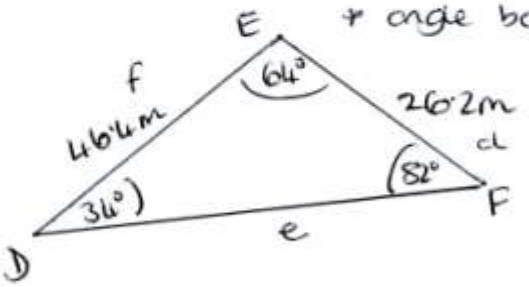
Trigonometry Past Paper Questions - Solutions

(1) 2010 Paper 2 Q8

Key words: • Complete circuits ⇒ all the way round ⇒ need to find DF.

Information • 2 sides, 2 angles ⇒ find 3rd angle ⇒ 2 sides + angle between ⇒ cosine rule.



• missing angle = 180 - 116
= 64°

Cosine Rule: $e^2 = d^2 + f^2 - 2df\cos E$

$e^2 = 26{\cdot}2^2 + 46{\cdot}4^2 - 2 \times 26{\cdot}2 \times 46{\cdot}4 \times \cos 64°$

$e^2 = 1773{\cdot}56$

$e = 42{\cdot}1$ m

total circuit = 46·4 + 26·2 + 42·1 = 114·7

Complete circuits = 1000 ÷ 114·7 = 8·7

Answer: 8 complete circuits.

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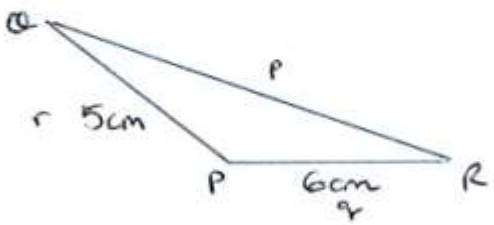
(2) 2010 Paper 2 Q.10



• Keywords: angle, obtuse (bigger than 90°) area.

• Information: Area = 12 cm², ∠QPR ⇒ ∠P.

• Area = $\frac{1}{2}qr\sin P$

$12 = \frac{1}{2} \times 6 \times 5 \times \sin P$

$12 = 15 \times \sin P$

$\sin P = 12/15$

$P = \sin^{-1}(12/15)$

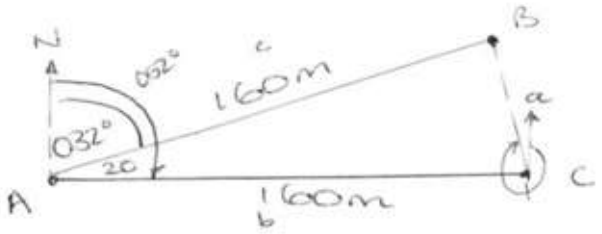
$P = 53{\cdot}1°$ or $180 - 53{\cdot}1 = 126{\cdot}9°$

S ✓	A ✓
T	C

Answer: ∠QPR = 126·9°

(3) 2009 Paper 2 Q.9

- Keywords - Bearings - measured clockwise from NORTH.
  - Length - lower case letter.



(a) $\angle BAC = 20°$ $(52° - 32°)$ ✓ (1KU)

(b) length of BC = a. => 2 sides + angle between => cosine rule

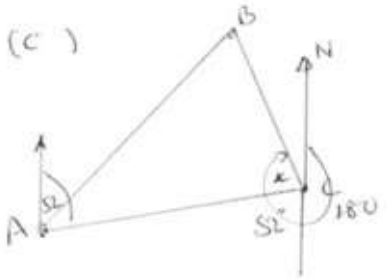
$a^2 = b^2 + c^2 - 2bc\cos A$ ✓

$a^2 = 160^2 + 160^2 - 2 \times 160 \times 160 \times \cos 20°$

$a^2 = 3087.74$ ✓

$a = 55.6\,m$ ✓ (3RE)

(c)



$x = \frac{1}{2}(180 - 20)$

$x = 80°$ (△ ABC is isosceles △)

✓ (so 2 angles the same)

- bearing of B from C = $180 + 52 + 80$ (2RE)

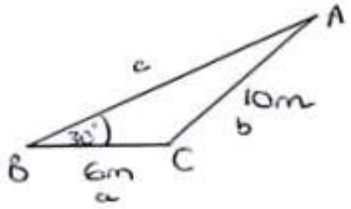
$= \underline{312°}$ ✓

(4) 2009 Paper 1 Q.11

Triangle ⇒ sine or cosine rule

⇒ Do not have 2 sides + angle between so sine rule

⇒ looking for sin ? so not cosine rule!



Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{6}{\sin A} = \frac{10}{\sin 30} = \frac{c}{\sin C}$ ✓

$\frac{6}{\sin A} = \frac{10}{\sin 30}$

$6 \sin 30 = 10 \sin A$

$\sin A = \frac{6 \sin 30}{10}$ ✓ $\quad \sin 30 = 0.5$

$\sin A = \frac{6 \times 0.5}{10}$

$\sin A = 3/10$ ✓ (3RE)

$\sin A = 0.3$

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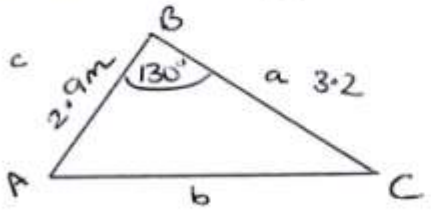
(5) 2008 Paper 2 Q.7

Keywords: length, angle

Information / Diagram: Triangle ⇒ Trig.

length of telegraph pole = 6.2m.



$BC = 6.2 - 2.9$

$BC = 3.3$ ✓

- 2 sides + angle between ⇒ cosine rule:

$b^2 = a^2 + c^2 - 2ac\cos B$ ✓

$b^2 = 3.2^2 + 2.9^2 - 2 \times 3.2 \times 2.9 \times \cos 130°$ ✓

$b^2 = 30.58$

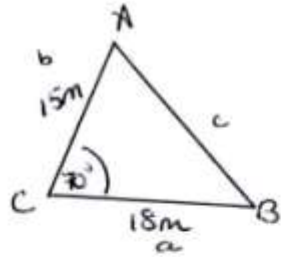
$b = 5.5$

length of AC = 5.5 m ✓ (4RE)

(6) 2008 Paper 2 Q.8

keywords: Area ⇒ $A = \frac{1}{2}ab\sin C$.

(a)



Area $= \frac{1}{2} \times 15 \times 18 \times \sin 70°$ ✓✓

Area $= 126.9\ m^2$ ✓ (3ku)

(b) largest area ⇒ max value of $\sin C = 1$.

So max area $= \frac{1}{2} \times 15 \times 18 \times 1$

max area $= 135\ m^2$ (1KE)

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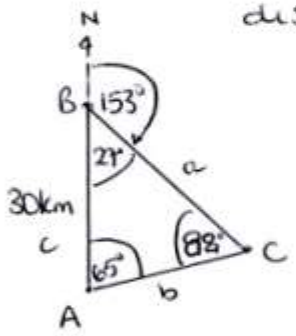
(7) 2007 Paper 2 Q.6

keywords: North, Bearings ⇒ measured clockwise from North.
distance ⇒ length.



- From Burton ⇒ start at B.
- find any missing angles.

$\angle ABC = 180 - 153 = 27$ (straight line)

$\angle BCA = 180 - 92 = 88$ (angles in △) ✓

- Haven't got 2 sides + angle between ⇒ sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

trying to find BC

$$\frac{a}{\sin 65°} = \frac{b}{\sin 27°} = \frac{30}{\sin 88}$$ ✓

$$\frac{a}{\sin 65} = \frac{30}{\sin 88}$$ (4RE)

$$a = \frac{30 \times \sin 65}{\sin 88}$$ ✓

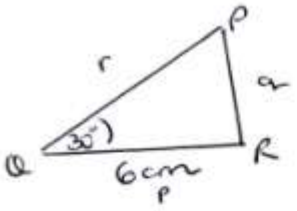
$$a = 27.2$$

Distance between Brunton + carlton = 27.2 km ✓

(8) 2008 Paper 2 Q.8

Keywords: triangle, length ⇒ Trig?

Area ⇒ ½ ab sin C .



• length PQ → r.

• Area = ½ pr sin Q

$15 = \frac{1}{2} \times 6 \times r \times \sin 30°$ ✓

$15 = 3 \times \sin 30 \times r$

$15 = 1.5 \times r$ ✓

$r = 15 \div 1.5$

$r = 10$ (3RE)

• length of PQ = 10cm ✓

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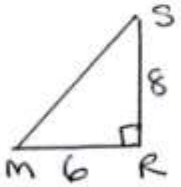
(9) 2006 Paper 2 Q.5

Keywords: • ST is vertical ⇒ 90° ⇒ Trig (SOH CAH TOA)

• rectangular garden ⇒ 90° ⇒ Trig / Pythagoras.

• midpoint ⇒ ½ way along.

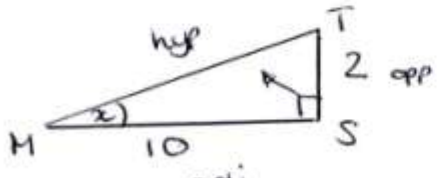


• By Pythagoras:

$MS^2 = 6^2 + 8^2$

$MS^2 = 100$

$MS = 10$. ✓



SOH CAH TOA

tan (angle) = opp/adj ✓

$\tan x = 2/10$

$x = \tan^{-1}(2 \div 10)$ ✓

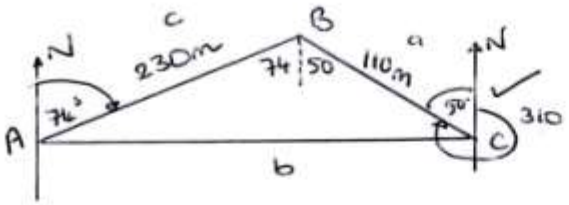
$x = 11.3°$

$\angle TMS = 11.3°$ ✓ (4ku)

(10) 2006 Paper 2 Q.6

Keywords: Bearings => measured clockwise from North.
Significant figures => rounding!

(a)



*from C, bearing of B = 310°
*360° in full turn.
*North lines are parallel -> Z + F shapes.

$\angle ABC = 74 + 50 = 124°$ ✓ (2RE)

(b) calculate AC => 2 sides + angle between => cosine rule.

$b^2 = a^2 + c^2 - 2ac\cos B$ ✓

$b^2 = 110^2 + 230^2 - 2 \times 110 \times 230 \times \cos 124°$ ✓

$b^2 = 93295.16$

$b = 305.4$ ✓ (4RE)

Distance from A to C = 305m ✓ (3 sig figs)

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(11) 2005 Paper 2 Q.3

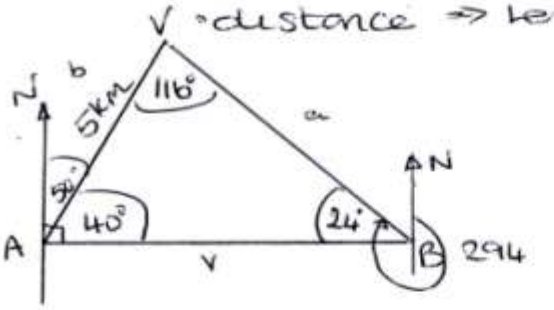
Keywords: Area of triangle => $\frac{1}{2}ab\sin C$ ✓

Area = $\frac{1}{2} \times 21 \times 19 \times \sin 110°$ ✓✓

Area = $187.5\ cm^2$ ✓ (4 KU)

(12) 2005 Paper 2 Q7

Keywords: • Bearing ⇒ measured clockwise from North.
• Due East ⇒ 090°.
• distance ⇒ length.



• 294° from B (North at B)
• 294 − 270 = 24° in △. ✓
• ∠ AVB = 180 − 64 = 116°
• length AB = v. ✓

• Don't have 2 sides + angle between ⇒ Sine rule

$$\frac{a}{\sin A} = \frac{v}{\sin V} = \frac{b}{\sin B} \quad ✓$$

$$\frac{a}{\sin 40} = \frac{v}{\sin 116} \quad \frac{5}{\sin 24°}$$

$$\frac{v}{\sin 116} = \frac{5}{\sin 24} \quad ✓$$

(5ku)

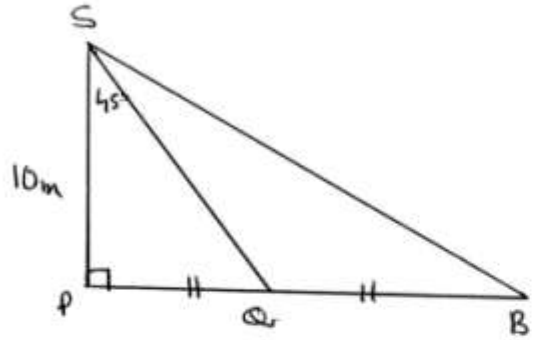
$$v = \frac{5 \times \sin 116}{\sin 24}$$

$$v = 11.05 \quad ✓$$

Distance between the 2 hostels is 11.05 km

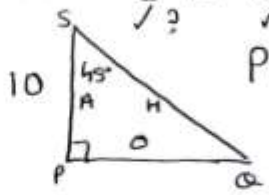
(14) 2004 Paper 2 Q.6

- Keywords: Angle, Mid-point, How many more degrees, directly above (ie right angles)
- Information: Vertical height = 10m
  $\angle PSQ = 45°$



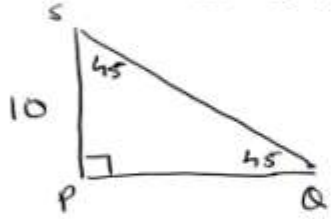
$\triangle SPQ$ is a R.A.T.

Use $S^{O}H$ $C^{A}H$ $T^{O}A$
✓? ✓✓ ✓?✓



$PQ = 10 \times \tan 45°$
$= 10m$

OR SPQ is an isosceles Triangle

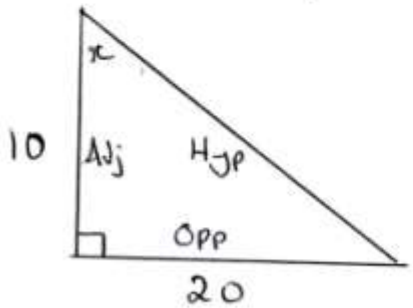


$|SP| = |PQ|$
$\Rightarrow |PQ| = 10m$

$|PQ| = |QB| \Rightarrow |PB| = 20m$

$\triangle SPB$ is a R.A.T.

Use $S^{O}H$ $C^{A}H$ $T^{O}A$ to find $\angle PSB$
?✓ ?✓ ?✓✓



$\tan x = \frac{O}{A} = \frac{20}{10} = 2$

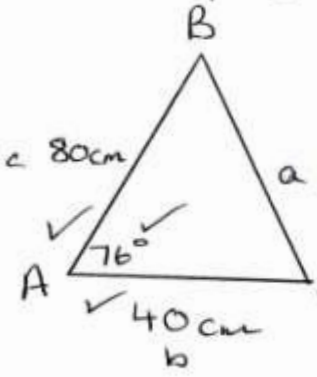
$x = \tan^{-1}(2)$
$= 63.4°$

Angle $\angle BSQ = 63.4° - 45°$
$= 18.4°$

$\angle PSB$ is $18.4°$ more than $\angle PSQ$

(15) 2004 Paper 2 Q.7

- Keywords: Angle, length of rod, Sig. figures
- Information: length of Side of Trapdoor = 80cm
  Angle made by Trapdoor and opening = 76°
  length at which rod is placed from trapdoor = 40 cm.



Given 2 sides and an included angle

=> Use Cosine Rule to find length of side

$a^2 = b^2 + c^2 - 2bc \cos A$

$= 40^2 + 80^2 - 2 \times 40 \times 80 \times \cos 76°$

$= 6451.69\ldots$ (leave in calculator!)

$a = \sqrt{6451.69\ldots}$

$= 80.32$ cm

$= 80$ cm (2 Sig. fig.)

(16) 2004 Paper 2 Q 9

· Keywords: Prism, Regular Pentagon, Vertex
Volume

· Information given: Depth (height) of Prism = 8 cm

Regular pentagon => Angle at centre for each Δ is $\frac{360^\circ}{5} = 72^\circ$

Triangle is Isosceles => OA = OB = 10 cm.

Volume = Cross Sectional Area × Height

Cross Sectional Area

Area of 1 Triangle $= \frac{1}{2} a \times b \times \sin C$

$= \frac{1}{2} \times 10 \times 10 \times \sin 72^\circ$

$= 47.55 \text{ cm}^2$

C.S.A. $= 47.55 \times 5$

$= 237.76 \text{ cm}^2$

Volume $= 237.76 \times 8$

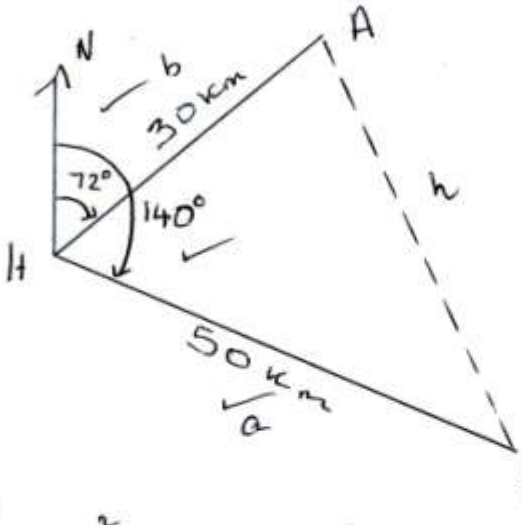
$= \underline{1902.11 \text{ cm}^3}$ or 1902 cm^3

(Ignore rounding)

(17) 2003 Paper 2 Q3

Key words: Bearing (3 figure), Far apart

Information given: Bearing from H to A is 072°
Bearing from H to B is 140°
Distance |AH| = 30km
Distance |BH| = 50km



$\angle AHB = 140° - 72°$
$= 68°$

Given 2 sides and an included angle
⇒ Use Cosine Rule to find Distance |AB|

$h^2 = a^2 + b^2 - 2ab\cos H$

$= 50^2 + 30^2 - 2 \times 50 \times 30 \times \cos 68°$

$= 2276.18022$ (Leave in calculator!)

$h = \sqrt{2276.18022} = \underline{47.7\text{km}}$

The yachts are 47.7km apart from each other when they stopped

(18) 2003 Paper 2 Q6

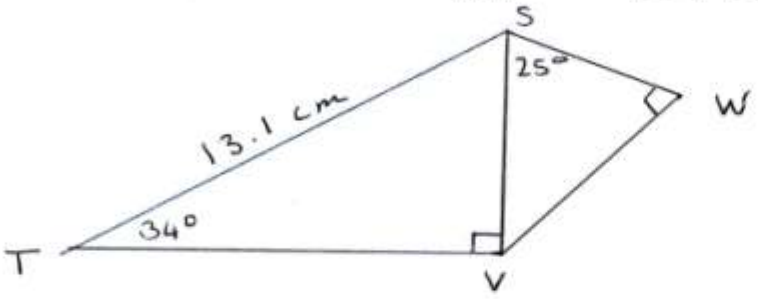
Keywords: Angle, 90° (right-angle), length

Information given: $\angle STV = 34°$

$\angle VSW = 25°$

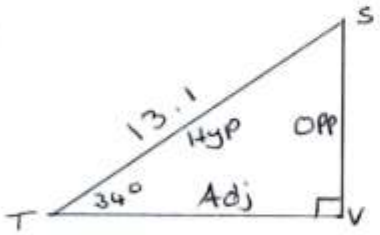
$\angle SVT = \angle SWV = 90°$

$|ST| = 13.1$ cm.



To find $|SW|$ we need to find $|SV|$ first using Triangle STV

STV is a R.A.T. $\Rightarrow$ Use SOH CAH TOA



$S^{O}H \quad C^{A}H \quad T^{O}A$ Use Sine Ratio

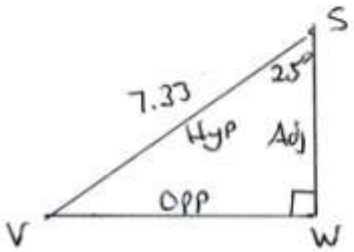
✓? ✓ ✓ ✓ ✓?

$Opp = Hyp \times \sin x$

$|SV| = 13.1 \times \sin 34°$

$= 7.33$ cm (2 d.p.)

SWV is a R.A.T. $\Rightarrow$ Use SOH CAH TOA



$S^{O}H \quad C^{A}H \quad T^{O}A$ Use Cos Ratio

✓ ✓ ✓? ✓ ✓ ?

$Adj = Hyp \times \cos x$

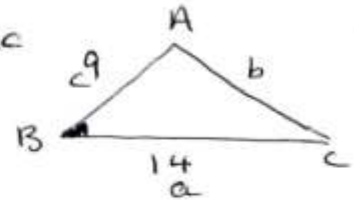
$= 7.33 \times \cos 25°$

$|SW| = \underline{6.64 \text{ km}}$ (2 d.p.)

(19) 2003 Paper 2 Q7

Key Words: Area of Triangle, Acute Angle

Information Given: $|AB| = 9$ cm
$|BC| = 14$ cm
Area $ABC = 38$ cm^2



Use Area of a Triangle Formula

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$\text{Area} = \frac{1}{2} \times a \times c \times \sin B$$

$$\frac{1}{2} \times 9 \times 14 \times \sin B = 38$$

$$\sin B = \frac{38}{63}$$

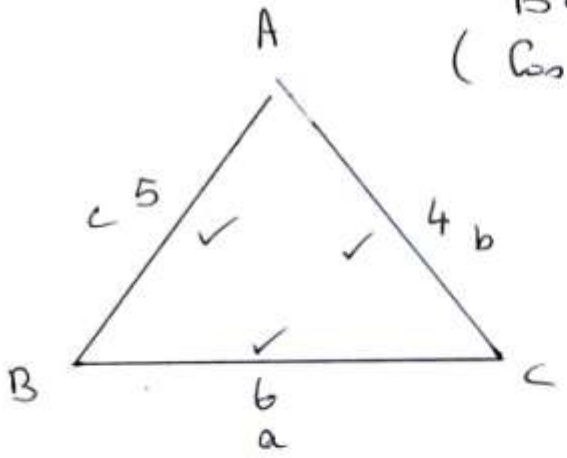
$$B = \sin^{-1}\left(\frac{38}{63}\right)$$

$$= 37.1^\circ \quad (1 \text{ d.p.})$$

(20) 2002 Paper 1 Q.7 (Non-Calc!)

Information Given:

AB = 4 units
AC = 5 units
BC = 6 units
($\cos A = \frac{1}{8}$) (Given to prove)



Find $\angle BAC$

Given Three sides

=> Use Cosine Rule to find angle

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{4^2 + 5^2 - 6^2}{2 \times 4 \times 5} = \frac{16 + 25 - 36}{40}$$

$$= \frac{5}{40} = \frac{1}{8} \quad \text{(Simplified)}$$

$$\Rightarrow \quad \underline{\cos A = \frac{1}{8}}$$

(21) 2002 Paper 2 Q4

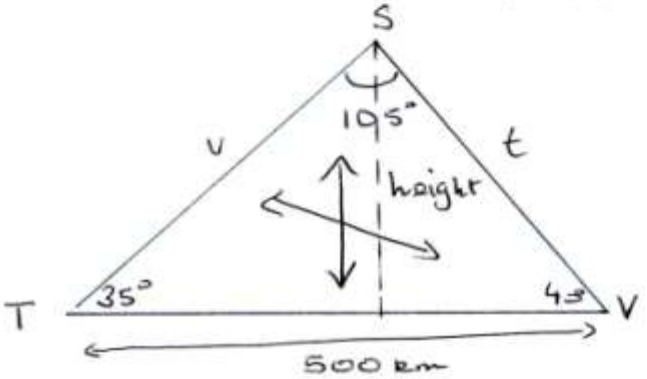
Keywords: Angle, height above the ground (Vertical distance)

Information Given: $|TV| = 500$ km

$\angle TVS = 40°$

$\angle VTS = 35°$

To find a vertical height of a non-R.A.T. you must know or calculate either side $|ST|$ or $|SV|$



Choosing to find $|ST|$

$\angle TSV = 180° - (35° + 40°)$

$= 105°$

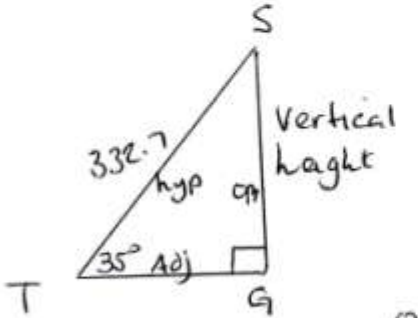
We have a pair of opposition

$\Rightarrow$ Use Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{V}{\sin 40°} = \frac{500}{\sin 105°}$$

$$V = \frac{500}{\sin 105°} \times \sin 40°$$

$$V = 332.7 \text{ km}$$



ΔSTG is a R.A.T.

$\Rightarrow$ Use SOH CAH TOA

S O H C A H T O A Use Sin Ratio

✓ ? ✓ ✓ ✓ ✓ ?

$$\text{OPP} = \text{Hyp} \times \sin \angle$$

$$\text{height} = 332.7 \times \sin 35°$$

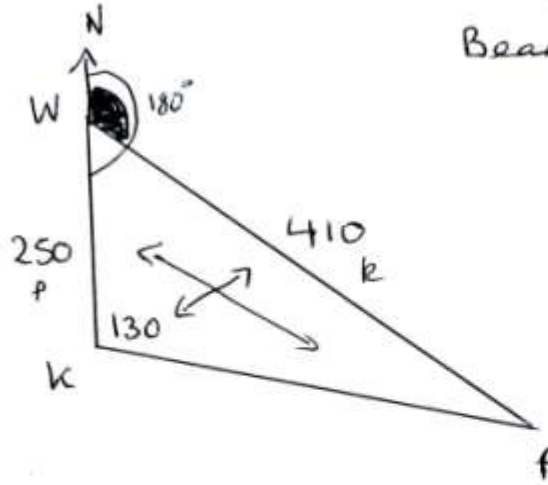
$$= \underline{190.85 \text{ km}} \quad (2 \text{ d.p.})$$

(22) 2001 Paper 2 Q6

- Keywords: Bearing (3-figure), Due South
- Information given: $|WK| = 250$ km
  $|WP| = 410$ km

Bearing from K to P is 130°

Bearing from W to K is 180° (due south)



Find Bearing of P from W (shaded angle)

Need to find $\angle KWP$

but not enough information

$\Rightarrow$ Find $\angle KPW$ using Sine Rule and then find $\angle KWP$

$$\frac{P}{\sin P} = \frac{KR}{\sin K} \qquad \frac{250^\circ}{\sin P} = \frac{410}{\sin 130^\circ}$$

$$410 \times \sin P = 250 \times \sin 130^\circ$$

$$\sin P = \frac{250 \times \sin 130^\circ}{410}$$

$$= 0.467\ldots \text{ (leave in calculator!)}$$

$$P = \sin^{-1}(0.467\ldots)$$

$$= 27.8^\circ$$

$$= 28^\circ \text{ (nearest degree)}$$

$\Rightarrow \angle KWP = 180^\circ - (130^\circ + 28^\circ)$

$= 22^\circ$

$\Rightarrow$ Bearing of P from W $= 180^\circ - 22^\circ$

$= 158^\circ$

(23) 2001 Paper 2 Q. 8.

· Keywords : Prism , Cross-Section , Volume

. Information given : height (depth) of prism = 5cm
: 2 Sides and included Angle of uniform cross-section face

Calculate Volume

Volume = C.S.A. × height

C.S.A. Area = $\frac{1}{2}$ a b Sin C

= $\frac{1}{2}$ × 8 × 14 × Sin 100°

= 55.15 cm² (2 d.p.)

Volume = 55.15 × 5

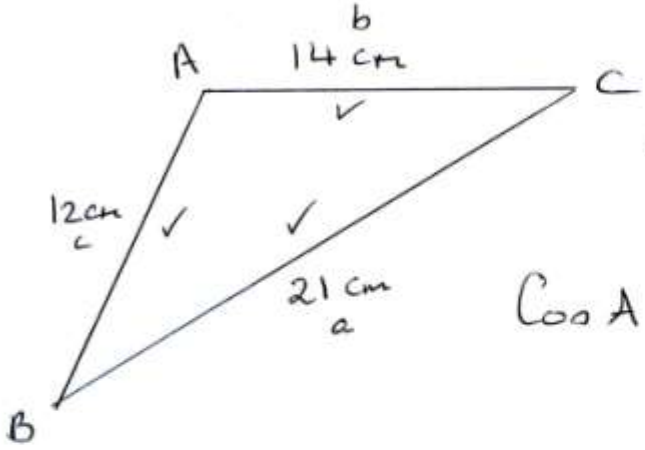
= 275.75 cm³ (2. d.p.)

(24) 2001 Paper 2 Q10

. Keywords: Obtuse Angle, height (Vertical) from ground

. Information given: 3 Sides of a Triangle

(a) Calculate Obtuse angle



Three Sides given
⇒ Use Cosine Rule to find Angle A

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

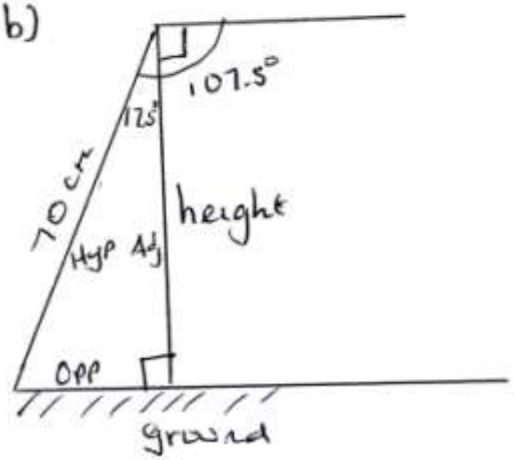
$$= \frac{14^2 + 12^2 - 21^2}{2 \times 14 \times 12} = -0.30\ldots$$

(leave in calculator)

$$A = \cos^{-1}(-0.3005\ldots)$$

$$= \underline{107.5^\circ} \quad (1\text{ d.p.}) \quad \underline{\text{obtuse}}$$

(b)



Looking for vertical height of Table from the ground

R.A.T. ⇒ Use SOH CAH TOA

S O H ✓ ✓ C A H ✓ ? T O A ✓ ? Use Cos Ratio

$$\text{Adj} = \text{Hyp} \times \cos x^\circ$$

$$\text{Height} = 70 \times \cos 17.5^\circ$$

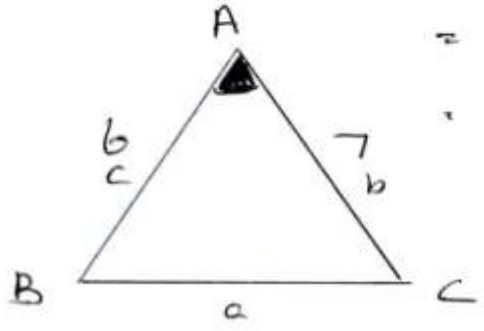
$$= \underline{66.8\text{ cm}} \quad (1\text{ d.p.})$$

(25) 2008 Paper 2 Q.6

. Keywords : Area, Possible sizes of angles

. Information Given : Area of a Triangle
length of 2 sides

Given 2 sides of a Triangle, find the included angle.



$$\text{Area} = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2} \times b \times c \times \sin A$$

$$= \frac{1}{2} \times 6 \times 7 \times \sin A = 14$$

$$21 \sin A = 14$$

$$\sin A = \frac{14}{21}$$

$$A = \sin^{-1}\left(\frac{14}{21}\right)$$

$$= 41.8^\circ$$

or $A = 180^\circ - 41.8^\circ$

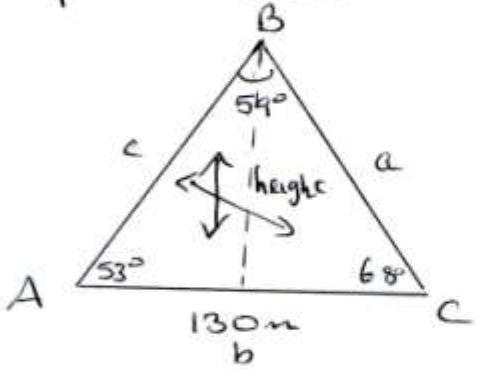
$$= 138.2^\circ$$

Ans $\angle BAC$ is either 41.8° or 138.2°

(26) 2000 Paper 2 Q.7

- Keywords: Angle of elevation, Distance, height above the ground (Vertical height)
- Information Given: Two internal Angles
  $\angle BAC = 53°$
  $\angle BCA = 68°$
  length of 1 side = 130 m

To find a vertical height of a non R.A.T we must find either |BA| or |BC| before we calculate the height



$\angle ABC = 180° - (53° + 68°)$
$= 59°$

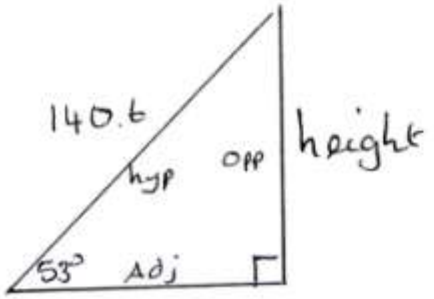
Choose to find |AB|

We have a pair of opposites
⇒ Use Sine Rule

$$\frac{c}{\sin C} = \frac{b}{\sin B} \qquad \frac{c}{\sin 68°} = \frac{130}{\sin 59°}$$

$$c = \frac{130}{\sin 59°} \times \sin 68°$$

$$= 140.6 \text{ m}$$



RAT ⇒ Use SOH CAH TOA

S O H ✓? ✓ C A H ✓ ✓ T O A ✓? Use Sine Rule

Opp = Hyp × Sin ∠

Height = 140.6 × Sin 53°

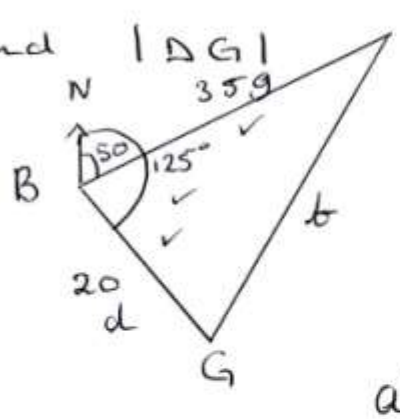
= 112.3 m (1 d.p.)

(27) 1999 Paper 2 & 3

· Keywords : bearing (3-figure), distance

· Information Given: Bearing of D from B is 050°
Bearing of G from B is 125°
|DB| = 35 km
|GB| = 20 km

Find |DG|



∠DBG = 125° − 50° = 75°

We have 2 sides and an included angle

⇒ Use Cosine Rule

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = g^2 + d^2 - 2gd \cos B$

$= 35^2 + 20^2 - 2 \times 35 \times 20 \times \cos 75°$

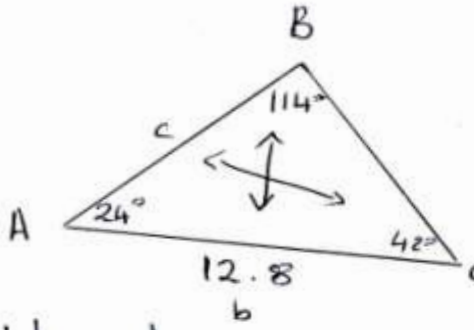
$= 1262.65...$ (leave in calculator)

$b = \sqrt{1262.6...} =$

$= \underline{35.5 \text{ km}}$ (1 d.p.)

28 1999 Paper 2 Q.9

- Keywords: Rectangle, Triangle, inclined, horizontal, longer sloping edge
- Information given: 2 internal Angles of Triangle
  1 side of rectangle (= length of longest side of triangle)



$\angle ABC = 180° - (24° + 42°)$
$= 114°$

longer sloping side = |AB|

We have a pair of opposites ⇒ Sine Rule

$$\frac{b}{\sin B} = \frac{c}{\sin C} \qquad \frac{12.8}{\sin 114°} = \frac{c}{\sin 42°}$$

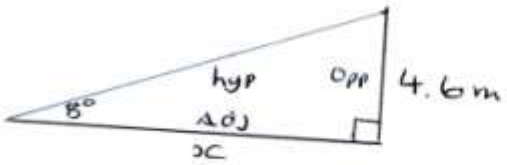
$$c = \frac{12.8}{\sin 114°} \times \sin 42°$$

$$\underline{c = 9.4 \text{ m}} \qquad (1 \text{ d.p.})$$

(29) 1999 Paper 2 Q10

- Keywords: Square, Angle of elevation
- Information given: height of statue = 4.6 m
  Angle of elevation = 8°
  Right Angle Triangle
  Square

a)



The Tan Ratio

R.A.T. => Use SOH CAH TOA

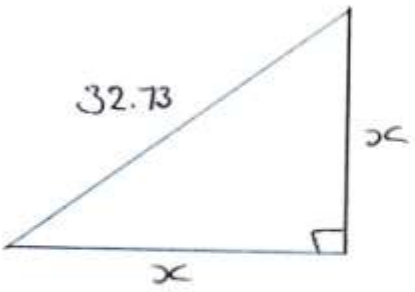
S O H ✓✓ C A H ✓? T O A ✓✓?

$$Adj. = \frac{Opp}{\tan x^\circ}$$

$$xc = \frac{4.6}{\tan 8^\circ} = 32.73\ m$$

b) Part (a) found the Diagonal of the Square

=> Use Pythagoras to find length of side



$$2xc^2 = 32.73^2$$

$$xc = \sqrt{\frac{32.73^2}{2}}$$

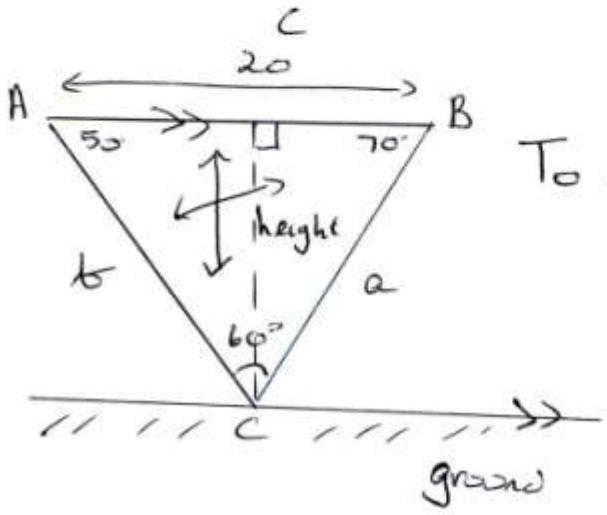
$$= 23.14$$

$$\approx \underline{23\ m}.$$

(30) 1998 Paper 2 Q.6

- Keywords: Parallel to ground, Angle of depression, height from ground (vertical height), Horizontal ground

- Information given: $\angle CAB = 50°$
  $\angle CBA = 70°$
  $|AB| = 20\,m$



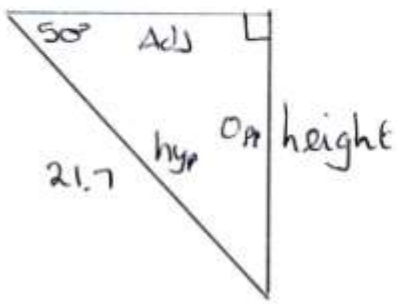
To find the vertical height of C to AB, we need to find either $|AC|$ or $|BC|$

$\angle ACB = 180° - (50° + 70°)$
$= 60°$

Choose to find $|AC|$
We have a pair of opposites

$$\frac{b}{\sin B} = \frac{c}{\sin C} \qquad \frac{b}{\sin 70°} = \frac{20}{\sin 60°}$$

$$b = \frac{20}{\sin 60°} \times \sin 70°$$
$$= 21.7\,m.$$



R.A.T. $\Rightarrow$ SOH CAH TOA
✓ ? ✓ ✓ ✓ ✓ ?

Use Sine Ratio

$Opp = Hyp \times \sin x°$

$height = 21.7 \times \sin 50°$
$= \underline{16.6\,m}$ (1 d.p.)