

**X100/301**

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NATIONAL  
QUALIFICATIONS  
2006

FRIDAY, 19 MAY  
9.00 AM – 10.10 AM

MATHEMATICS  
HIGHER

Units 1, 2 and 3

Paper 1

(Non-calculator)

**Read Carefully**

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



## FORMULAE LIST

### Circle:

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

**Scalar Product:**  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

**Trigonometric formulae:**  $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

**Table of standard derivatives:**

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

**Table of standard integrals:**

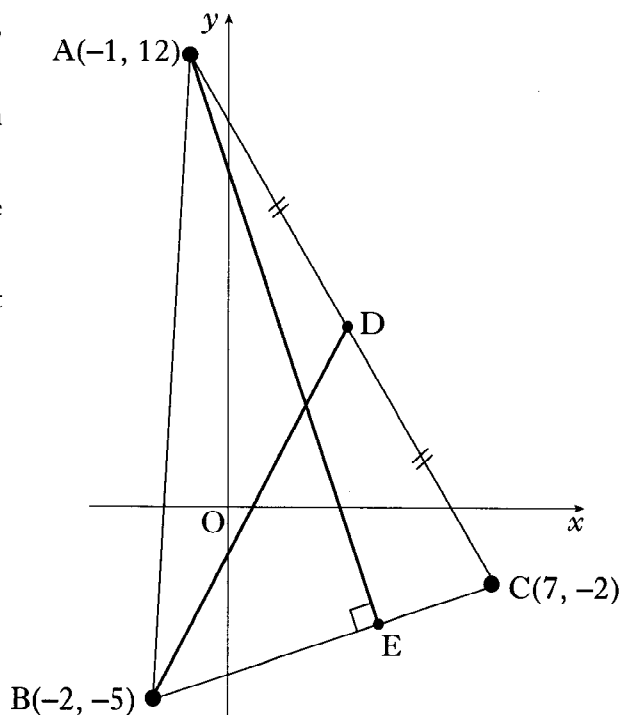
$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

**ALL questions should be attempted.**

*Marks*

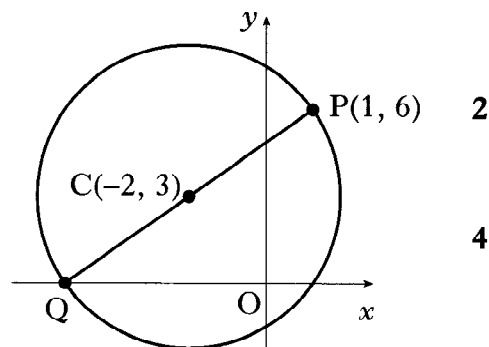
1. Triangle ABC has vertices  $A(-1, 12)$ ,  $B(-2, -5)$  and  $C(7, -2)$ .

- (a) Find the equation of the median BD.
- (b) Find the equation of the altitude AE.
- (c) Find the coordinates of the point of intersection of BD and AE.



2. A circle has centre  $C(-2, 3)$  and passes through  $P(1, 6)$ .

- (a) Find the equation of the circle.
- (b) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q.



3. Two functions  $f$  and  $g$  are defined by  $f(x) = 2x + 3$  and  $g(x) = 2x - 3$ , where  $x$  is a real number.

- (a) Find expressions for:

(i)  $f(g(x))$ ;

(ii)  $g(f(x))$ .

- (b) Determine the least possible value of the product  $f(g(x)) \times g(f(x))$ .

**[Turn over**

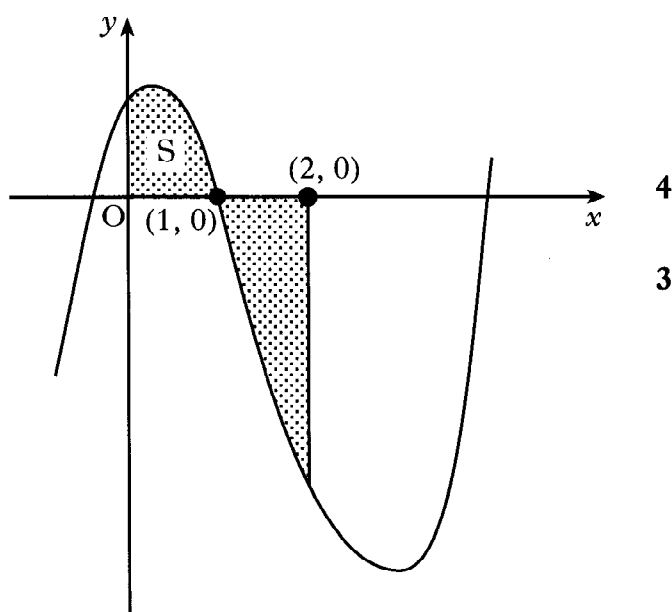
4. A sequence is defined by the recurrence relation  $u_{n+1} = 0.8u_n + 12$ ,  $u_0 = 4$ .
- (a) State why this sequence has a limit. 1
- (b) Find this limit. 2

5. A function  $f$  is defined by  $f(x) = (2x - 1)^5$ .
- Find the coordinates of the stationary point on the graph with equation  $y = f(x)$  and determine its nature. 7

6. The graph shown has equation  $y = x^3 - 6x^2 + 4x + 1$ .

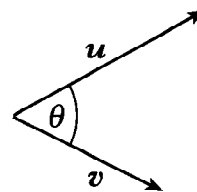
The total shaded area is bounded by the curve, the  $x$ -axis, the  $y$ -axis and the line  $x = 2$ .

- (a) Calculate the shaded area labelled S.
- (b) Hence find the total shaded area.

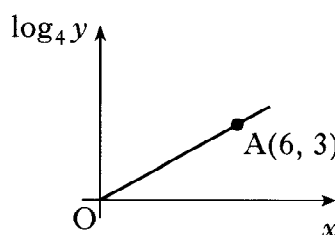


7. Solve the equation  $\sin x^\circ - \sin 2x^\circ = 0$  in the interval  $0 \leq x \leq 360$ . 4
8. (a) Express  $2x^2 + 4x - 3$  in the form  $a(x + b)^2 + c$ . 3
- (b) Write down the coordinates of the turning point on the parabola with equation  $y = 2x^2 + 4x - 3$ . 1

9.  $\mathbf{u}$  and  $\mathbf{v}$  are vectors given by  $\mathbf{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$  and  $\mathbf{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$ , where  $k > 0$ .



- (a) If  $\mathbf{u} \cdot \mathbf{v} = 1$ , show that  $k^3 + 3k^2 - k - 3 = 0$ . 2
- (b) Show that  $(k + 3)$  is a factor of  $k^3 + 3k^2 - k - 3$  and hence factorise  $k^3 + 3k^2 - k - 3$  fully. 5
- (c) Deduce the only possible value of  $k$ . 1
- (d) The angle between  $\mathbf{u}$  and  $\mathbf{v}$  is  $\theta$ . Find the exact value of  $\cos \theta$ . 3
10. Two variables,  $x$  and  $y$ , are connected by the law  $y = a^x$ . The graph of  $\log_4 y$  against  $x$  is a straight line passing through the origin and the point A(6, 3). Find the value of  $a$ . 4



[END OF QUESTION PAPER]

# **X100/303**

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NATIONAL  
QUALIFICATIONS  
2006

FRIDAY, 19 MAY  
10.30 AM – 12.00 NOON

**MATHEMATICS  
HIGHER**  
Units 1, 2 and 3  
Paper 2

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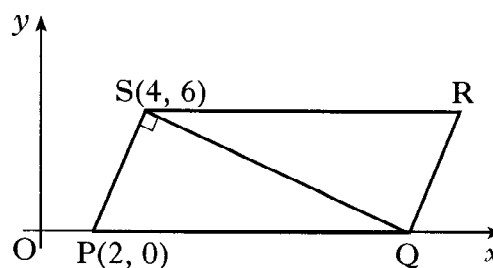
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*Marks*

1. PQRS is a parallelogram. P is the point (2, 0), S is (4, 6) and Q lies on the x-axis, as shown.

The diagonal QS is perpendicular to the side PS.

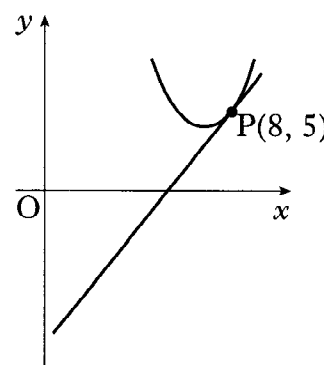


- (a) Show that the equation of QS is  $x + 3y = 22$ . 4
- (b) Hence find the coordinates of Q and R. 2

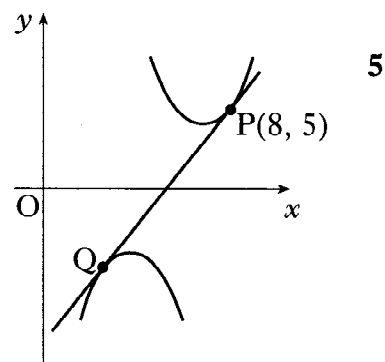
2. Find the value of  $k$  such that the equation  $kx^2 + kx + 6 = 0$ ,  $k \neq 0$ , has equal roots. 4

3. The parabola with equation  $y = x^2 - 14x + 53$  has a tangent at the point P(8, 5).

(a) Find the equation of this tangent. 4



(b) Show that the tangent found in (a) is also a tangent to the parabola with equation  $y = -x^2 + 10x - 27$  and find the coordinates of the point of contact Q.



4. The circles with equations  $(x - 3)^2 + (y - 4)^2 = 25$  and  $x^2 + y^2 - kx - 8y - 2k = 0$  have the same centre.

Determine the radius of the larger circle. 5



5. The curve  $y = f(x)$  is such that  $\frac{dy}{dx} = 4x - 6x^2$ . The curve passes through the point  $(-1, 9)$ . Express  $y$  in terms of  $x$ . 4

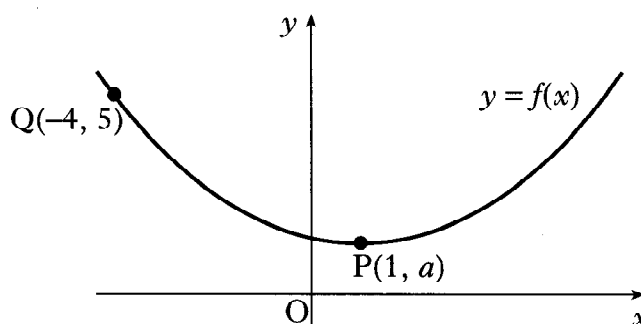
6. P is the point  $(-1, 2, -1)$  and Q is  $(3, 2, -4)$ .
- (a) Write down  $\vec{PQ}$  in component form. 1
- (b) Calculate the length of  $\vec{PQ}$ . 1
- (c) Find the components of a unit vector which is parallel to  $\vec{PQ}$ . 1

7. The diagram shows the graph of a function  $y = f(x)$ .

Copy the diagram and on it sketch the graphs of:

(a)  $y = f(x - 4)$ ; 2

(b)  $y = 2 + f(x - 4)$ . 2

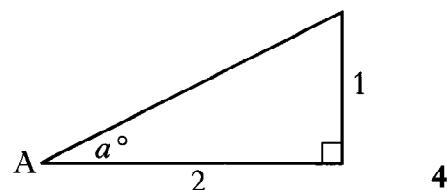


8. The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of  $a^\circ$  at A.

(a) Find the exact values of:

(i)  $\sin a^\circ$ ;

(ii)  $\sin 2a^\circ$ .



- (b) By expressing  $\sin 3a^\circ$  as  $\sin(2a + a)^\circ$ , find the exact value of  $\sin 3a^\circ$ . 4

9. If  $y = \frac{1}{x^3} - \cos 2x$ ,  $x \neq 0$ , find  $\frac{dy}{dx}$ . 4

10. A curve has equation  $y = 7\sin x - 24\cos x$ .

(a) Express  $7\sin x - 24\cos x$  in the form  $k\sin(x - a)$  where  $k > 0$  and  $0 \leq a \leq \frac{\pi}{2}$ . 4

(b) Hence find, in the interval  $0 \leq x \leq \pi$ , the  $x$ -coordinate of the point on the curve where the gradient is 1. 3

11. It is claimed that a wheel is made from wood which is over 1000 years old.

To test this claim, carbon dating is used.

The formula  $A(t) = A_0 e^{-0.000124t}$  is used to determine the age of the wood, where  $A_0$  is the amount of carbon in any living tree,  $A(t)$  is the amount of carbon in the wood being dated and  $t$  is the age of the wood in years.

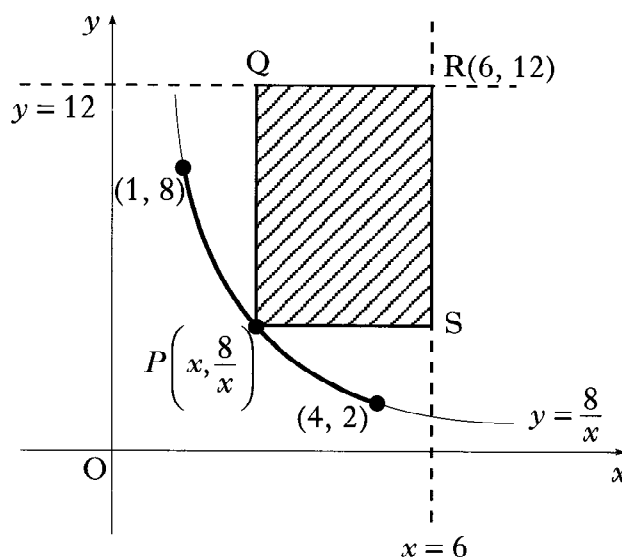
For the wheel it was found that  $A(t)$  was 88% of the amount of carbon in a living tree.

Is the claim true?

5

12. PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines  $x = 6$  and  $y = 12$
- P lies on the curve with equation  $y = \frac{8}{x}$  between  $(1, 8)$  and  $(4, 2)$
- R is the point  $(6, 12)$ .



- (a) (i) Express the lengths of PS and RS in terms of  $x$ , the  $x$ -coordinate of P.  
 (ii) Hence show that the area,  $A$  square units, of PQRS is given by

$$A = 80 - 12x - \frac{48}{x}$$

3

- (b) Find the greatest and least possible values of  $A$  and the corresponding values of  $x$  for which they occur.

8

[END OF QUESTION PAPER]