Advanced Higher Mechanics 2003 Paper

1. A particle, initially at rest, is projected from the origin with acceleration $(12 - 3t^2)\mathbf{i}\ ms^{-2}$, where i is the unit vector in the direction of motion, and t is the time measured in seconds from the start of the motion.

(a) Determine the position of the particle when it next comes to rest. **4**

(b) Find the velocity of the particle when it returns to the origin. **3**

2.

Motorcyclist A has uniform acceleration $-2\mathbf{j}\ ms^{-2}$, initial velocity $\mathbf{i}\ ms^{-1}$ and initial position $-\mathbf{i}$ metres relative to a rectangular coordinate system with unit vectors **i, j** in the x, y directions respectively.

(a) Find the position $r_A(t)$ of the motorcyclist A at time t seconds, where t is measured from the start of the motion.

The position of a second motorcyclist B relative to the same coordinate system as A is **1**

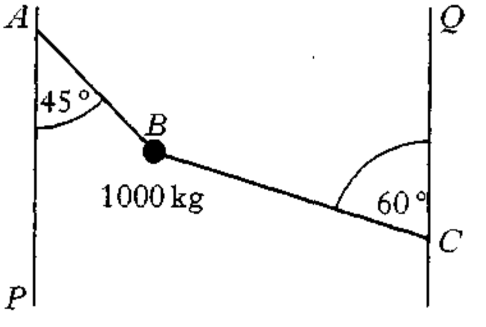
$$r_B(t) = (2t - 3)\mathbf{i} + (1 - t^2)\mathbf{j}.$$

(b) (i) Find the position of A relative to B.

(ii) Calculate the minimum distance between the motorcyclists. **3**

3.

On a construction site, a 1000kg concrete block is supported in equilibrium by two light inextensible chains AB and BC, attached to the block at B, as shown below.

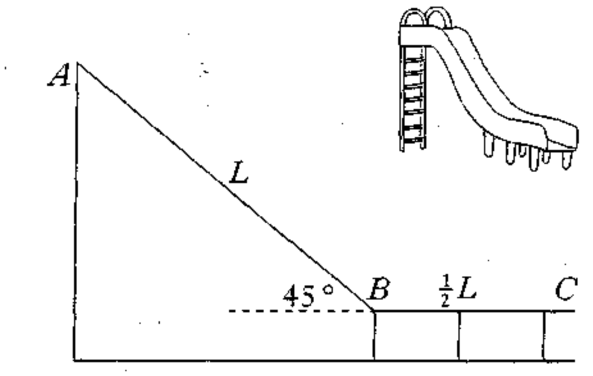


PA and CQ are vertical with *angle* $PAB = 45°$ and *angle* $BCQ = 60°$. The tensions in the chains over sections AB and BC are denoted by T_1 and T_2, respectively. **2**

(a) By resolving the forces horizontally, find a relationship between T_1 and T_2 **3**

(b) Calculate the tension T_2

4. The diagram below shows a slide in a playground. The section AB of the chute has length L metres and is inclined at an angle of 45° to the horizontal, whereas section BC is horizontal and has length $^1/_2L$ metres.



Starting from rest, Jill slides down the chute from A to C. Over both sections of the chute a frictional force acts on Jill where the coefficient of friction between her and the chute is $^1/_2$

.*(a)* Find the speed of Jill at the point *B*. **4**

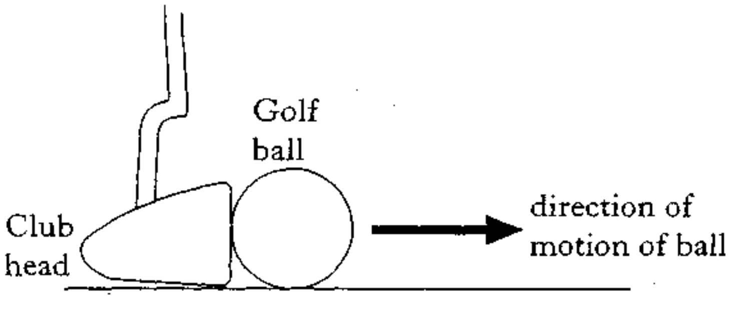
(b) Assuming that there is no change of speed as Jill moves from the sloping part of the slide to its horizontal part, show that her speed at *C* is given by

$$\sqrt{\frac{gL(\sqrt{2}-1)}{2}}\ \text{m s}^{-1},$$ **3**

where g ms^{-2} is the magnitude of the acceleration due to gravity.

5.

An experiment is performed to test a new design of golf club. The club head exerts a constant force of magnitude F newtons for T seconds on an initially stationary golf ball of mass m kg. The golf ball moves off in a horizontal direction as shown. The time t seconds is measured from the moment that the club head comes in contact with the golf ball.



(a) When $0 < t < T$, find an expression for the speed of the golf ball in terms of F, m and t. **2**

Write down an expression for the speed of the golf ball for $t > T$. **1**

(b) Find, in terms of F, m and T, the total work done by the club on the golf ball. **3**

6. An ice puck of mass *m kg is* projected across a horizontal ice rink with initial velocity $2\mathbf{i}$ ms^{-2} , where $\mathbf{i}$ is the unit vector in the direction of motion. A resistive force of $-0.05\, mv\, \boldsymbol{i}$ newtons acts on the puck, where v $\mathbf{i}$ ms^{-1} is the velocity of the puck at time t seconds from the start of the motion.

(a) Write down a differential equation for v, and hence find v in terms of t. **4**

(b) Calculate the time taken for the velocity of the puck to reduce to half of its initial value. **2**

7.

A missile is launched from ground level with speed V ms^{-1} at an angle of 30° to the horizontal.

(a) Show that the height y metres of the missile at time t is given by

$$y = \tfrac{1}{2}t\,(V - gt),$$

where g ms^{-2} is the magnitude of the acceleration due to gravity, and t is measured in seconds from the moment of launch. **3**

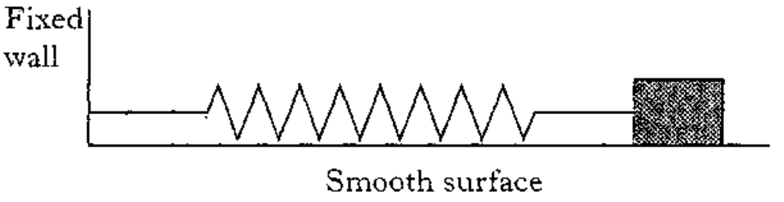
(b) Find the maximum height H attained by the missile, giving your answer in terms of V and g **2**

(c) A missile is detected on radar if $y \geq \frac{1}{4}H$.

Show that the missile appears on radar for $\sqrt{3}V/(2g)$ seconds. **5**

8.

A block of mass *m kg* is at rest on a smooth horizontal surface, as shown below. A spring with stiffness constant k is attached to the block and to a rigid wall.



The block is displaced to the right by a small distance a metres from the equilibrium position and then released.

(a) Show that the displacement, x metres, $(|x| < a)$, of the block from the equilibrium position at time t seconds after it is released satisfies the differential equation

$$\frac{d^2x}{dt^2} = -\omega^2 x.$$

Express ω^2 in terms of k and m.

Hence show that

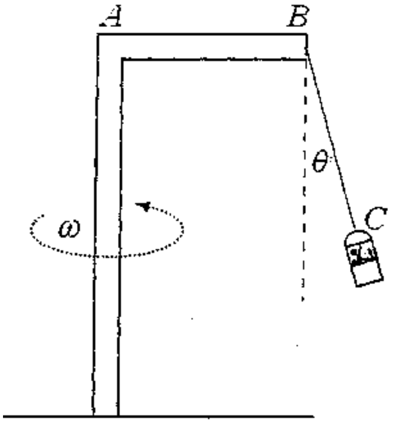
$$v^2 = \omega^2(a^2 - x^2),$$

where v ms^{-1} is the speed of the block at time t seconds.

(b) Calculate the positions of the block where the kinetic energy of the block equals the potential energy stored in the spring.

Find the time taken for the block to travel once between these two positions, expressing your answer in terms of ω.

9. A fairground ride consists of a "car" which is rotaced at angular speed ω radians per second about a vertical pole, as shown below. The rotating mechanism consists of a horizontal arm *AB of* length $L_{()}$ metres and a light chain *BC* of length L_1 metres to which the car is attached at *C*. The chain makes an angle θ *to the* vertical when the angular speed is ω. Assume that the chain remains taut throughout the motion during which *A*, *B* and *C* always lie in the vertical plane through the vertical pole.



(a) Show that the angular speed ω is related to L_0, L_1 and θ by the equation

$$\omega^2 = \frac{g\tan\theta}{L_0 + L_1\sin\theta},$$

where g ms^{-2} is the magnitude of the acceleration due to gravity.

(b) The operator of the ride wishes to compare the angular speeds required when $0 = 30°$ with $0 = 60°$ when $L_1 = 2L_0$. Denoting the angular speeds at 30° and 60° by ω_1 and ω_2, respectively, show that

$$\omega_2^2 = \frac{6}{1+\sqrt{3}}\omega_1^2.$$