

(note: there is an error in the question - units are miles + km/hr !)

(b) $t = x/50$ (B to A)

• whole journey: $d = 2x$ ✓

$t = \left(\frac{x}{75} + \frac{x}{50}\right)$

$t = \frac{2x}{150} + \frac{3x}{150} = \frac{5x}{150} = \frac{x}{30}$. ✓

speed = D/t

$= 2x \div \frac{x}{30}$ ✓

$= 2x \times \frac{30}{x}$

$= \frac{60x}{x}$ ✓ (4RE)

$= 60 \text{ km/hr}$.

---

(39) 2002 Paper 1 Q.3

$5 - x > 2(x + 1)$

$5 - x > 2x + 2$ ✓

$-3x > -3$ ✓ (3KU)

$x < 1$ ✓

note: remember to change $>$ to $<$ when ÷ by negative no.

(40) 2002 Paper 1 Q.4

$f(x) = x^2 + 5x$

$f(-3) = (-3)^2 + 5.(-3)$ ✓

$f(-3) = 9 - 15$

$f(-3) = -6$ ✓ (2KU)

---

(41) 2002 Paper 1 Q.5

(a) $p^2 - 4q^2 = (p-2q)(p+2q)$ ✓ (1KU)

(b) $\frac{p^2-4q^2}{3p+6q} = \frac{(p-2q)(p+2q)}{3(p+2q)}$ ✓

$= \frac{(p-2q)}{3}$ ✓ (2KU)

---

(42) 2002 Paper 1 Q.6

$L = \frac{1}{2}(h-t)$

$\frac{1}{2}(h-t) = L$

$h - t = 2L$ ✓

$h = 2L + t$ ✓ (2KU)

---

(43) 2002 Paper 1 Q.10

$\sqrt{27} + 2\sqrt{3} = \sqrt{9 \times 3} + 2\sqrt{3}$

$= 3\sqrt{3}$ ✓ $+ 2\sqrt{3}$

$= 5\sqrt{3}$ ✓ (2KU)

(44) 2002 Paper 1 Q11

$y^8 \div (y^3)^{-2} = y^8 \times y^{-6}$ ✓

$= y^2$ ✓ (2ku)

---

(45) 2002 Paper 2 Q.9

(a) Easy call: $25p \times 3 + 5p \times 7 = 75p + 35p = £1.10$ ✓ (1ku)

(b) Easy call: $25 \times 3 + 5 \times (m-3) = 75 + 5m - 15$

$= (60 + 5m)\ p$ ✓ (1ku)

$=$

(c) Greer call: $40 \times 2 + 2 \times (m-2) = 80 + 2m - 4$

$= (76 + 2m)\ p$ (1ku)

$=$

(d) Greer call < Easy call

$76 + 2m < 60 + 5m$ ✓

$-3m < -16$

$m > 16/3$ (3RE)

$m > 5\frac{1}{3}$ ✓

least no of minutes required = 6 ✓ minutes

---

(46) 2001 Paper 1 Q.3

$f(m) = m^2 - 3m$

$f(-5) = (-5)^2 - 3 \times (-5)$ ✓

$f(-5) = 25 + 15$ (2ku)

$f(-5) = 40$ ✓

(47) 2001 Paper 1 Q.4

$$2x - \frac{(3x-1)}{4} = 4 \qquad (\times 4)$$

$$8x - (3x-1) = 16 \quad \checkmark$$

$$8x - 3x + 1 = 16$$

$$5x = 15 \quad \checkmark$$

$$\underline{\underline{x = 3}} \quad \checkmark \qquad (3KU)$$

note: solve algebraically means you are not allowed to guess + check.

---

(48) 2001 Paper 1 Q.10

$$\frac{\sqrt{3}}{\sqrt{24}} = \sqrt{\frac{3}{24}}$$

$$= \sqrt{\frac{1}{8}}$$

$$= \frac{1}{\sqrt{8}} \quad \checkmark$$

$$= \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \quad \checkmark \text{ (rationalise denominator)} \qquad (3KU)$$

$$= \underline{\underline{\frac{\sqrt{2}}{4}}} \quad \checkmark$$

---

(49) 2001 Paper 1 Q.11

$$I = \frac{20}{2^c} \qquad c \geq 0$$

(a) $c = 3$, $I = \frac{20}{2^3}$

$I = 20/8$

$\underline{\underline{I = 2\frac{1}{2}}}$ ✓ (1KU)

(b) $I = 10$, $10 = \frac{20}{2^c}$ ✓

$2^c = 20/10$

$2^c = 2$

$\underline{\underline{c = 1}}$ ✓ (2KU)

(c) maximum intensity: $c = 0$, $I = 20/2^0 = 20/1 = 20$ ✓

If $c > 0$, Intensity will reduce eg $c = 1$, $I = 20/2 = 10$.

maximum intensity = $\underline{\underline{20}}$ ✓ (3RE)

(50) 2000 Paper 1 Q3

$f(x) = 2x - 5x^2$

$f(-2) = 2\times(-2) - 5\times(-2)^2$ ✓ $[(-2)^2 = 4]$

$f(-2) = -4 - 20$

$f(-2) = \underline{-24}$ ✓ (2ku)

---

(51) 2000 Paper 1 Q.4

(a) $x^2 - 16 = \underline{(x-4)(x+4)}$ ✓ (1ku)

(b) $\frac{5(2x-3)}{4x^2-9} = \frac{5(2x-3)}{(2x-3)(2x+3)}$ ✓

$= \underline{\underline{\frac{5}{(2x+3)}}}$ ✓ (2ku)

---

(52) 2000 Paper 1 Q.8

$2y < 3-(y+6)$

$2y < 3-y-6$ ✓

$3y < -3$ ✓ (3ku)

$\underline{y < -1}$ ✓

---

(53) 2000 Paper 1 Q.9

(a) $a^{\frac{1}{2}}(a + \frac{1}{a}) = a^{\frac{3}{2}} + \frac{a^{\frac{1}{2}}}{a}$ ✓

$= \underline{a^{\frac{3}{2}} + a^{-\frac{1}{2}}}$ ✓ (2ku)

(b) $\sqrt{18} - \sqrt{2} = \sqrt{9\times2} - \sqrt{2}$

$= 3\sqrt{2} - \sqrt{2}$ ✓ (2ku)

$= \underline{2\sqrt{2}}$ ✓

(54) 1999 Paper 1 Q.2

$x = -1$
$y = 3$

$20 - 4x^2y = 20 - 4.(-1)^2.(3)$ ✓
$= 20 - 12$
$= \underline{\underline{8}}$ ✓ (2ku)

---

(55) 1999 Paper 1 Q.4

$3x^2 - 5x - 2 = \underline{\underline{(3x + 1)(x - 2)}}$ ✓✓ (2ku)

---

(56) 1999 Paper 1 Q.9

$5x - 4 < 2(1 - 2x)$
$5x - 4 < 2 - 4x$ ✓
$9x < 6$ ✓
$x < 6/9$ (3ku)
$\underline{\underline{x < 2/3}}$ ✓

---

(57) 1999 Paper 1 Q.10

(a) $f(x) = 3^x$
$f(4) = 3^4$
$\underline{\underline{f(4) = 81}}$ ✓ (1ku)

(b) $f(x) = \sqrt{27}$
$3^x = \sqrt{27}$ ✓
$(3^x)^2 = 27$ (3ku)
$3^{2x} = 27$ ✓ $(3^3 = 27)$
$2x = 3$
$\underline{\underline{x = 3/2}}$ ✓

(58) 1998 Paper 1 Q.2

$a = -5$
$b = -4$

$a^2 + 2ab$
$= (-5)^2 + 2(-5)(-4)$ ✓
$= 25 + 40$
$= \underline{65}$ ✓ (2ku)

---

(59) 1998 Paper 1 Q.3

$f(x) = 3/x^2$

$f(1/3) = 3/(1/3)^2$ ✓
$= 3/1/9$
$= 3 \div 1/9$ (2ku)
$= 3 \times 9/1$
$= \underline{27}$ ✓

---

(60) 1998 Paper 1 Q.9

(a) $\sqrt{2}(\sqrt{6} - \sqrt{2}) = \sqrt{2}\sqrt{6} - \sqrt{2}\sqrt{2}$ ✓
$= \sqrt{12} - 2$
$= \sqrt{4 \times 3} - 2$
$= \underline{2\sqrt{3} - 2}$ ✓ (2ku)

(b) $\dfrac{b^{1/2} \times b^{3/2}}{b} = \dfrac{b^2}{b}$ ✓ $= \underline{b}$ ✓ (2ku)