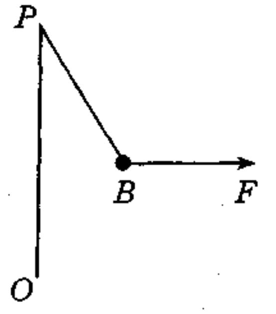
Advanced Higher Mechanics 2005 Paper

1. A ball B of weight 9 newtons is attached to one end of a light inextensible string. The other end of the string is attached to P, the top of a fixed vertical pole OP.



By exerting a horizontal force of magnitude F newtons, the ball is held in equilibrium, with the string taut and.OPB = 30 °.
Calculate:

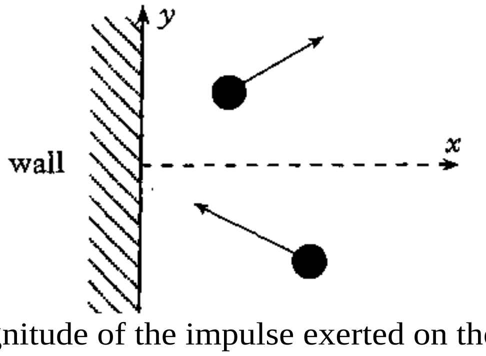
(a) the tension in the string; **2**

(b) the value of F. **2**

2. A ball is projected vertically from ground level. The ball attains a maximum height of 49 metres before returning to the ground.
Assuming only the action of gravity, calculate the time of flight of the ball. **5**

3. A particle executes simple harmonic motion about a point O. The magnitude of the maximum acceleration is 1 m s^{-2} and the maximum speed is 4ms^{-1}.
Calculate the period of the motion. **4**

4. A ball of mass 0.01 kg collides with a fixed vertical wall. Immediately before the collision the velocity of the ball is -3**i** + 4**j**, and just after the collision the velocity is 2**i** + 3**j**, where **i** and **j** are unit vectors in the x and y directions of the rectangular coordinate system shown below and speeds are measured in ms^{-1}.



Calculate the magnitude of the impulse exerted on the ball by the wall. **3**

5. The velocity of an ice skater relative to a rectangular coordinate system with origin *O*, is given by

$$v = 3(t^2 - 4t + 2)\mathbf{i} + 4\mathbf{j},$$

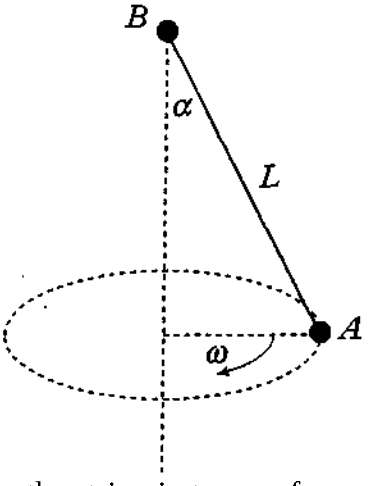
where **i**, **j** are unit vectors in the *Ox* and *Oy* directions, *t* seconds is the time and the speed is measured in ms^{-1}. Initially the skater has position vector $-4\mathbf{j}$.

(a) Find the time at which the acceleration is instantaneously equal to zero. **2**

(b) Calculate the distance of the skater from *O* when the acceleration is instantaneously equal to zero. **4**

6. A ball of mass *m* kg is attached to one end *A* of a light inextensible string of length *L* metres. The other end of the string is attached to a fixed point *B*. The ball moves, with string taut, in a horizontal circle with constant angular speed ω radians per second as shown in the diagram. During this motion, the string is inclined at an angle α to the downward vertical through *B* where

$\tan \alpha = 5/12$



(a) Find the tension in the string in terms of *m* and *g*. **2**

(b) Find an expression for ω in terms of *g* and *L*. **3**

7. A particle of mass 2 kg is accelerated horizontally from rest at a point O by a force $8t\,\mathbf{i}$, whose magnitude is measured in newtons and where **i** is the unit vector in the direction of motion and *t* seconds is the time from the start of the motion.

(a) Find the velocity, **v**, of the particle as a function of time *t*. **2**

(b) Calculate the work done on the particle in the first second of the motion **3**

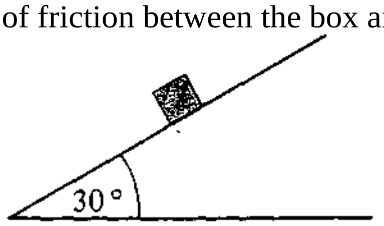
8. A bungee jumper of mass *m* kg falls vertically from rest from a high bridge. One end of an elastic rope is attached to the jumper, the other end to the bridge at the point where the jumper commences her fall. The natural-length of the rope is *l* metres and the modulus of elasticity is 12 *mg* newtons.
At the moment when the jumper is brought instantaneously to rest by the rope, the extension of the rope is *a* metres.

(a) Neglecting the effect of air resistance, use conservation of energy to show that the extension satisfies **3**

$$6a^2 - la - l^2 = 0.$$

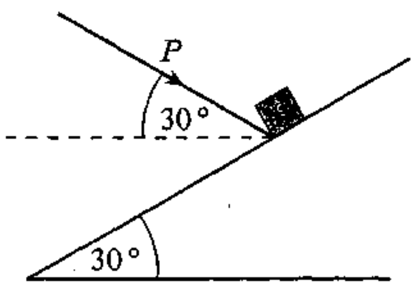
(b) Hence find, in terms of *l*, the distance the jumper falls before first coming instantaneously to rest. **3**

9. *(a)* A box of mass *m* kg is placed on a rough plane inclined at 30° to the horizontal. The coefficient of friction between the box and the plane is μ.



Given that the box remains in equilibrium, show that $\mu > 1/\sqrt{3}$

(b) The same box is kept in equilibrium on another rough plane, which is also inclined at 30° to the horizontal, by the action of a force of magnitude *P* newtons as shown in the diagram below. This force is acting up the plane at an angle of 30 ° to the horizontal. The coefficient of friction between the box and this plane is 0.5 and the box is on the point of slipping down the plane.



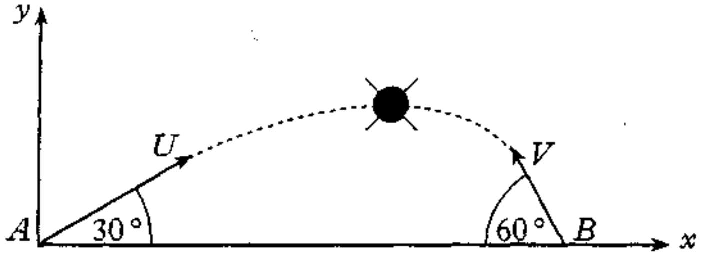
$$R = \frac{\sqrt{3}}{2}(mg + P) \text{ newtons.}$$

(ii) Show further that

$$P = \frac{(2-\sqrt{3})mg}{2+\sqrt{3}} \text{ newtons.}$$

(i) Show that the reaction force normal to the inclined plane has magnitude given by

10 Two points A and B are a distance L metres apart on horizontal ground. A ball is thrown from A towards B with speed U ms-1 at an angle of projection of 30°. Simultaneously, a second ball is thrown from B towards A with speed V ms^{-1} and angle of projection 60°.



(a) Using the coordinate system shown in the diagram:

(i) write down expressions in terms of U and t for the x and y coordinates of the ball thrown from A at time t seconds after projection; **2**

(ii) show that at time t, the x-coordinate of the ball thrown from B is $x = L - \frac{1}{2} Vt$ and write down the corresponding expression for the y-coordinate. **2**

(b) The balls collide before reaching the ground,

(i) Show that $U = \sqrt{3}\, V$. **2**

(ii) Find an expression for the horizontal distance from A at which the collision takes place, giving your answer in terms of L. **4**

11 A particle is projected horizontally from the origin, O, along the positive x-axis with initial speed 1 ms^{-1}. The particle has acceleration $4(4x - 1)\mathbf{i}$ ms^{-2}, where x metres ($0 < x < \frac{1}{4}$) is the distance of the particle from O after time t seconds and **i** is a unit vector in the direction of the x-axis.

(a) Show that the speed, v ms^{-1}, of the particle is given by **5**

$$v = 1 - 4x.$$

(b) Hence show that

$$x = \frac{1}{4}(1 - e^{-4t})$$ **5**