Higher Unit 1 Prep Booklet

App 1.1

① (i) $A(4,-3)$ $B(-6,2)$ $\quad m = \frac{2+3}{-6-4} = \frac{5}{-10} = -\frac{1}{2}$

$y-2 = -\frac{1}{2}(x+6)$

$2y-4 = -x-6$

$\underline{x+2y+2=0}$

(ii) $m = \tan\theta$

$m = \tan 40^\circ$

$m = 0.839$

(iii) $y = 4x+1$

a) $m=4$ $(2,1)$

$y-1 = 4(x-2)$

$y-1 = 4x-8$

$\underline{4x-y-7=0}$

b) $m = -\frac{1}{4}$ $(2,1)$

$y-1 = -\frac{1}{4}(x-2)$

$4y-4 = -x+2$

$\underline{x+4y-6=0}$

② (i) $A(2,-7)$ $B(6,1)$ $\quad m = \frac{1+7}{6-2} = \frac{8}{4} = 2$

$y-1 = 2(x-6)$

$y-1 = 2x-12$

$\underline{2x-y-11=0}$

(ii) $m = \tan 55^\circ$

$m = 1.19$

(iii) $y = 3x-1$ $\quad (4,2)$

a) $y-2 = 3(x-4)$

$y-2 = 3x-12$

$\underline{3x-y-10=0}$

b) $y-2 = -\frac{1}{3}(x-4)$

$3y-6 = -x+4$

$\underline{x+3y-10=0}$

③(i) $A(-1,3)$ $B(-4,2)$ $m = \frac{2-3}{-4+1} = \frac{-1}{-3} = \frac{1}{3}$

$y - 3 = \frac{1}{3}(x+1)$

$3y - 9 = x + 1$

$\underline{x - 3y + 10 = 0}$

(ii) $m = \tan 75°$

$m = 3.73$

(iii) $y = \frac{1}{4}x + 1$ $(3,-1)$

a) $y + 1 = \frac{1}{4}(x-3)$

$4y + 4 = x - 3$

$\underline{x - 4y - 7 = 0}$

b) $y + 1 = -4(x-3)$

$y + 1 = -4x + 12$

$\underline{4x + y - 11 = 0}$

④(i) $A(4,-4)$ $B(2,6)$ $m = \frac{6+4}{2-4} = \frac{10}{-2} = -5$

$y + 4 = -5(x-4)$

$y + 4 = -5x + 20$

$\underline{5x + y - 16 = 0}$

(ii) $m = \tan 30°$

$m = 0.577$

(iii) $y = -\frac{3}{4}x - 2$ $(-2,4)$

a) $y - 4 = -\frac{3}{4}(x+2)$

$4y - 16 = -3x - 6$

$\underline{3x + 4y - 10 = 0}$

b) $y - 4 = \frac{4}{3}(x+2)$

$3y - 12 = 4x + 8$

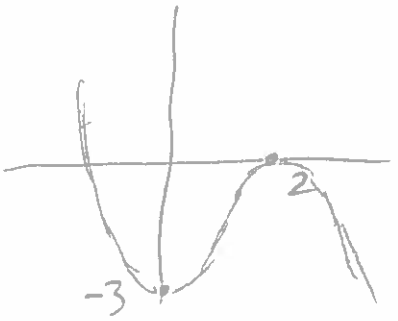
$\underline{4x - 3y - 20 = 0}$

E+F 1.3

① a) $y = -f(x)$

$(0,3) \xrightarrow{y\times -1} (0,-3)$

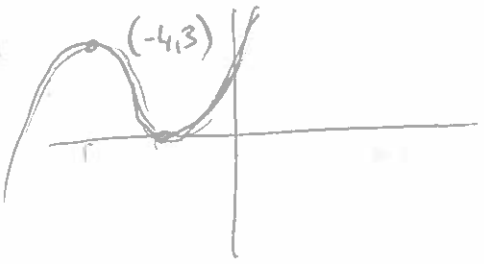
$(2,0) \rightarrow (2,0)$



b) $y = f(x+4)$

$(0,3) \xrightarrow{x-4} (-4,3)$

$(2,0) \rightarrow (-2,0)$



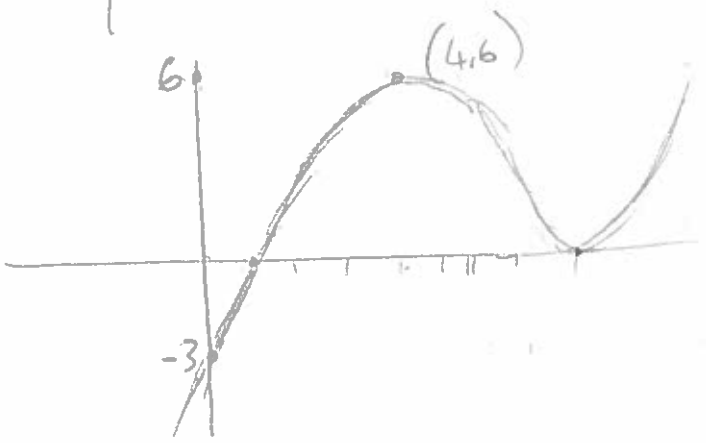
② a) $y = -f(x)$

$(0,3) \xrightarrow{y\times -1} (0,-3)$

$(1,0) \rightarrow (1,0)$

$(4,-6) \rightarrow (4,6)$

$(8,0) \rightarrow (8,0)$



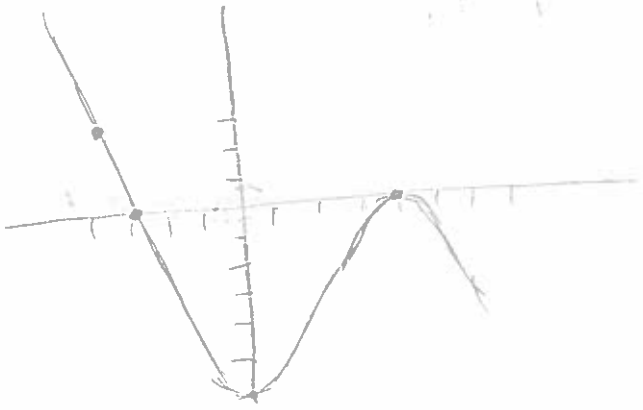
b) $y = f(x+4)$

$(0,3) \xrightarrow{x-4} (-4,3)$

$(1,0) \rightarrow (-3,0)$

$(4,-6) \rightarrow (0,-6)$

$(8,0) \rightarrow (4,0)$

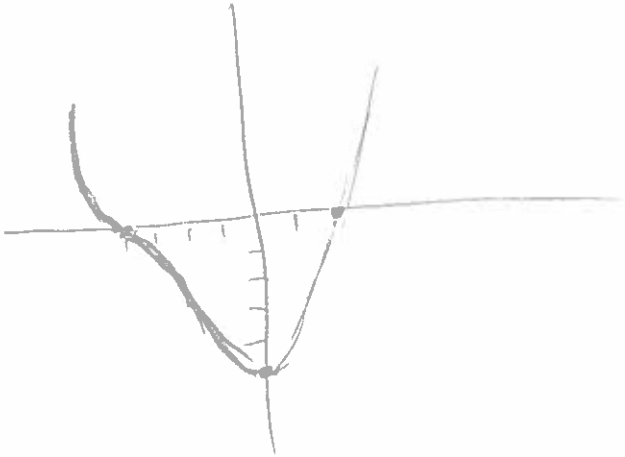


③ a) $y = -f(x)$

$(-4,0) \xrightarrow{y\times -1} (-4,0)$

$(0,5) \rightarrow (0,-5)$

$(2,0) \rightarrow (2,0)$



b) $y = f(x-4)$ ($x+4$)

$(-4,0) \to (0,0)$

$(0,5) \to (4,5)$

$(2,0) \to (6,0)$

④ $y = -f(x) + 2$ ($y \times -1$, $y+2$)

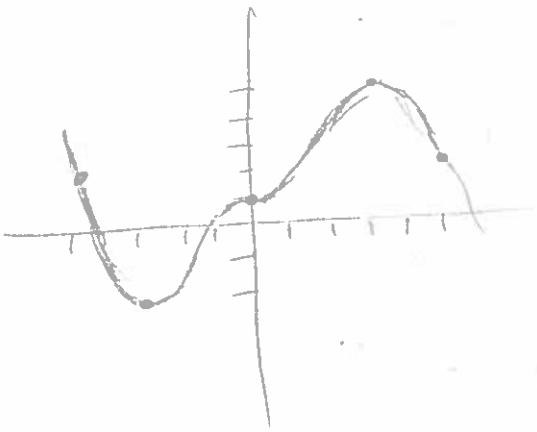
$(-5,0) \to (-5,0) \to (-5,2)$

$(-3,4) \to (-3,-4) \to (-3,-2)$

$(0,1) \to (0,-1) \to (0,+)$

$(3,-3) \to (3,3) \to (3,5)$

$(5,0) \to (5,0) \to (5,2)$



b) $y = f(x-3)$ ($x+3$)

$(-5,0) \to (-2,0)$

$(-3,4) \to (0,4)$

$(0,1) \to (3,1)$

$(3,-3) \to (6,-3)$

$(5,0) \to (8,0)$

⑤ $y = a^x$ $(1,3)$

$3 = a^1$

$\underline{a = 3}$

⑥ $y = a^x$ $(2,9)$

$9 = a^2$

$\underline{a = \sqrt{9} = 3}$

⑦ $y = a^x$ $(1,6)$

$6 = a^1$

$\underline{a = 6}$

⑧ $y = a^x$ $(1,2)$

$2 = a^1$

$\underline{a = 2}$

(9) $y = 2^x$ inverse $y = \log_2 x$

(10) $y = 3^x$ inverse $y = \log_3 x$

(11) $y = 7^x$ inverse $y = \log_7 x$

(12) $y = 6^x$ inverse $y = \log_6 x$

(13) $f(x) = x^3$ $g(x) = 2x - 4$

$f(g(x)) = (2x-4)^3$

(14) $f(x) = x^2 - 1$ $g(x) = 3x - 1$

$f(g(x)) = (3x-1)^2 - 1$

(15) $f(x) = 2x^2$ $g(x) = x + 1$

$f(g(x)) = 2(x+1)^2$

(16) a) $f(x) = x^2 + x$ $g(x) = 3x + 1$

$f(g(x)) = (3x+1)^2 + 3x + 1$

$= 9x^2 + 6x + 1 + 3x + 1$

$= \underline{9x^2 + 9x + 2}$

(17) a) $y = \sin x^\circ - 2$

b) $y = \frac{1}{2}\cos x^\circ$

(18) a) $y = \sin x - 1$

b) $y = \sin 3x$

(19) a) $y = \sin x + 1$

b) $y = \sin 4x$

(20) a) $y = 2\sin x + 1$

b) $y = \sin \frac{1}{2}x$

(b) $h(x) = 5x$ $k(x) = \tan x$

$k(h(x)) = \tan 5x$

b) $h(x) = 4x$ $k(x) = \cos x$

$k(h(x)) = \cos 4x$

b) $h(x) = \sin x$ $k(x) = \frac{1}{2}x$

$k(h(x)) = \frac{1}{2}\sin x$

b) $h(x) = 2x + \pi$ $k(x) = \cos x$

$k(h(x)) = \cos(2x + \pi)$

(21) $f(x) = 2x-4 \quad g(x) = \frac{1}{2}x+2$

$h(x) = g(f(x)) = \frac{1}{2}(2x-4)+2$

$= x-2+2$

$= x$ $g(x)$ is inverse to $f(x)$

(22) $f(x) = 3x+9 \quad g(x) = \frac{1}{3}x-3$

$h(x) = g(f(x)) = \frac{1}{3}(3x+9)-3$

$= x+3-3$

$= x$ $g(x)$ is inverse to $f(x)$

(23) $f(x) = 2x-1 \quad g(x) = \sqrt{x}$

$g(f(x)) = \sqrt{(2x-1)}$ $2x-1 \geq 0$

$2x \geq 1$

$x \geq \frac{1}{2}$

(24) $f(x) = \frac{1}{x^2-4} \quad g(x) = 2x+1$

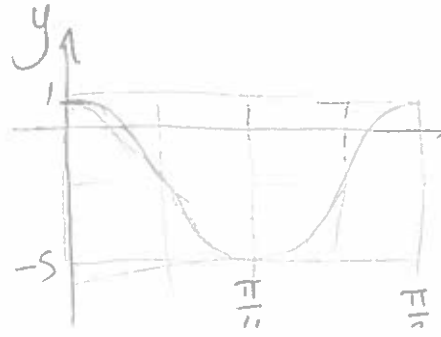
$g(f(x)) = 2\left(\frac{1}{x^2-4}\right)+1$ $x^2-4 \neq 0$

$= \frac{2}{x^2-4}+1$ $x^2 \neq 4$

$x \neq \pm 2$.

(25) max $(0,1)$ $y = a\cos bx + c$

min $\left(\frac{\pi}{4}, -5\right)$ $a = \frac{1-(-5)}{2} = 3$



$c = -2$.

Period $= \frac{\pi}{2}$ $\therefore b = 4$

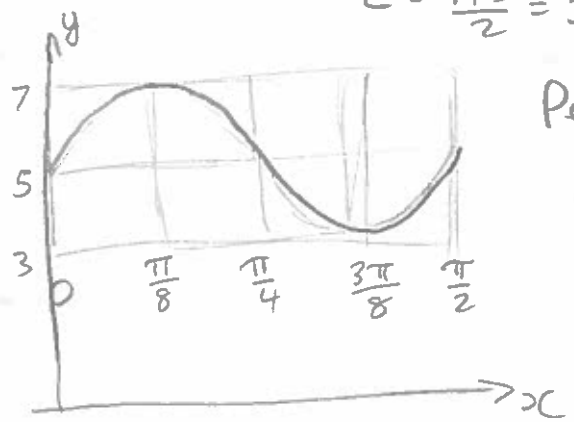
(26) max $\left(\frac{\pi}{8}, 7\right)$

min $\left(\frac{3\pi}{8}, 3\right)$

$y = a \sin bx + c$

$a = \frac{7-3}{2} = 2.$

$c = \frac{7+3}{2} = 5$



Period $= \frac{\pi}{2} \therefore b = 4.$

Kel 1.3

① $y = \frac{x^5 - 3}{x^3} = \frac{x^5}{x^3} - \frac{3}{x^3}$

$= x^2 - 3x^{-3}$

$\frac{dy}{dx} = 2x + 9x^{-4}$

$= 2x + \frac{9}{x^4}$

② $y = \frac{x^4 + 2}{x^3} = \frac{x^4}{x^3} + \frac{2}{x^3}$

$= x + 2x^{-3}$

$\frac{dy}{dx} = 1 + 6x^{-4}$

$= 1 - \frac{6}{x^4}$

③ $y = \frac{1 + x^4}{x^2} = \frac{1}{x^2} + \frac{x^4}{x^2}$

$= x^{-2} + x^2$

$\frac{dy}{dx} = -2x^{-3} + 2x$

$= -\frac{2}{x^3} + 2x$

④ $y = \frac{3 + x^6}{x^4} = \frac{3}{x^4} + \frac{x^6}{x^4}$

$= 3x^{-4} + x^2$

$\frac{dy}{dx} = -12x^{-5} + 2x$

$= -\frac{12}{x^5} + 2x$

⑤ $y = \frac{6}{x^5} + 3x^{\frac{1}{2}}$

$= 6x^{-5} + 3x^{\frac{1}{2}}$

$\frac{dy}{dx} = -30x^{-6} + \frac{3}{2}x^{-\frac{1}{2}}$

$= -\frac{30}{x^6} + \frac{3}{2x^{\frac{1}{2}}}$

$= -\frac{30}{x^6} + \frac{3}{2\sqrt{x}}$

⑥ $y = \frac{3}{x^4} + 2x^{\frac{1}{5}}$

$= 3x^{-4} + 2x^{\frac{1}{5}}$

$\frac{dy}{dx} = -12x^{-5} + \frac{2}{5}x^{-\frac{4}{5}}$

$= -\frac{12}{x^5} + \frac{2}{5x^{\frac{4}{5}}}$

$= -\frac{12}{x^5} + \frac{2}{5\sqrt[5]{x^4}}$

(7) $y = \frac{2}{x^3} + 8x^{\frac{1}{4}}$

$= 2x^{-3} + 8x^{\frac{1}{4}}$

$\frac{dy}{dx} = -6x^{-4} + 2x^{-\frac{3}{4}}$

$= -\frac{6}{x^4} + \frac{2}{x^{3/4}}$

$= -\frac{6}{x^4} + \frac{2}{\sqrt[4]{x^3}}$

(8) $y = \frac{5}{x^2} + 6x^{\frac{1}{3}}$

$= 5x^{-2} + 6x^{\frac{1}{3}}$

$\frac{dy}{dx} = -10x^{-3} + 2x^{-\frac{2}{3}}$

$= -\frac{10}{x^3} + \frac{2}{x^{2/3}}$

$= -\frac{10}{x^3} + \frac{2}{\sqrt[3]{x^2}}$

(9) $y = 3\sin x$

$\frac{dy}{dx} = 3\cos x$

(10) $y = \frac{3}{4}\cos x$

$\frac{dy}{dx} = -\frac{3}{4}\sin x$

(11) $y = 5\sin x$

$\frac{dy}{dx} = 5\cos x$

(12) $y = 8\cos x$

$\frac{dy}{dx} = -8\sin x$

(13) $y = x^2 - 5x + 6$ (5,6)

$\frac{dy}{dx} = 2x - 5$

let $x = 5$ $\frac{dy}{dx} = 10 - 5 = 5$

$y - 6 = 5(x - 5)$

$y - 6 = 5x - 25$

$5x - y - 19 = 0$

(14) $y = x^2 - 6x + 8$ P(5,3)

$\frac{dy}{dx} = 2x - 6$

let $x = 5$ $\frac{dy}{dx} = 10 - 6 = 4$

$y - 3 = 4(x - 5)$

$y - 3 = 4x - 20$

$4x - y - 17 = 0$.

(14) b) $\frac{dy}{dx} = 2x - 6$

let $x=3$ $= 6-6 = 0$

$m=0$ ∴ parallel to x axis

(15) $y = x^2 - 10x + 24$ $P(7,3)$ b) let $x=5$

$\frac{dy}{dx} = 2x - 10$ $\frac{dy}{dx} = 10 - 10 = 0$

let $x=7$ $= 14 - 10 = 4$ $m=0$ ∴ parallel to x axis

$y - 3 = 4(x-7)$

$y - 3 = 4x - 28$

$4x - y - 25 = 0$

(16) $y = x^2 - 12x + 35$ $P(4,3)$ b) let $x=6$

$\frac{dy}{dx} = 2x - 12$ $\frac{dy}{dx} = 12 - 12 = 0$

let $x=4$ $= 8 - 12 = -4$ $m=0$ ∴ parallel to x axis

$y - 3 = -4(x-4)$

$y - 3 = -4x + 16$

$4x + y - 19 = 0$

(17) $y = 3x^2$ $y = 3(2)^2 = 12$

$\frac{dy}{dx} = 6x$ $y - 12 = 12(x-2)$

$y - 12 = 12x - 24$

let $x=2$ $\frac{dy}{dx} = 12$. $12x - y - 12 = 0$

(18) $y = x^2 + 2x \quad x = 1 \quad y = 1 + 2 = 3$

$\frac{dy}{dx} = 2x + 2$

$= 2 + 2 = 4$

$y - 3 = 4(x - 1)$

$y - 3 = 4x - 4$

$\underline{4x - y + 1 = 0}$.

$x = -1 \quad \frac{dy}{dx} = -2 + 2 = 0$. parallel to x axis

(19) $h(t) = 2 + 10t + 5t^2$

$h(0) = 2$ ~~seconds~~

b) $2 + 10t - 5t^2 = 2$.

~~$(5t \quad)(t - 2) = 0$~~

$10t - 5t^2 = 0$

$-5t(-2 + t) = 0$

$\therefore \quad t = 0$ or $t = 2$ seconds

c) $h'(t) = 10 - 10t$

$h'(1) = 10 - 10 = 0 \text{ ms}^{-1}$

The ball is stationary at the top of the arc

d) $h'(0) = 10 - 0 = 10 \text{ms}^{-1}$

$h'(2) = 10 - 20 = -10 \text{ms}^{-1}$

(20) $x(t) = 1 + 4t + t^2$

a) $x'(t) = 4 + 2t$

b) $4 + 2t = 0$ The tram has stopped.

$-2t = +4 \quad t = +2$ seconds

c) $x'(3) = 4 - 6 = -2 \text{ms}^{-1}$

It is reversing.

App 1.3

① ↓68% +600 million

a) $u_{n+1} = 0.32\,u_n + 600$

b) $L = \frac{b}{1-a} = \frac{600}{1-032} = 882$ million.

Yes it can sustain the bacteria as $882 < 900$

② ↓ $\frac{1}{8}$ +24

a) $u_{n+1} = \frac{7}{8}u_n + 24$

b) $L = \frac{b}{1-a} = \frac{24}{1-\frac{7}{8}} = 192$ rabbits

No it cannot sustain the rabbits as $192 > 180$

③ ↓20% + 400

a) $u_{n+1} = 0.8\,u_n + 400$

b) $L = \frac{400}{1-0.8} = 2000$ The population of insects will settle at 2000 in the long term.

④ ↓6% +560

a) $u_{n+1} = 0.94\,u_n + 560$

$L = \frac{560}{1-0.94} = 9000$ The population of ants will settle at 9000 in the long term.