Advanced Higher Mechanics 2004 Paper

1. The position of a power sledge on a frozen lake at time t seconds, relative to a rectangular coordinate system, is

$$r(t) = (2t^2 - t)\boldsymbol{i} - (3t + l)\boldsymbol{j},$$

where i, j are unit vectors in the x, y directions respectively and distances are measured in metres.

Calculate the time at which the speed is 5 m s^{-1} **4**

2.

At 2pm, a ferry leaves port O travelling at $25\sqrt{2}$ km/h in a north-easterly direction. At the same time, a liner is 10km east of O and travelling due north at 20 km/h. Both velocities remain constant.

(a) By choosing an appropriate rectangular coordinate system with origin O, find the position of the ferry relative to the liner at time t, measured in hours from 2 pm. **4**

(b) Calculate the distance between the ferry and the liner at 3 pm. **2**

3.

A piston connected to a water wheel oscillates about a point O with simple harmonic motion of period 8π seconds and maximum acceleration 0.25 ms^{-2}.

(a) Calculate the amplitude of the motion. **3**

(b) Calculate the positions, relative to O, of the piston when it is moving with half its maximum speed. **4**

4.

A ramp consists of a rough plane inclined at angle θ to the horizontal, where $\tan\theta = {}^5/_{12}$

A box of mass m kg is given a push up the line of greatest slope of the ramp, which gives the box an initial speed of $\sqrt{(gL)}$ ms^{-1}, where L metres is the distance travelled before the box comes to rest.

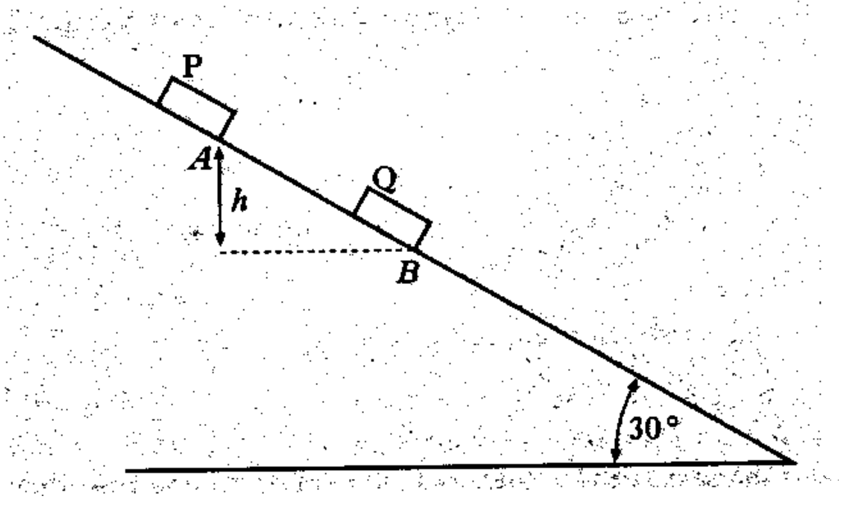
Calculate the value of the coefficient of friction between the box and the surface of the ramp. **7**

5. An unladen helicopter of mass M kilograms can hover at a constant height above the ground when the engine exerts a lift force of P newtons.

The helicopter is loaded with cargo which increases its mass by 1%. When airborne, the engine now exerts a lift force 5% greater than P to accelerate the helicopter vertically upwards. Calculate this vertical acceleration. **5**

6.

The diagram shows a ramp, inclined at 30° to the horizontal, which has a smooth section above *B* and a rough section below B. Identical blocks, P and Q, each has weight *W* newtons. Block Q is stationary at *B*, held by friction, and block P is held at rest at *A*. Block P is a vertical height of A metres above block Q (where the dimensions of the blocks should be ignored).



When block P is released, it slides down, the ramp colliding and coupling with block Q. The combined blocks then move down the rough section of the ramp, coming to rest at a vertical height $^1/_2h$ metres below B ;

(i) Find in terms of *g* ancl *h* the speed of the combined block immediately after the collision.

(ii) Using the work/energy principle, show that the constant frictional ,force acting on the combined block whilst it is moving has magnitude $^3/_2$W newtons.

7. A football is kicked from a point O on a horizontal plane, giving the ball an initial speed V ms^{-1} at an angle α to the horizontal. Assuming that gravity is the only force acting on the ball:

(a) Show that the maximum height, *H* metres, attained by the football is given by **3**

$$H = \frac{V^2}{2g}\sin^2\alpha.$$

(b) A second identical football is kicked from O with the same initial speed V ms^{-1} but at angle of projection *2* α to the horizontal *($2a < \frac{1}{2}\pi$).* The maximum height attained by this football is *h* metres,

(i) Show that **3**

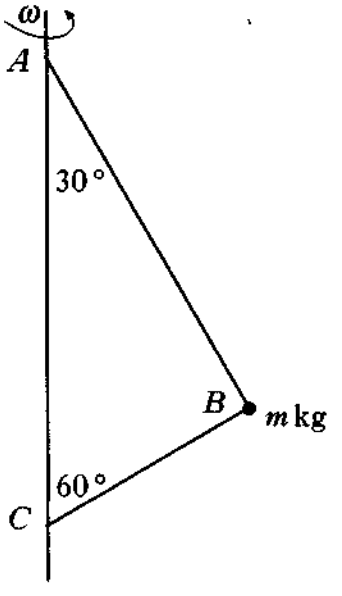
$$h = 4H\left(1 - \frac{2gH}{V^2}\right).$$

[Note that sin *2* α = 2sin α cos α.]

(ii) Given that the maximum height attained by the second football is three times that attained by the first, find the angles of projection of each of the two footballs. **4**

8.

A bead of mass *m* kilograms is attached to a vertical rotating column by two strings, as shown below. String *AB* is elastic, with natural length L metres and modulus of elasticity *2mg* newtons. The string is attached to the column at *A* and to the bead at *B*. String *BC* is inextensible and has length L metres. The vertical column is rotating at ω rads^{-1}, such that the strings *AB* and *BC* are taut and remain in a vertical plane. Angles *ACB* and *BAG* are 60° and 30° respectively.



(a) Show that the tension in the string *AB* is $2(\sqrt{3} - 1)$ *mg* newtons.

(b) Find, in terms of *m* and *g*, an expression for the tension in the string *BC*.

(c) Given that $L = 1$, calculate ω.

9. A particle of mass *m* kg moves in a horizontal straight line from the origin O with initial velocity U**i** ms^{-1}, where **i** is the unit vector in the direction of motion.
A resistive force $-mkv^3$**i** acts on the particle, where *k* is a constant and *v***i** is the velocity of the particle at time *t* seconds measured from the start of the motion.

(i) Show that the velocity of the particle satisfies the differential equation

$$\frac{dv}{dx} = -kv^2,$$

2

where *x* is the distance of the particle from O.

Hence show that **3**

$$v = \frac{U}{1 + kUx}.$$

(ii) Using (i), or otherwise, show that **3**

$$kUx^2 + 2x = 2Ut.$$

(iii) Find an expression, in terms of *k* and *U*, for the time taken for the speed of the particle to reduce to half its initial value. **3**