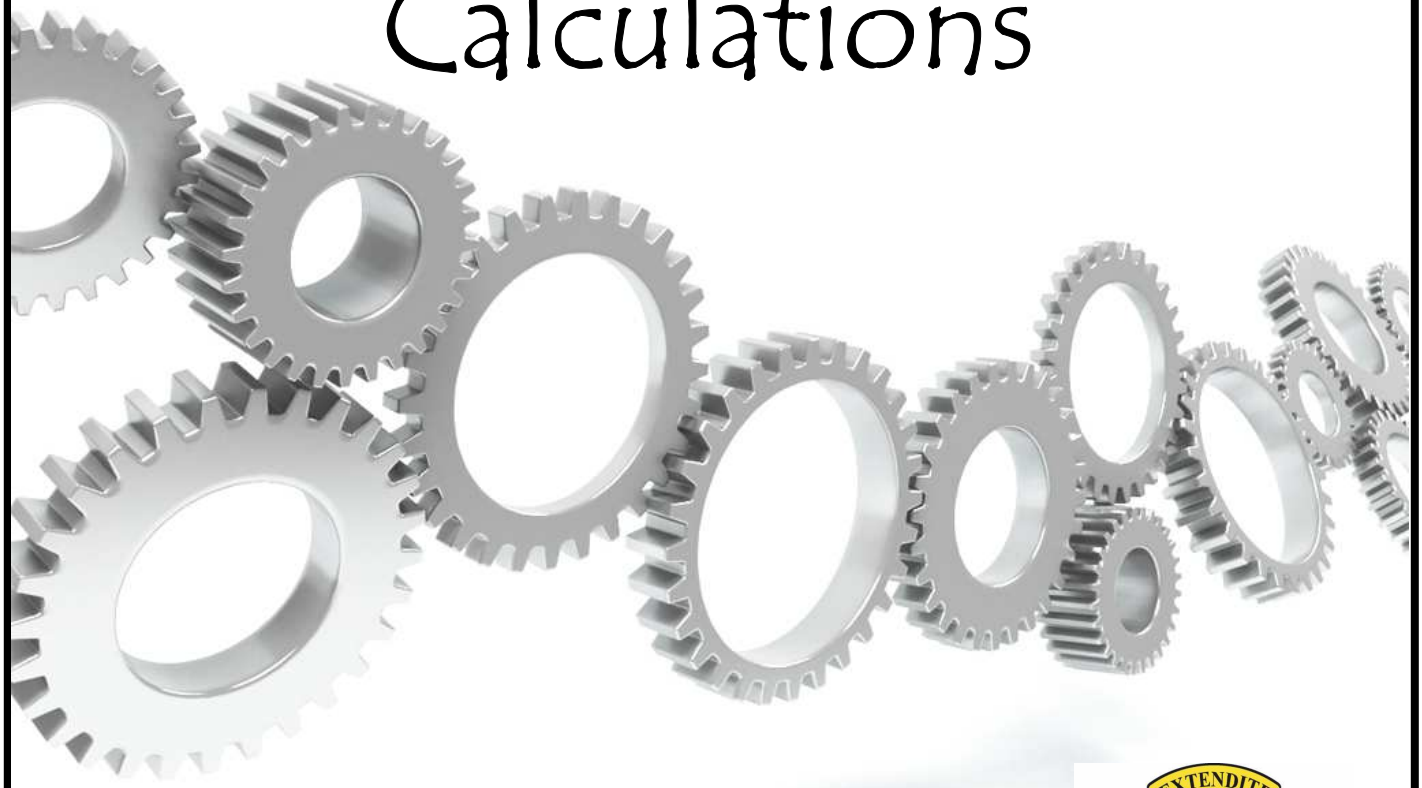
Engineering Science

Mechanical & Structural Calculations



Name _____

Class _____

Teacher _____

Ellon Academy
Technical Faculty

Learning Intentions

- To build up my knowledge of mechanisms and structures
- Investigate a range of mechanical and structural systems
- To be able to produce diagrams of a range of structures and mechanisms
- How to develop mechanical solutions to solve problems
- To understand moments and reactions

Success Criteria

- I can produce diagrams to show the change in motion
- I can work out the efficiency of a system
- I understand the terms; equilibrium, reaction forces, friction, torque and efficiency.
- I can produce diagrams of a range of structures
- I can use the moments calculation to solve problems.

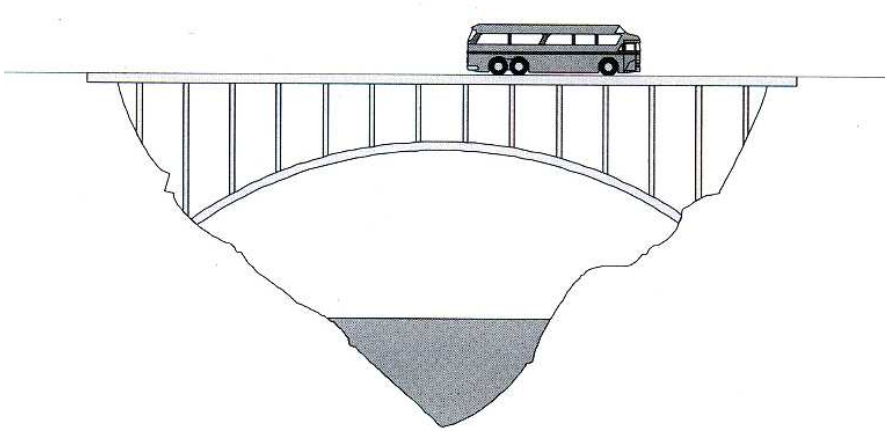
To access video clips that will help on this course go to www.youtube.com/MacBeathsTech

Structures

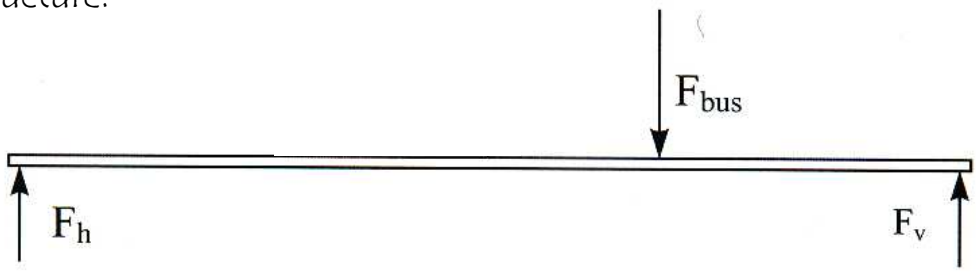
Free body diagrams

It is important to isolate different parts of a structure or body form from its adjacent surroundings. In a line diagram this can be done by drawing a free-body diagram, which represents all or part of the structure, showing the forces that affect it.

Example



If all the visual components acting on the structure or body were removed and replaced with their force value, a simplified diagram would allow a better understanding of how the forces are affecting the structure.



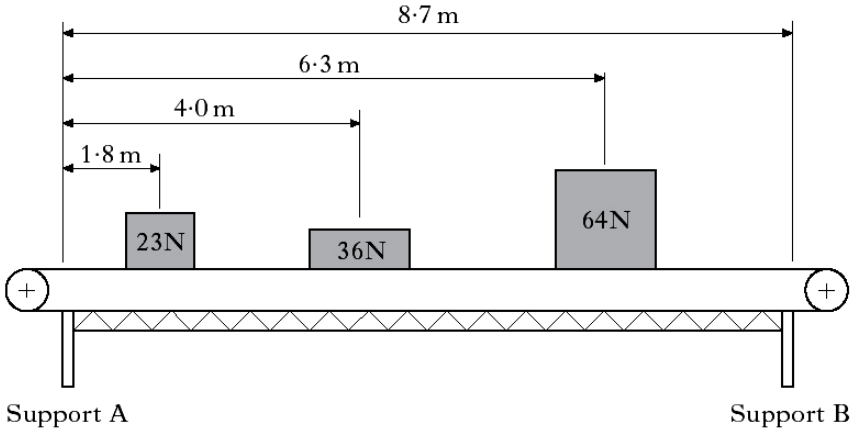
This diagram is a simplified free-body diagram of the upper picture. The forces representing the bus are shown through where its centre is, and the weight of the bridge is represented through the centre of the bridge. The forces F_h and F_v represent the forces that the supports have on the structure; therefore they also have to be shown.

If distances are known they MUST also be shown

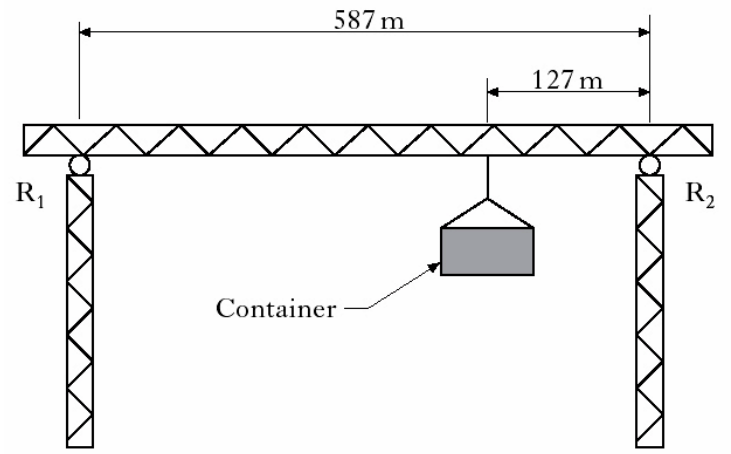
Task 1

Draw a free body diagram of the picture shown below, indicating clearly all forces and reactions.

a)

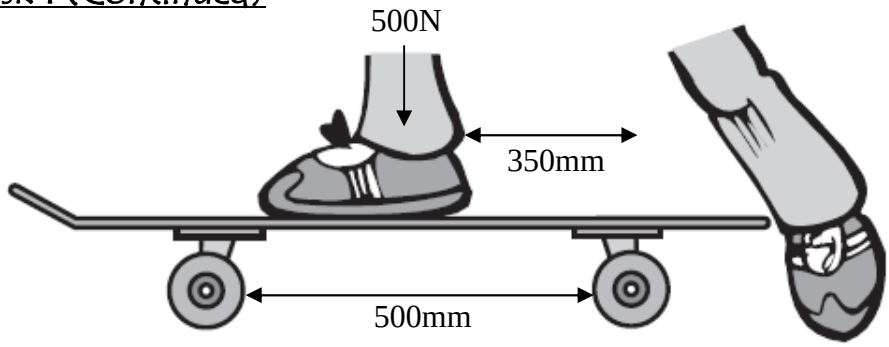


b)

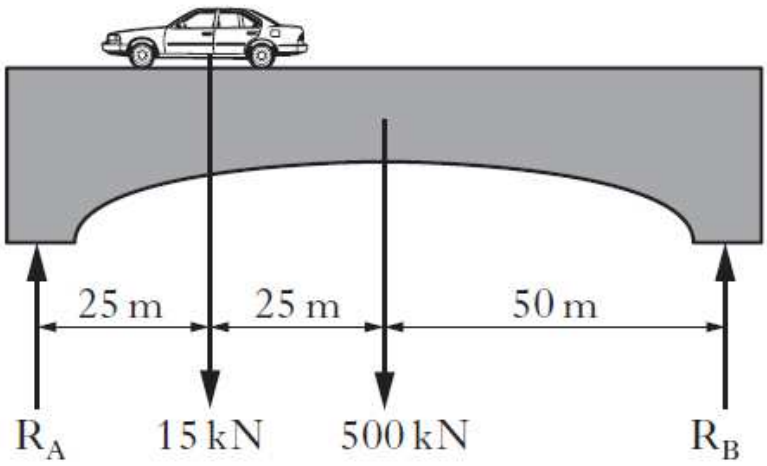


Task 1 (Continued)

c)

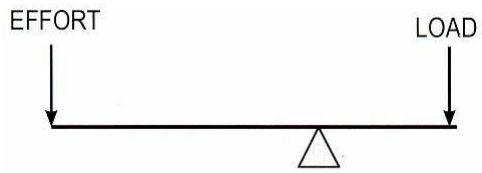


d)



Moments

The turning effect that occurs to due force is called a moment. A moment is measured in Newton Metres.



When any system is in perfect balance and is in a steady state such as this diagram, it is said to be in equilibrium.

The effort wants to turn the lever anticlockwise and the load wants to turn the lever clockwise. If the lever/beam is in equilibrium, but these moments **must** be equal.

Moment = force x distance
(M) (f) (d)

So…..

↺ M = M ↻

f x d = f x d

The Principle of Moments

The 'principle of moments' state that the sum of the moments must equal 0

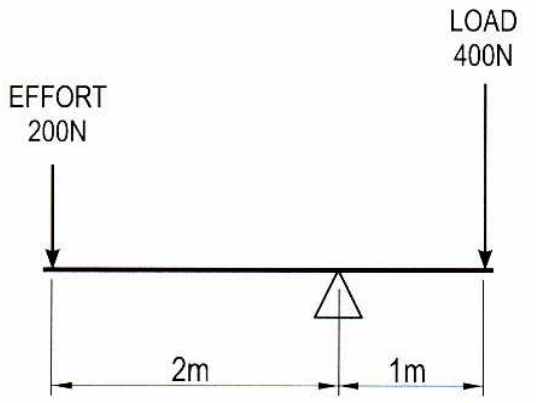
$$\Sigma M = 0$$

Or the sum of the clockwise moments must equal the sum of the anti-clockwise movements

$$\Sigma CWM = \Sigma ACWM$$
$$(f_1 \times d_1 = F_2 \times d_2)$$

Example

Use the principal of moments to prove that the lever is in equilibrium



Clockwise moment = $f_1 \times d_1$

= 200N x 2m = 400 Nm

Anti-clockwise moment = $f_2 \times d_2$

= 400 N x 1 m = 400 Nm

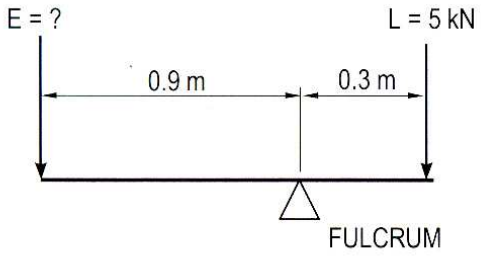
$$\Sigma CWM = \Sigma ACWM$$
$$400Nm = 400Nm$$

Therefore the lever is in a state of equilibrium

Task 2

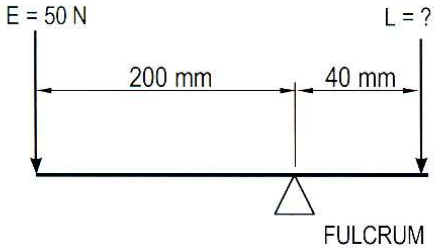
Use the principal of moments to find the missing force or distance.

a)



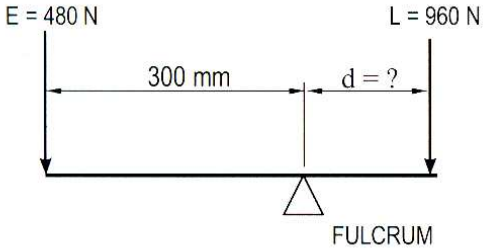
$\Sigma CWM = \Sigma ACWM$

b)



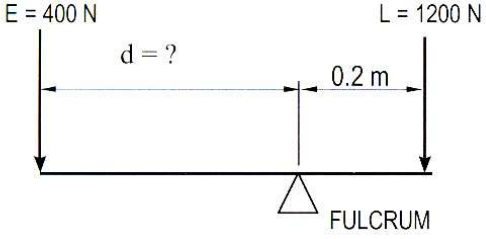
$\Sigma CWM = \Sigma ACWM$

c)



$\Sigma CWM = \Sigma ACWM$

d)

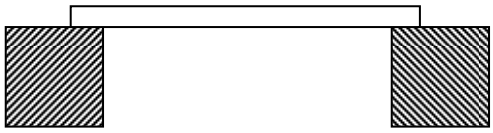


$\Sigma CWM = \Sigma ACWM$

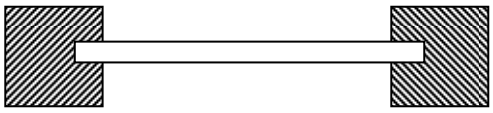
Beam Reactions

Beams have to be supported differently from lever applications and this determines beam support reactions. Beams, therefore, are supported in a number of ways.

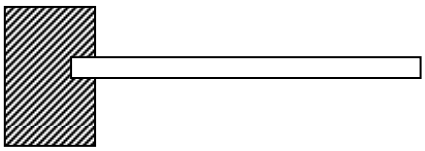
a) Simply supported at both ends



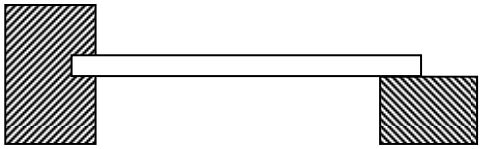
b) Encastre, built in a both ends



c) Cantilever, built in at one end only



d) Built in at one end and simply supported at the other



At the point of support, the downward forces acting on the beam are resisted by the forces acting upwards. These upward forces are known as beam reactions, or simply the 'reactions'.

Beams

Apart from levers, structural beams and beam related objects are also affected by forces and turning moments. For a horizontal structure to be stable (in equilibrium) when being affected by forces in a vertical plane, the following conditions must be satisfied.

1. The sum of the downward forces acting upwards must equal the sum of the forces acting downward.

   Σ upward forces = Σ downward forces

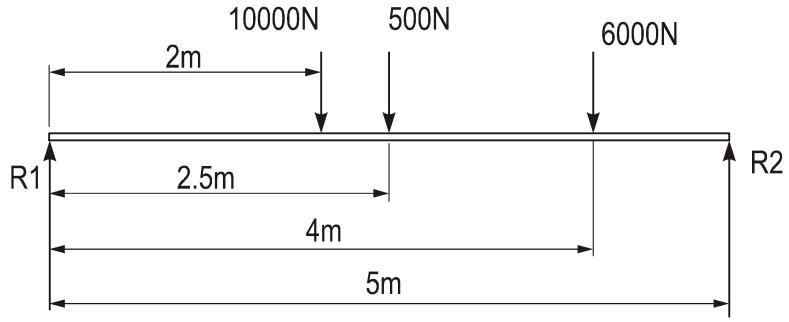
2. The sum of the clockwise moments about any point must equal the sum of the anticlockwise moments about the same point. That is….

   $\Sigma CWM = \Sigma ACWM$

   (Principle of moments)

Example

Determine the reactions R_1 and R_2 for the simply supported beam



Take moments about R_1

$$\Sigma \text{ clockwise moments} = \Sigma \text{ anticlockwise moments}$$

$$(10{,}000\ \text{N} \times 2\ \text{m}) + (500\ \text{N} \times 2.5\ \text{m}) + (6000\ \text{N} \times 4\ \text{m}) = R_2 \times 5\ \text{m}$$

$$R_2 = \frac{20{,}000\ \text{Nm} + 1250\ \text{Nm} + 24{,}000\ \text{Nm}}{5\ \text{m}}$$

$$R_2 = 9050\ \text{N}$$

Also Σ upwards forces = Σ downwards forces

$$R_1 + 9050\ \text{N} = 10{,}000\ \text{N} + 500\ \text{N} + 6000\ \text{N}$$

$$R_1 = 16{,}500\ \text{N} - 9050\ \text{N}$$

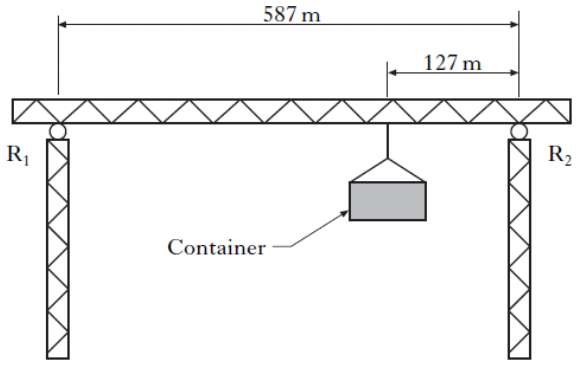
$$R_1 = 7450\ \text{N}$$

Therefore the reactions for the beam supports are

$$R_1 = 7450\ \text{N and } R_2 = 9050\ \text{N}$$

Task 3

The structure shown below is used to transport a 5000 kg container



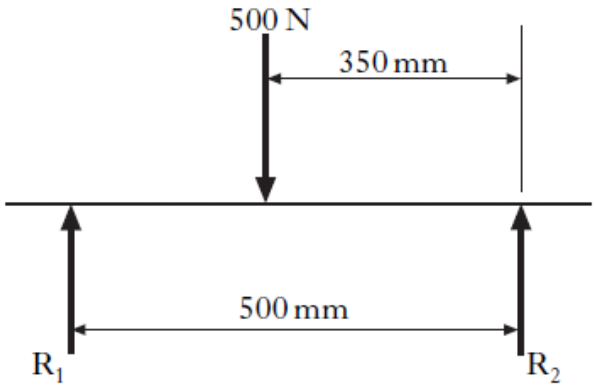
(*a*) Calculate the weight of the 5000 kg container.

(*b*) Draw the Free Body Diagram of the structure, showing all forces and distances

(*c*) (i) Calculate, by taking moments about R2, the size of the reaction R1.

(ii) Determine the size of the reaction R2.

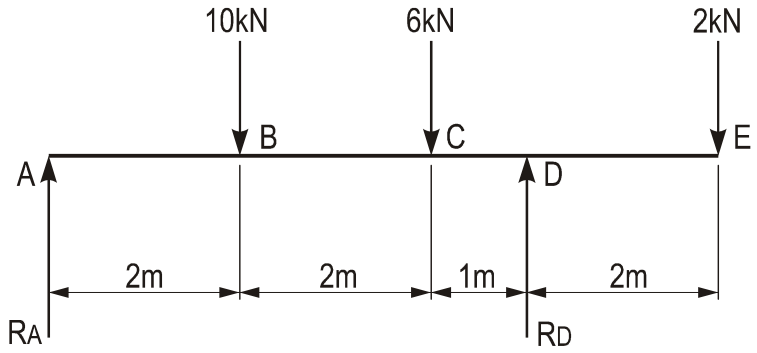
Task 4



(i) Calculate, by taking moments about R2, the size of the reaction R1.

(ii) Determine the size of the reaction R2.

Task 5



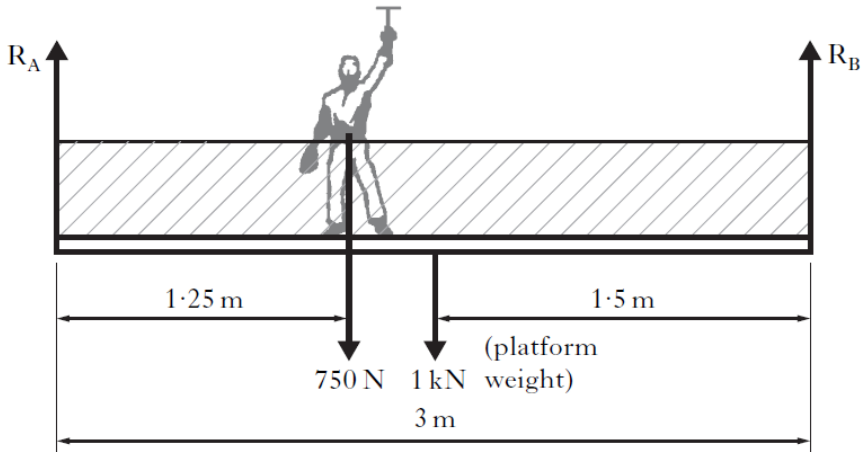
(i) Calculate, by taking moments about R2, the size of the reaction R1.

(ii) Determine the size of the reaction R2.

http://www.youtube.com/watch?v=bgBt7scLIo4

Task 6

Below shows a window cleaner's platform.



(*a*) Draw a free body diagram of the system shown in Figure Q10(*a*).

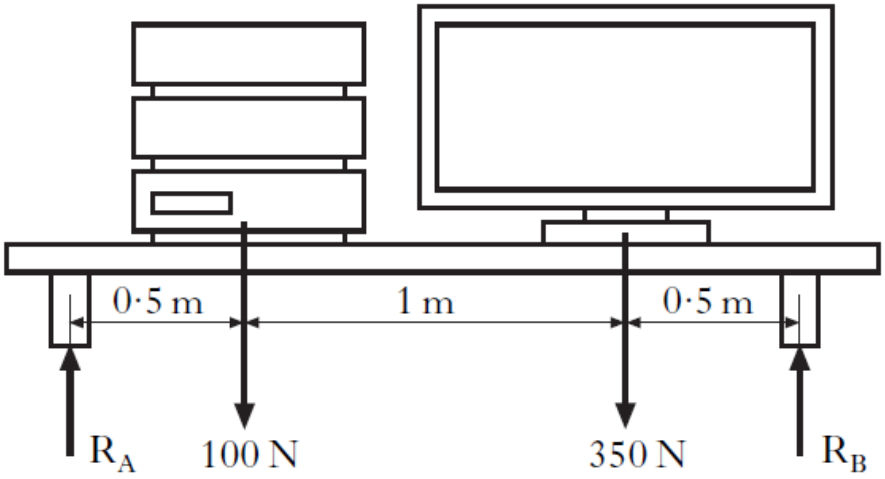
(*b*) Calculate:

(i) the reaction force RA (take moments about RB);

(ii) the reaction force RB.

Task 7

A unit used to support the DVD player, an entertainment system and television is shown



a) Draw a free body diagram for the unit shown.

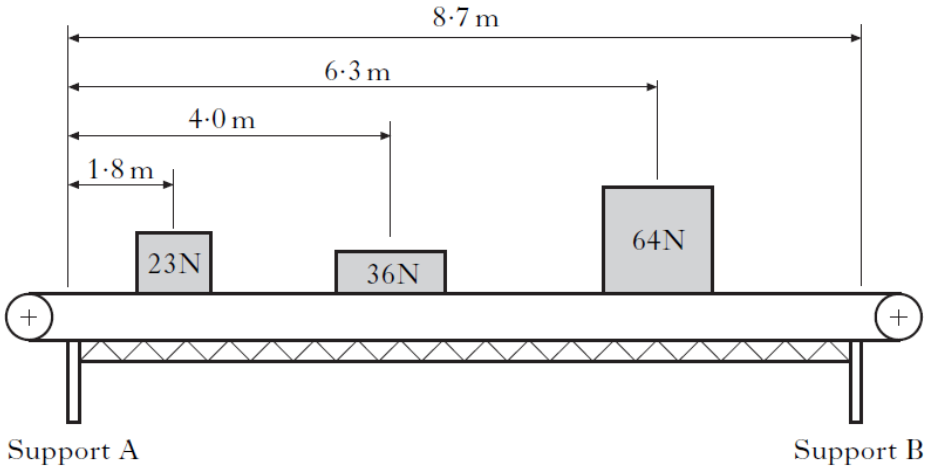
b) Calculate:

(i) the reaction force RA (take moments about RB);

(ii) the reaction force RB.

Task 8

A conveyor belt is used to move packages in a warehouse. When the conveyor belt stopped, packages were in the position shown.



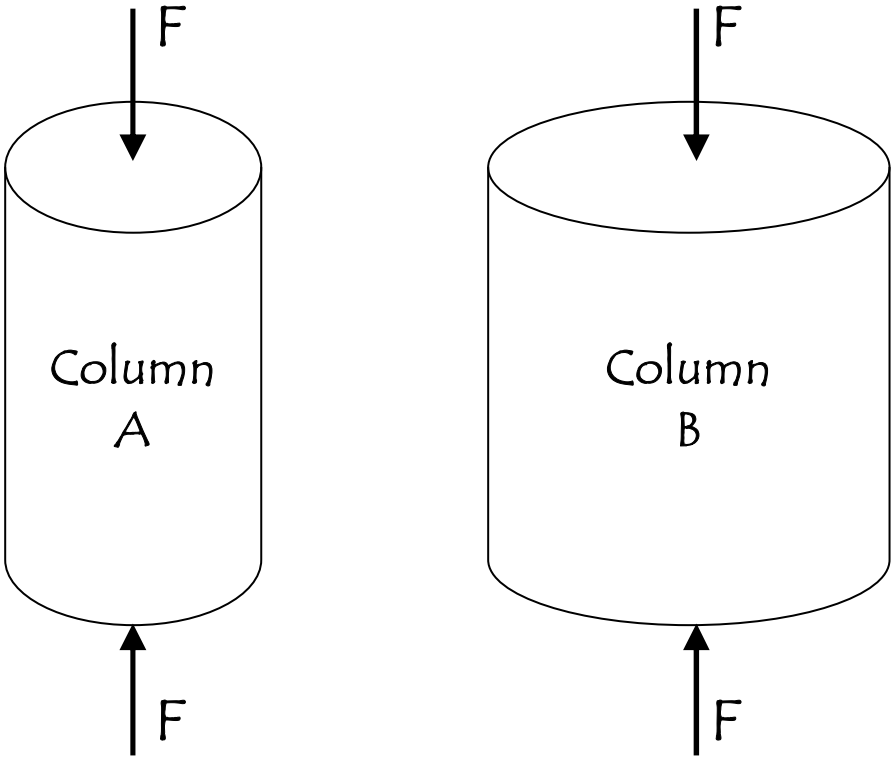
(*a*) Draw the free body diagram for the system.

(*b*) (i) Calculate, by taking moments about support A, the reaction at support B.

(ii) Determine the reaction at support A.

Stress on Members

When building structures an Engineer has to take stress into consideration. When a load is applied to a structure, there is a force put onto its member. This is known as the **stress**. This is important for Engineers to know as if too large a load is placed on a structure it will bend, buckle or break.



The stress can vary by the force put on it, but also by the size of the structure. For example if the same load is applied to column A and column B, then Column A will suffer more from stress. The reason why is because Column A has a far smaller cross sectional area.

Stress is calculated using the formula:

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} \qquad \sigma = \frac{F}{A}$$

Force is measured in Newtons, Area is measured in mm^2, therefore **Stress is measured in N/mm^2**

Task 9

(a) A square bar of 20mm x 20mm cross section is subjected to a tensile load of 500N. Calculate the stress in the bar.

(b) A wire 4mm in diameter is subjected to a force of 300N. Finds the stress of the wire.

(c) The stress in a steel wire supporting a load of 8kN should not exceed 200n/$mm^{2.}$ Calculate the minimum diameter of wire required to support the load.

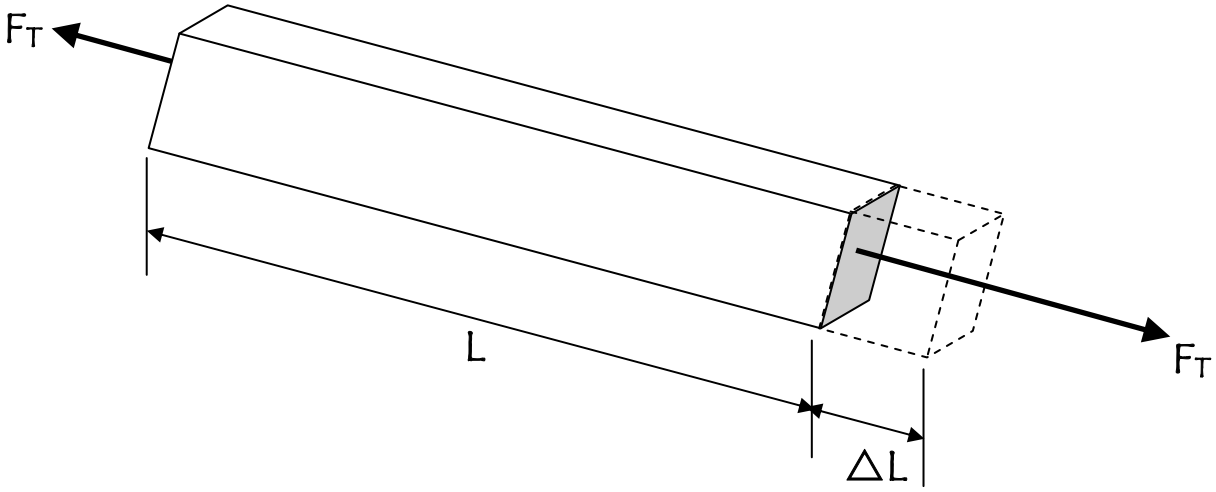
Task 10

The figure shows an oil platform used in the North Sea. The platform is designed so that each of the three legs always carries an equal share of the 120 MN load. The legs are 15m in diameter and are made of steel reinforced concrete.

Work out the stress that is applied on each leg

Strain on Members

While some members of structured get compressed (put under stain), others get pulled apart. This is called a tensile load, and it is put under strain.



Strain is calculated using the formula:

$$\text{Strain} = \frac{\text{Change in Length}}{\text{Original Length}} \qquad \varepsilon = \frac{\triangle L}{L}$$

The length in both parts of the equation cancel each other out. Because of this strain is dimensionless and DOES NOT have a unit.

Task 11

(a) A steel wire 5m long is used to support a load. When the load is applied the wire stretches by 2.5mm. Calculate the strain of the wire.

(b) A wire 10m long stretches 5mm when a force is applied at one end. Calculate the strain.

(c) The allowable strain on a bar is 0.0075 and its length is 2.5m. Find the change in length.

(d) During testing, a steel rod stretches 0.6mm. If the resulting strain was 0.00012, what is the original length of the rod?

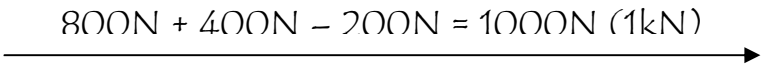
Vectors

Force is a vector quantity and has both magnitude and direction. This means force is often represented by a line, or '**vector quantity.**'
The direction of the force may be indicated by an arrow-headed line, with the length of the line drawn in scale to represent the size of force. This line is called a '**vector.**'

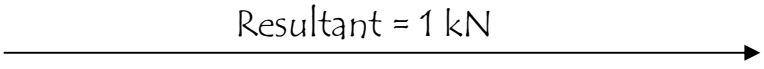
In this diagram the cyclist is pedalling with a force of 800 N and is being assisted by a tail wind of 400 N, but the friction from the road surface measures 200 N.

The overall effect will be 800 N + 400 N – 200 N = 1000 N
(or 1 kN)

A suitable scale can then be selected (possibly 10mm to represent 100 N) and by using this scale each force is drawn in turn, one following the other. This is called a '**vector diagram**'.



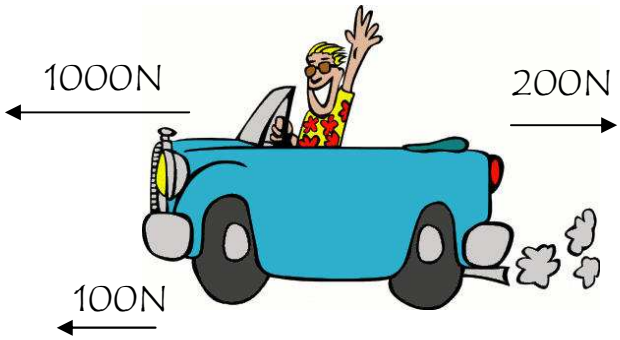
When the 3 forces are added together, they can be replaced by a single force that has the same effect. This is known as the '**resultant**'.



Task 12

Draw the vector diagrams for each picture, using a suitable scale.

a)



b)

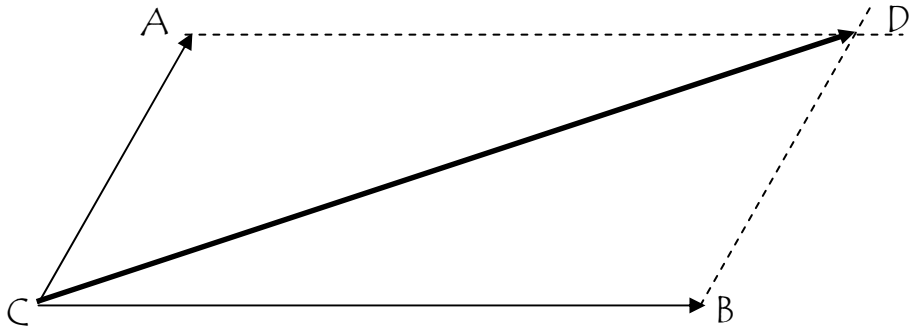
c)

Vectors at angles

Vectors can also be used to find the resultant of 2 forces that are inclined at an angle to each other.
In this example the resultant of the 2 forces can be found by drawing 2 vectors. First you have to choose a suitable scale and draw the 2 vectors CA and CB. the bigger the scale you use the more accurate the vectors.

Scale : 10mm = 10 N
28mm = 28 N = CA
60 mm = 60 N = CB.

From A, draw a line parallel to CB, and from B, draw a line parallel to CA. call the point where the two lines intersect point D. now draw a line from C to D.



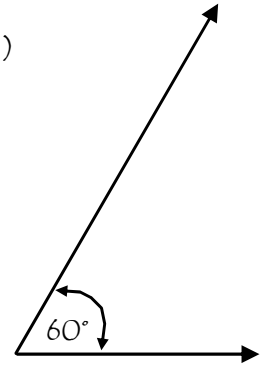
Line CD is the resultant of the two forces CA and CB.

By this measurement, the resultant has a magnitude of 77N

Task 13

Work out the resultant for each of these diagrams. (10mm = 10N)

a)



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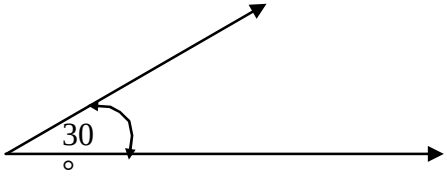

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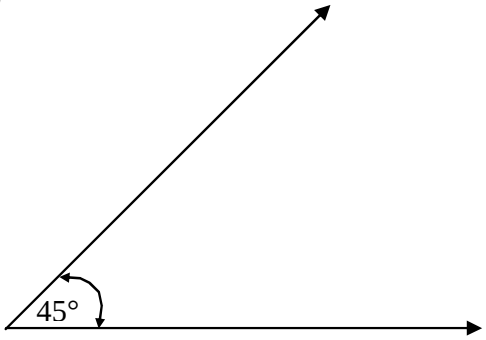
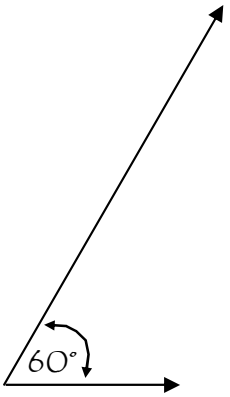


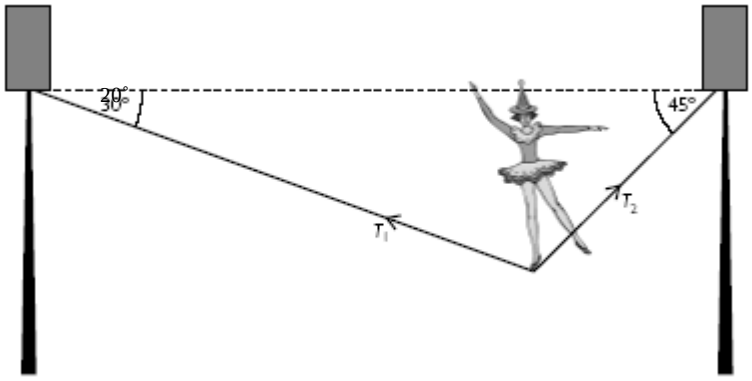
d)



e)

Task 14

A circus high wire act is shown.



The high wire performer has a weight of 750N and is at rest.

Determine graphically the tension (T_1 and T_2) on the wire.

Movement-multiplier ratio in gears

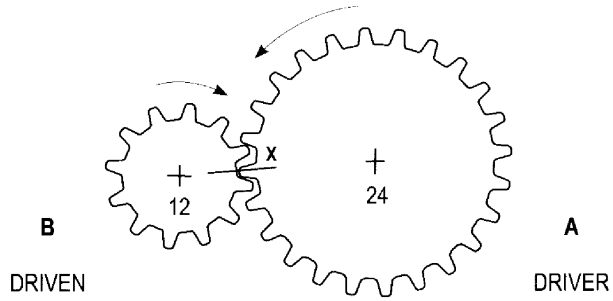
The ratio of change in speeds between the gears is called the movement-multiplier ratio. The ratio of a gear system is found by dividing the number of teeth on the driven gear by the number of teeth on the driver gear. This can be used to calculate the output speed of a gear system.

$$\text{Movement ratio} = \frac{\text{Number of teeth on driven gear}}{\text{Number of teeth on driver gear}}$$

http://www.bbc.co.uk/schools/gcsebitesize/design/systemscontrol/mechanismsrev5.shtml

<u>Example</u>

To work out this gear system the gear multiplier ratio is:



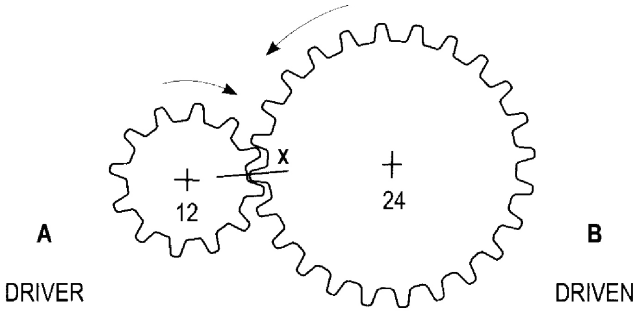
Gear Ratio = $\frac{12}{24} = \frac{1}{2}$ <u>OR</u> 1:2

The Driver will be going at <u>TWICE</u> the speed of the driven

This means that if gear A was rotating at 100 rpm clockwise then gear B would rotate at 200rpm anti-clockwise.

Gears can also be used to **decrease** the speed of a mechanism.

Gear ratio = $\frac{24}{12} = \frac{2}{1}$ OR 2:1

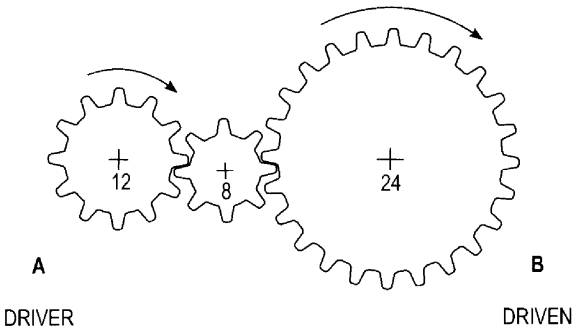


The Driven will be going at HALF the speed of the driver

If gear A is still rotating at 100 rpm in a clockwise direction then gear B will now rotate at 50 rpm in an anticlockwise direction.

Idler Gears

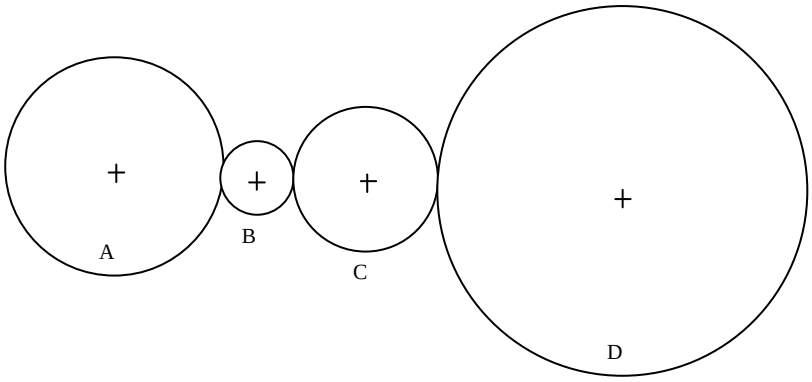
To get the driven gear to rotate in the same direction as the driver a third gear is inserted in the system. The idler gear has no effect on the speed of the driven gear wheel.



The multiplier ratio for the simple gear train in this diagram is still 2:1. If Gear A still rotates at 100rpm clockwise then the output of Gear B will rotate at 50 rpm clockwise.

Task 15

A = 50 teeth
B = 10 teeth
C = 25 teeth
D = 100 teeth



For this simple gear train, find the following:

a) The gear that rotates in the same direction as A

b) The multiplier ratio of A to B, A to C and A to D

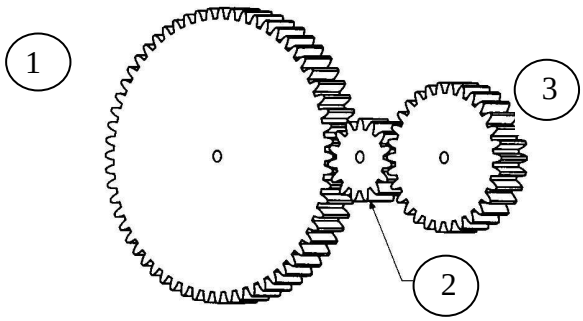
A to B A to C a to D

c) The speed of B, C and D if A rotates at 500rpm.

B= C= D=

Task 16

A simple gear train used in a food processor is shown.



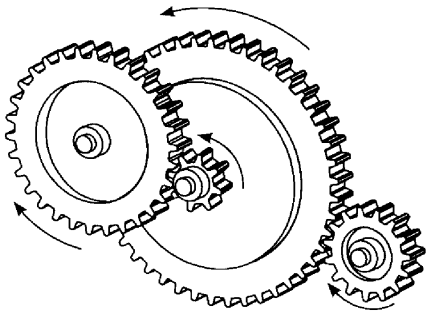
(a) Draw, using the correct symbols, this simple gear train.

Gear 2 in the simple gear train is an idler.

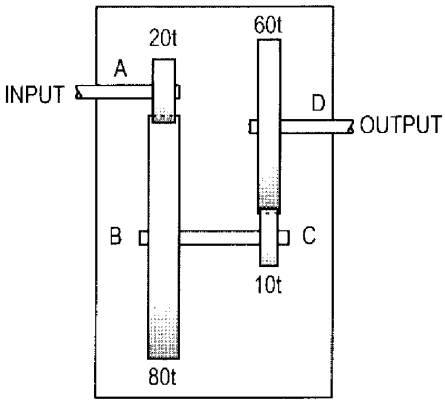
(b) Explain the function of an idler gear.

Compound Gears

If gears are required to produce a very large change in speed, for example 100:1 then problems can arise with the size of gear wheels if a simple gear train is used. The problem can be overcome by mounting pairs of gears on the same shaft as shown.



This diagram shows how the shafts are connected between the 'pairs' of gears. Gears B and C are connected and rotate at the same speed. To calculate the multiplier ratio for the gear train it is necessary to calculate the ratio for each pair of meshing gears.



Example

The multiplier ratio for this gear system would be…

The multiplier ratio for the first pair of meshing teeth is:

Ratio of AB = $\frac{\text{driven}}{\text{Driver}} = \frac{80}{20} = \frac{4}{1} = \frac{4:1}{}$

The multiplier ratio for the second pair of meshing teeth is:

Ratio of CD = $\frac{\text{driven}}{\text{Driver}} = \frac{60}{10} = \frac{6}{1} = \frac{6:1}{}$

The total multiplier ratio is calculated by multiplying both ratios:

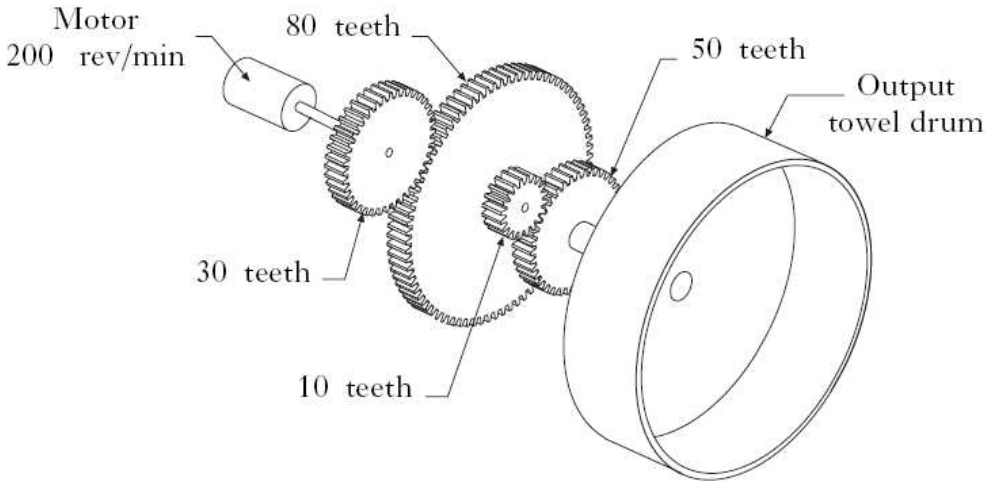
Total ratio = $\frac{4}{1} \times \frac{6}{1} = \frac{24}{1} = \frac{24:1}{}$

For an input speed of 100 rpm, the output speed would be **4.17 rpm**

(100 ÷ 24)

Task 17

A motorised towel dispenser uses a compound gear train as shown.



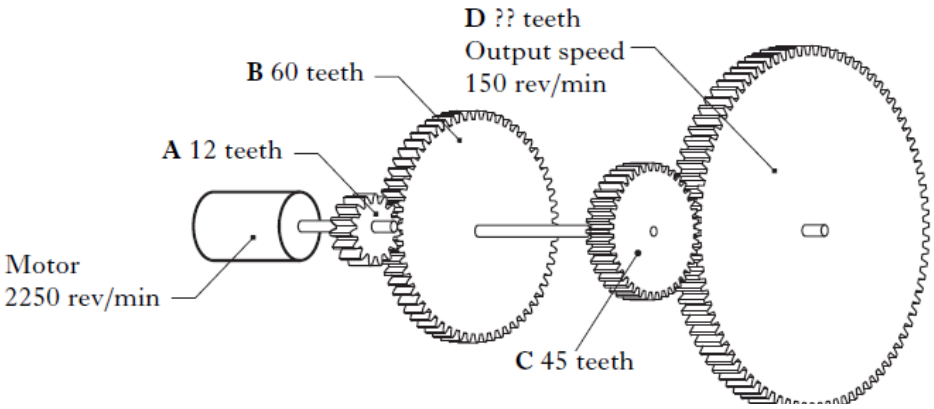
a) Calculate the multiplier ratio for the gear system

b) Describe the difference between the input and output speed.

c) Explain the difference between the input and output direction produced by this drive mechanism.

Task 18

A gear system for operating a theatre curtain is shown



(*a*) State the name of the type of gear system.

(*b*) Calculate:

(i) the velocity ratio of the system when gear D rotates at 150 rev/min.

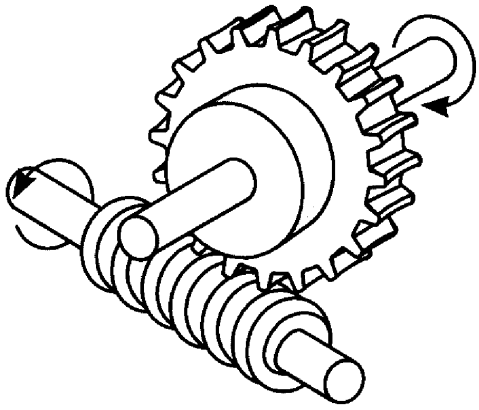
(ii) the ratio of gear B to gear A;

(iii) the number of teeth on gear D to produce the correct output speed.

(*c*) State **one** advantage of a gear system made from plastic rather than steel gears.

Worm and Wheel

Another way of making larger speed reductions is to use a worm and wheel. The worm, which looks rather like a screw thread, is fixed to the driver shaft (sometimes directly onto the motor shaft). It meshes with a worm wheel, which is fixed to the driven shaft. The driven shaft runs at 90° to the driver shaft.



You should think of the worm wheel as a gear with only 1 tooth. This allows a huge reduction in speed which takes up very little space.

Example

The multiplier ratio between the gears in the diagram would be:

$$\text{Multiplier ratio} = \frac{\text{driven}}{\text{Driver}} = \frac{20}{1} = 20:1$$

This would mean that for a motor rotating at 100 rpm, the output driven gear would rotate at only 5 rpm (100 ÷ 20)

Task 19

A motor with a single worm wheel output rotates at 500 rpm. You are given the following sizes of worm gears which to select.

a) 10 teeth
b) 25 teeth
c) 50 teeth

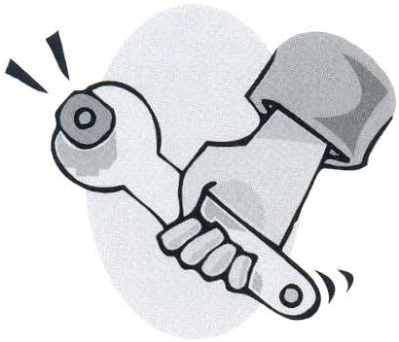
Explain which gear should be connected to the motor to give the **slowest** output speed and why? Use maths to prove this.

Torque and Drive Systems

Torque is the amount of turning produced by a force. The turning or twisting action or exerted by a force, or a number of forces, will cause, or tend to cause rotary motion.

Example 1

How much torque is required to tighten the nut if the force required is 45N and the radius of the tool is 200 mm.



torque = force x radius

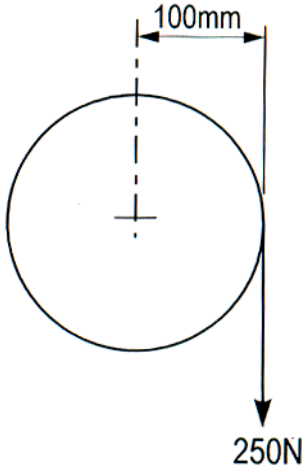
= 45 N x 200 mm

Example 2

A belt drives a pulley with a diameter of 200 mm. if the effective belt tension tending to turn the pulley is 250 N, find the work done per revolution.

When a force of *P* newtons acts at the rim of a pulley of *r* metres radius, then the work done per revolution is $P \times \pi D$

(P newtons x circumference)



Therefore, the work done per revolution

= torque x circumference

= 250 x (3.14 x 0.2)

= 250 x 0.628

= 157 J

Power Transmitted by a Belt Drive

Example

The effective pull on a belt drive is 420 N when driving a 500 mm diameter pulley. The speed of the rotation is 220 revolutions per minute. Find the power.

When a force, *P* newtons, acts at the rim of a pulley, of *r* metres radius, revolving at *n* revolutions per second, the power or work done per second is given by:

$$P \times 2\pi r n.$$

Power = force (*P* newtons) x circumference ($2\pi r$) x revolutions/s (*n*)

Thus power, or work done/s = torque (Pr) x angle rotated through /s ($2\pi n$)

$$= 2\pi nT$$

The effective driving torque = force x radius

$$= (T_1 - T_2) \frac{\text{diameter (d)}}{2}$$

T_1 is the tension on the tight side
T_2 is the tension on the slack side

Remember!

Change numbers to its proper units!

500mm = 0.5m

Therefore power transmitted = π dn $(T_1 - T_2)$

Power = π dn $(T_1 - T_2)$

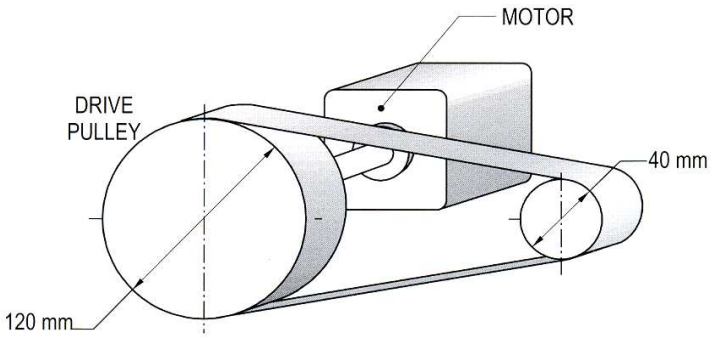
$$= 3.14 \times 0.5 \times \frac{220}{60} \times 420$$

= 2418 watts

= 2.42 kW

Multiplier Ratio for Belt Drives

Pulley systems can be used to transmit rotary motion over a large distance. The input rotary motion is often from a fixed speed and fixed torque electric motor. Torque is a turning force produced by mechanisms and is measured in Newton metres. Changing the ratio of the diameters of the pulleys can very both the speed of the output and the torque at the output.



This motor is connected to a pulley of 120mm ø. This is the driver pulley. The driven pulley is 40 mm ø. To work out the multiplier ratio for a pulley system, use this equation:

$$\text{Multiplier ratio} = \frac{\text{diameter of driven pulley}}{\text{diameter of driver pulley}}$$

For the system in the diagram above, the multiplier ratio is

$\frac{40}{120} = \frac{1}{3}$ or $1 : 3$

Multiplier ratio can also be found by using this equation

$$\text{Multiplier Ratio} = \frac{\text{input speed}}{\text{Output speed}}$$

Therefore this equation can be used to work out speeds at different sections of the system

Example

If the motor speed is 1200 rpm, then what is the output speed?

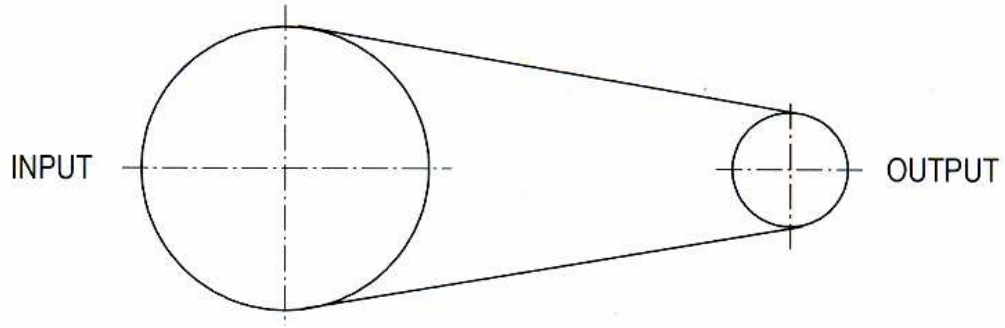
$$\text{Output speed} = \frac{\text{input speed}}{\text{Multiplier ratio}} = \frac{1200 \text{ rpm}}{1/3} = 3600 \text{ rpm}$$

Task 20

a) Label the line diagram of the belt showing:

- The driver pulley
- The driven pulley
- The belt

http://www.bbc.co.uk/schools/gcsebitesize/design/systemscontrol/mechanismsrev7.shtml

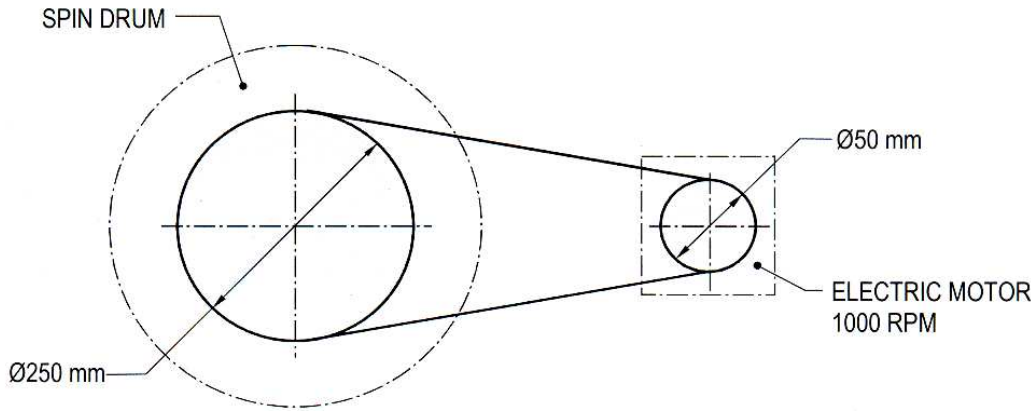


b) In the system above, when the driver is turned, does the driven pulley turn faster or slower than the driver?

c) If the diameter of the driver pulley is 40mm and the diameter of the driven pulley is 10mm, what I the multiplier ratio?

Task 21

This is a belt drive system for a washing MACHING DRUM. It is for transmitting rotary motion from an electric motor to a spin dryer system.



The motor has an output force of 800Nm at 1000rpm.
Calculate:

a) The multiplier ratio of the system

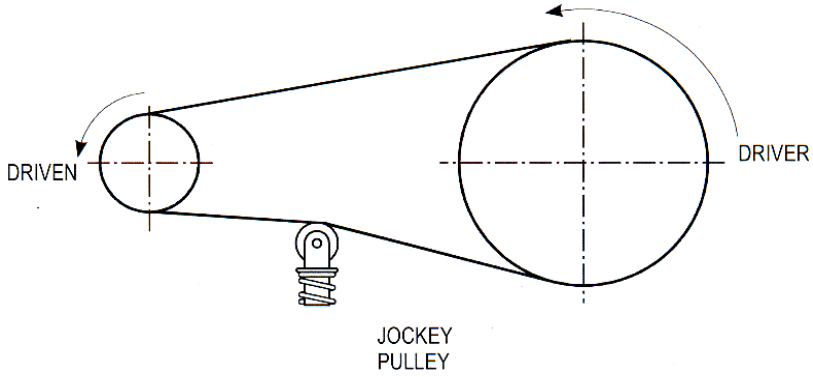
b) The speed of the drum

Drive systems

Jockey pulley

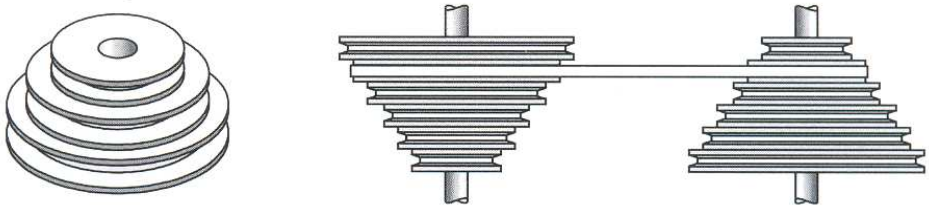
The advantage of a belt drive system is that it will absorb shock as it is allowed to slip. It is used in machines where you would not want the motor to stop or cease.

However, excessive slipping will create inefficiency in the system. At the same time, if the belt is too tight the pulley bearings will get damaged. One way of keeping the correct tension is to use a spring loaded jockey pulley.



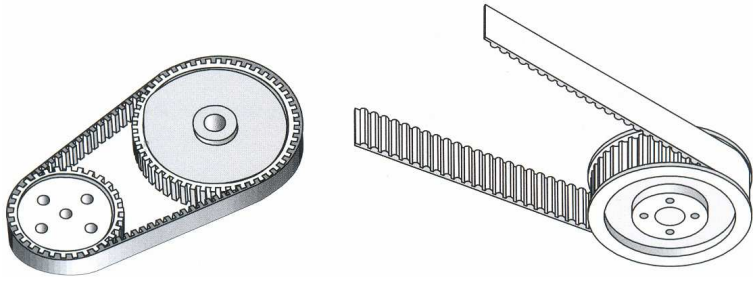
Stepped Cone Pulley system

A variety of output speeds and torques can be achieved using this system. The drive motor is attached to one set of pulleys and the drive belt can be moved between various pairs of pulleys to give a selection of speeds.



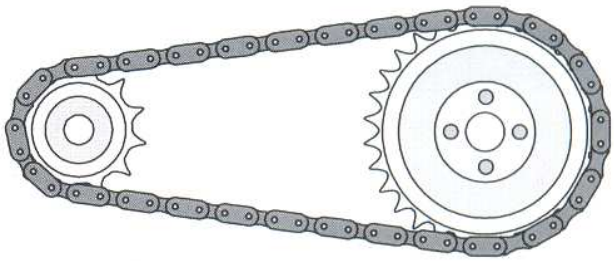
Toothed belts

Belt drives tend to use their ability to slip to their advantage, however where slippage would damage a mechanism toothed belts have been developed that retain the advantages of normal belts but do not slip.



Chain Drives

Where large forces have to be transmitted and there can be no slippage allowed chain drives are used. Instead of a pulley a toothed wheel known as a sprocket is used to drive a chain.



Bicycle Chain Drive

This is an application of a chain drive that should be familiar to everyone. When cycling, if you want to go suddenly faster, you stand up and put extra weight into the pedals. The force is transmitted to the back wheel by means of the chains. Unless the chain or sprockets are damaged the chain will not slip and the extra force will carry out its task in allowing you to go faster.

Chains are very strong, and unless they are badly worn, they will not slip. However, they do have to be oiled regularly, and both the chain and sprockets are prone to wear. They are also more expensive and much noisier than belt drives.

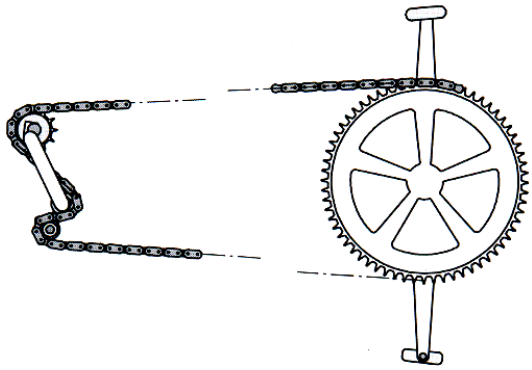
Multiplier ratio for chain drives

Calculating the multiplier ratio, output speed and torque of a chain drive is very similar to calculating them in belt drive systems. The calculation is:

$$\text{Multiplier ratio} = \frac{\text{no. of teeth on driven sprocket}}{\text{no. of teeth on driver sprocket}}$$

Chain Tension

Chain drive systems must also have a means to tension the chain. If the chain is over-tensioned there will be extensive wear on the chain, sprockets and bearings in the system. To give a greater control, a spring loaded jockey wheel, such as that used in Derailleur gears on mountain bikes is used.



Task 22

A pedal cycle has 60 teeth on the driver sprocket and 10 teeth on the driven sprocket. What is the multiplier ratio of the chain-drive system?