Mathematics of Mechanics (Advanced Higher) Force, Energy and Periodic Motion		
1.1 Applying skills to principles of momentum, impulse, work, power and energy		
Sub-skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Working with impulse as the change in momentum, and/or force as the rate of change of momentum	♦ Use impulse appropriately in a simple situation, making use of the equations $\mathbf{I} = m\mathbf{v} - m\mathbf{u} = \int \mathbf{F}dt$ and $I = Ft$	Force has been referred to in the *Linear and Parabolic Motion Unit,* section 1.4. Examples may include bouncing balls, collisions of objects etc.
Working with the concept of conservation of linear momentum	♦ Use the concept of the conservation of linear momentum: $m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$ or $m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = (m_1 + m_2)\mathbf{v}$ for bodies that coalesce. ➢ Solve problems on linear motion in lifts, recoil of a gun, pile drivers, etc.	Familiarity with Newton's second law, $F = ma$ and force as the rate of change of momentum, is essential. Equations of motion with constant acceleration may occur.
Determining work done by a constant force in one or two dimensions, or a variable force during rectilinear motion	♦ Evaluate appropriately the work done by a constant force, making use of the equations $W = Fd$ (one dimension) ➢ $W = \mathbf{F} \cdot \mathbf{d}$ (two dimensions) ➢ Determine the work done in rectilinear motion by a variable force, using integration: $W = \int \mathbf{F} \cdot \mathbf{i}\, dx = \int \mathbf{F} \cdot \mathbf{v}dt$ where $\mathbf{v} = \frac{dx}{dt}\mathbf{i}$	Learners should appreciate that work can be done by or against a force. Examples may be taken from transport, sport, fairgrounds etc.

	♦ Apply to practical examples the concept of power as the rate of doing work: $P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$ (constant force)	Problems involving inclined planes should be studied.
Using the concepts of kinetic (E_K) and/or potential (E_P) energy to applying the work-energy principle	♦ $E_K = \frac{1}{2}mv^2$, $E_P = mgh$ for a uniform gravitational field ♦ Work done = change in energy ➢ $E_P = \frac{\lambda x^2}{2l}$ for elastic strings/springs ➢ $E_P = \frac{GMm}{r}$ associated with Newton's Inverse Square Law	Learners should be familiar with the difference between kinetic and potential energy, and the meaning of conservative forces such as gravity, and non-conservative forces such as friction. This can be linked with motion along an inclined plane within *Linear and Parabolic Motion 1.4.* Link with Simple Harmonic Motion from *Force, Energy and Periodic Motion 1.3* Link with horizontal circular motion from *Force, Energy and Periodic Motion 1.2.*
Using the concepts of kinetic (E_K) and/or potential (E_P) energy within the concept of conservation of energy	♦ $E_K + E_P$ is constant for simple problems involving motion in a plane ➢ Use of this within a situation involving vertical circular motion	Conditions required to perform full circles should be considered, including cases with a particle attached to an inextensible string, a particle on the end of a light rod, a bead running on the inside or the outside of a cylinder and a bead on a smooth circular wire. Examples should include calculating the initial speed of projection required for each of these cases. For particles of equal mass, describing circles of equal radius, consideration should be given to the requirement for a greater speed of projection in the case of an inextensible string versus a light rod.

1.2 Applying skills to motion in a horizontal circle with uniform angular velocity		
Sub-skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Applying equations to motion in a horizontal circle with uniform angular velocity	♦ Solve problems involving motion in a circle of radius r, with uniform angular velocity ω, making use of the equations: $\theta = \omega t$ $v = r\omega = r\dot{\theta}$ $a = r\omega^2 = r\dot{\theta}^2 = \frac{v^2}{r}$ $\mathbf{a} = -\omega^2\mathbf{r}$ $T = \frac{2\pi}{\omega}$ ➢ Apply these equations to motion including skidding, banking and other applications.	Terms used will include: angular velocity, angular acceleration, radial and tangential components. Vectors should be used to establish these equations, starting from $\mathbf{r} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j}$, where r is constant and θ is varying, before considering the special case where $\theta = \omega t, \omega$ being constant: Hence, if $\boldsymbol{e}_r = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}$ and $\boldsymbol{e}_\theta = -\sin\theta\mathbf{i} + \cos\theta\mathbf{j}$ are the unit vectors in the radial and tangential directions respectively, it follows that the radial and tangential components of velocity are $\mathbf{0}$ (zero vector) and $r\dot{\theta}\boldsymbol{e}_\theta$ respectively, and those of acceleration are $-r\dot{\theta}^2 e_r$ and $r\ddot{\theta}\boldsymbol{e}_\theta$ respectively. Examples could include motion in a horizontal circle around a banked surface, including skidding, the 'wall of death', the conical pendulum.
Using equations for horizontal circular motion alongside Newton's Inverse Square Law of Gravitation	♦ Solve a simple problem using Newton's Inverse Square Law, $F = \frac{GMm}{r^2}$ Identify modelling assumptions made in particular contexts. Examples include applying this to simplified motion of satellites and moons, making use of the equations of motion for horizontal circular motion to find the time for one orbit, the height of the satellite above the planet's surface etc.	Appreciation is needed that the magnitude of the gravitational force of attraction between two particles is inversely proportional to the square of the distance between the two particles. Motion here will consider circular orbits only, and additional effects, such as the rotation of a moon about its own axis whilst orbiting a planet, can be ignored. Link with gravitational potential energy from *Force, Energy and Periodic Motion 1.1*. Link with escape velocity.

1.3 Applying skills to Simple Harmonic Motion		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Working with the concept of Simple Harmonic Motion (SHM)	♦ Understand the concept of SHM and use the basic equation $\ddot{x} = -\omega^2 x$, and the following associated equations, knowing when and where they arise in order to solve basic problems involving SHM in a straight line: $v^2 = \omega^2(a^2 - x^2)$ where $v = \dot{x}$ $T = \frac{2\pi}{\omega}$ $\lvert v \rvert_{\max} = \omega a$ $\lvert \ddot{x} \rvert_{\max} = \omega^2 a$ ➢ Apply the solutions $x = a\sin(\omega t + \alpha)$ and the special cases $x = a\sin\omega t$ and $x = a\cos\omega t$ to solve problems	Terms used will include: oscillation, centre of oscillation, period, amplitude, frequency, maximum velocity and maximum acceleration. $v^2 = \omega^2(a^2 - x^2)$ can be derived from the solution of a separable first order differential equation. Linked with *Mathematical Techniques for Mechanics 1.4.* At this stage these solutions can be verified or established from $\mathbf{r} = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j}$ rotating round a circle.
Applying Hooke's Law to problems involving SHM	♦ Make use of the equation for Hooke's Law $T = \frac{\lambda x}{l}$, to determine an unknown tension/thrust, modulus of elasticity or extension/compression of natural length.	Terms used should include: tension, thrust, natural length, stiffness constant, modulus of elasticity, extension, compression, position of equilibrium, oscillation. Learners should appreciate that the tension in the string or spring is directly proportional to the extension from the natural length: ie $T = kx$, where the stiffness constant (k) is equivalent to $\frac{\lambda}{l}$, with λ being the modulus of elasticity and l being the natural length of the string/spring.

	➢ Consider the position of equilibrium and the equation of motion for an oscillating mass, and apply these to the solution of problems involving SHM, including problems involving shock absorbers, small amplitude oscillations of a simple pendulum etc	These will include problems involving elastic strings and springs, and a simple pendulum, but not the compound pendulum. Learners should be aware that SHM and linear motion could arise in the same context for a stretched string.
1.4 Applying skills to Centres of Mass		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Determining the turning effect of force	♦ Evaluate the turning effect of a single force or a set of forces acting on a body, considering clockwise and anticlockwise rotation: Moment of force about point *P* = magnitude of force *x* perpendicular distance from *P* *and/or* Understand that for a body in equilibrium the sum of the moments of the forces about any point is zero	Practical investigations on closing a door, using a spanner, balancing a seesaw etc should provide good opportunity for discussion. The effect of both a single force, in changing its point of application, and the effect of several forces should be considered. Only uniform rods in equilibrium with at most 3 forces acting should be considered.
Using moments to find the centre of mass of a body	♦ Equate the moments of several masses acting along a line to that of a single mass acting at a point on the line $\sum m_i x_i = \bar{x}\sum m_i$ where $(\bar{x}, 0)$ is the centre of mass of the system ➢ Extend this to two perpendicular directions to find the centre of mass of a set of particles arranged in a plane. ie $\sum m_i x_i = \bar{x}\sum m_i$ and $\sum m_i y_i = \bar{y}\sum m_i$ where $(\bar{x}, \bar{y})$ is the centre of mass of the system	Horizontal or vertical rods with up to three particles placed on them. Take moments about an axis forming a boundary of the body so that all moments are acting in the same sense.

	♦ Find the positions of centres of mass of standard uniform plane laminas, including rectangle, triangle, circle and semicircle. For a triangle, the centre of mass will be $\frac{2}{3}$ along median from vertex. For a semicircle, the centre of mass will be $\frac{4r}{3\pi}$ along the axis of symmetry from the diameter.	Centres of mass will lie on any axis of symmetry. Learners could investigate centres of mass of solid shapes but these **will not** be assessed in this Course.
	➢ Apply integration to find the centre of gravity of a uniform composite lamina of area A, bounded by a given curve $y = f(x)$ and the lines $x = a$ and $x = b$ $A\bar{x} = \int_a^b xy\,dx \quad A\bar{y} = \int_a^b \frac{1}{2} y^2\,dx$	Split the composite shape into several standard shapes. Identify the centre of mass of each shape and its position from a fixed point. Replace the lamina by separate particles and consider moments. Finding the centre of mass of a logo, perforated sheet, loaded plate etc.

Mathematics of Mechanics (Advanced Higher): Linear and Parabolic motion		
1.1 Applying skills to motion in a straight line		
Skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Working with time dependent graphs	♦ Sketch and annotate, interpret and use displacement/time, velocity/time and acceleration/time graphs. ♦ Determine the distance travelled using the area under a velocity/time graph.	Velocity/time graphs for both constant and variable acceleration should be considered. Displacement as area under a velocity/time graph may be linked to *Mathematical Techniques for Mechanics 1.3.* Learners could be encouraged to sketch a displacement/time graph from a velocity/time graph.
Working with rates of change with respect to time in one dimension	♦ Use calculus to determine corresponding expressions connecting displacement, velocity and acceleration eg If $s = 2t^3 - 21t^2 + 60t$ find expressions for velocity and acceleration given specific conditions. eg If the acceleration of a body is given by $a = 4t - t^2$, find the velocity and displacement when $t = 4$ seconds, given that the initial velocity is 3ms^{-1} when the body is 1 m from the origin.	The dot notation for differentiation with respect to time $\dot{x} = \frac{dx}{dt}$ and $\ddot{x} = \frac{d^2x}{dt^2}$ may be used. ↓ Differentiate ↓ — Displacement: x — Velocity: $\dot{x} = \frac{dx}{dt}$ — Acceleration: $\ddot{x} = \frac{d^2x}{dt^2}$ — ↑ Integrate ↑ The solution of differential equations in **this** unit will only require simple integration.

Using equations of motion in one dimension under constant acceleration	➢ Using calculus derive the equations of motion: $v = u + at$ and $s = ut + \frac{1}{2}at^2$ and use these to establish the equations: $v^2 = u^2 + 2as$ $s = \frac{u+v}{2}t$ $s = vt - \frac{1}{2}at^2$. ♦ Use these equations of motion in relevant contexts eg Given the initial and final velocities of a particle moving with constant acceleration, find the time of motion and the displacement. ➢ eg A stone is dropped from the top of a tower. In the last second of its motion, it falls one fifth of the height of the tower. Find the height of the tower. ♦ Identify modelling assumptions made in particular contexts.	Equations of motion under ***constant*** acceleration could be derived from definition of constant acceleration and displacement, as well as deriving these equations using calculus. One-dimensional motion and freefall under gravity must be considered. Stopping distances at traffic lights, speed cameras, etc can be investigated. Air friction (resistance) in vertical motion, constant velocity and interpretation of negative velocity, variation in the value of g, and the need to model all bodies as particles should be discussed.

1.2 Applying skills to vectors associated with motion		
Sub-skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Using vectors to define displacement, velocity and acceleration	♦ Give the displacement, velocity and acceleration of a particle as a vector and understand speed is the magnitude of the velocity vector If $\underline{r} = \begin{pmatrix} x \\ y \end{pmatrix}$ where *x* and *y* are functions of *t* then $\underline{v} = \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$ and $\underline{a} = \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix}$ eg If $\mathbf{v} = \begin{pmatrix} \sin 2t \\ t+1 \\ \cos t + \sin t \end{pmatrix}$ find expressions for displacement and acceleration given specific conditions.	Vectors can be expressed as column vectors or using $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ notation: $\begin{pmatrix} a \\ b \\ c \end{pmatrix} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ Learners should be familiar with the notation: $\mathbf{r}_{\mathrm{P}}$ for the position vector of P $\mathbf{v}_{\mathrm{P}} = \dot{r}_{\mathrm{P}}$ for the velocity vector of P $\mathbf{a}_{\mathrm{P}} = \dot{\mathbf{v}}_{\mathrm{P}} = \ddot{\mathbf{r}}_{\mathrm{P}}$ for the acceleration vector of P $\mathbf{i}, \mathbf{j}, \mathbf{k}$, as the unit vectors in x, y and z directions Speed $= \lvert \mathbf{v} \rvert = \lvert \dot{\mathbf{r}} \rvert$ Learners will meet this process when studying implicit differentiation in *Mathematical Techniques for Mechanics 1.2.* This can be extended to three dimensions. Differentiation and integration can be used to find displacement, velocity and acceleration given one of these as a function of time.
Finding resultant velocity, relative velocity or relative acceleration of one body with respect to another	♦ Resolve position, velocity and acceleration vectors into 2 and 3 dimensions and use these to consider resultant or relative motion eg A man can travel at $3{\cdot}5\,\mathrm{ms}^{-1}$ in still water. A river is $80\,\mathrm{m}$ wide and its current flows at $2\,\mathrm{ms}^{-1}$. Find the shortest time taken to cross the river and the distance downstream that	**Two dimensions:** If a body is travelling in xy-plane with speed $v\,\mathrm{ms}^{-1}$ making an angle $\theta°$ with OX, then its velocity vector can be expressed as $v\cos\theta\mathbf{i} + v\sin\theta\mathbf{j}$. **Three dimensions:** If body is travelling in xyz-plane with speed $v\,\mathrm{ms}^{-1}$ making an angle $\theta_1°$ with OX, $\theta_2°$ with OY and

	the boat is carried. ➢ eg A man can travel at $3\cdot5\,\mathrm{m\,s^{-1}}$ in still water. A river is $80\ \mathrm{m}$ wide and its current flows at $2\,\mathrm{m\,s^{-1}}$. Find the course set to cross the river directly and the time taken for such crossing ♦ Apply position, velocity and acceleration vectors to practical problems, including navigation, the effects of winds and currents and other relevant contexts. eg To a man driving due North at $40\,\mathrm{km\,h^{-1}}$ the wind appears to come from N60°W with a speed of $30\,\mathrm{km\,h^{-1}}$. What is the actual velocity of the wind? ➢ When a ship travels due North at $40\,\mathrm{km\,h^{-1}}$ the wind appears to come from N40°E. When it travels South at $50\,\mathrm{km\,h^{-1}}$ the wind appears to come from South East. Find the true velocity of the wind.	$\theta_3°$ with OZ, then its velocity vector can be expressed as $v\cos\theta_1\mathbf{i}+v\cos\theta_2\mathbf{j}+v\cos\theta_3\mathbf{k}$. Learners should be familiar with the notation: ${}_q\mathbf{r}_p=\overrightarrow{PQ}=\mathbf{q}-\mathbf{p}$ for the position vector of Q relative to P ${}_q\mathbf{v}_p=\mathbf{v}_q-\mathbf{v}_p=\dot{\mathbf{r}}_q-\dot{\mathbf{r}}_p$ for the velocity of Q relative to P ${}_q\mathbf{a}_p=\mathbf{a}_q-\mathbf{a}_p=\dot{\mathbf{v}}_q-\dot{\mathbf{v}}_p=\ddot{\mathbf{r}}_q-\ddot{\mathbf{r}}_p$ for the acceleration of Q relative to P. Contexts such as crossing a river, flying between airports etc can be used explore the effects of currents and winds.
Applying understanding of relative motion	♦ Solve a simple problem involving collision eg given position and velocity vectors for two bodies prove that they will collide. ➢ eg find the course needed for interception of one body by another ➢ Consider conditions for nearest approach eg Find the shortest distance between two moving bodies. eg Find the time to closest approach. eg Find the time for which one moving body is within a certain range of another.	For collision, both positions after time t being equal and ${}_a v_b$ to be in the direction of original relative position should be investigated. For nearest approach, both 'least separation' by differentiation and the vector condition ${}_p r_q \bullet {}_p v_q = 0$ can be explored. Suitable contexts could include shipping and aircraft movement and sports contexts.

1.3 Applying skills to projectiles moving in a vertical plane.		
Sub-skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Establishing the conditions of motion in horizontal and vertical directions involved in parabolic motion	♦ Derive the formulae $T = \frac{2u\sin\alpha}{g}$ $H = \frac{u^2\sin^2\alpha}{2g}$ $R = u\cos\alpha \times T = \frac{u^2\sin 2\alpha}{g}$ where T refers total time of flight H refers to greatest height R refers to the horizontal range	Learners should derive these formulae from equations of motion or by using calculus. Learners should appreciate that gravity only affects motion in the vertical plane and so motion of the projectile will be approached by considering vertical motion and horizontal motion separately. Learners can be reminded of the properties of the parabola, eg relate time of flight, T, with time to greatest height. This can be done by either: solving the vector equation $\ddot{\mathbf{r}} = -g\mathbf{j}$ to obtain expressions for $\dot{x}, \dot{y}, x$ and y in a particular case or using the equations of motion under constant acceleration.
Using the equations of motion in parabolic flight	♦ Use these formulae to find the time of flight, greatest height reached, or range of a projectile including maximum range of a projectile and the angle of projection to achieve this. ➢ Derive and use the equation of the trajectory of a projectile: $y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$ ♦ Solve problems in two-dimensional motion involving projectiles under a constant gravitational force. Projection will be considered in one vertical plane but point of projection can be from a different horizontal plane than that of landing.	Learners should be able to derive the equation of the trajectory. Sport will provide good context for this work. Projection from an inclined plane is not required.

1.4 Applying skills to forces associated with dynamics and equilibrium.		
Sub-skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Using Newton's first and third laws of motion to understand equilibrium	♦ Resolve forces in two dimensions to find their components. eg If a body is in equilibrium under three forces of which one is unknown, resolve vertically and horizontally to find the magnitude of the third force. eg Consider the equilibrium of a body sitting on an inclined plane. ➢ Consider the equilibrium of connected particles. ♦ Combine forces to find the resultant force, eg by resolving vertically and horizontally to find the resultant	Learners should understand the concepts of weight, friction, tension, resistance, normal reaction and gravity as expressions of force. When there is more than one force acting on a body, we choose to find the effects of all forces in two mutually perpendicular directions. Tension in the elastic string will be investigated in *Force, Energy and Periodic Motion 1.3.* Learners could investigate pulley systems but these **will not** be assessed in this Course. Lami's Theorem may be used but learners should understand that it is of limited application.
Understanding the concept of static friction, dynamic friction and limiting friction	♦ Know and use the relationships $F = \mu R$ and $\mu = \tan\theta$ for bodies on a slope. eg A particle is held on a rough slope inclined at 20° to the horizontal. Find the coefficient of friction between the particle and the plane. ➢ For stationary bodies $F \le \mu R$ ♦ Solve problems involving a particle or body in equilibrium under the action of certain forces. eg Bodies in equilibrium on rough planes. eg State modelling assumptions in questions. ➢ eg Apply an external force to keep a body in equilibrium on a slope and consider limiting equilibrium for movement along the line of greatest slope.	Consider a body in equilibrium on a plane and resolved forces will lead to $\mu = \tan\theta$, where θ is the angle between the slope and the horizontal. If $\tan\theta > \mu$ the body will accelerate down the slope. Understand that there is a limiting value of friction, $F_{\max} = \mu R$ during motion and this implies that $F \le \mu R$ where bodies are stationary.

Using Newton's Second Law of motion	♦ Use $F = ma$ to form equations of motion to model practical problems of motion in a straight line.	Learners should understand that the acceleration of a body is proportional to the resultant external force and takes place in the direction of the force. When $\mathbf{F} = m\mathbf{a}$ is a vector equation, the acceleration produced is in the direction of the applied or resultant force. Links with parabolic motion of projectiles are encouraged.
	♦ Solve problems involving motion on inclined planes, possibly including friction.	Equilibrium on inclined planes will have been considered earlier in this Unit. Both smooth and rough planes should be included. These questions can also be solved using energy considerations — *Force, Energy and Periodic Motion 1.1*.

Mathematics of Mechanics: Mathematical Techniques for Mechanics (Advanced Higher)		
1.1 Applying algebraic skills to partial fractions		
Sub-skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Expressing rational functions as a sum of partial fractions (denominator of degree at most 3 and easily factorised)	♦ Express a proper rational function as a sum of partial fractions where the denominator may contain: distinct linear factors, an irreducible quadratic factor, a repeated linear factor: eg i) $\frac{7x+1}{x^2+x-6}$ (linear factors) ii) $\frac{5x^2-x+6}{x^3+3x}$ (irreducible quadratic factor) iii) $\frac{3x+10}{(x+3)^2}$ (repeated linear factor) ➢ Reduce an improper rational function to a polynomial and a proper rational function by division or otherwise eg $\frac{x^3+2x^2-2x+2}{(x-1)(x+3)}$ eg $\frac{x^2+3x}{x^2-4}$	This is also required for integration of rational functions and may be used with differential equations where the solution requires separating the variables. Some discussion of horizontal and vertical asymptotes in relation to graph sketching should occur with this work but will not be assessed.

1.2 Applying calculus skills through techniques of differentiation		
Sub-skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Differentiating exponential and logarithmic functions	♦ Differentiate functions involving e^x, $\ln x$ eg $y = e^{x^2} + 4$ eg $f(x) = \ln(x^3 + 2)$	
Differentiating functions using the chain rule	♦ Apply the chain rule to differentiate the composition of at most 3 functions eg $y = \sqrt{e^{x^2} + 4}$ eg $f(x) = \sin^3(2x - 1)$	
Differentiating functions given in the form of a product and/or in the form of a quotient	♦ Differentiate functions of the form $\left(f(x)g(x)\right)$ and/or $\left(\frac{f(x)}{g(x)}\right)$ eg $y = 3x^4 \sin x$ eg $f(x) = x^2 \ln x,\ x > 0$ eg $y = \frac{2x - 5}{3x^2 + 2}$ eg $f(x) = \frac{\cos x}{e^x}$ ➢ Use the derivative of $\tan x$ ➢ Know the definitions of $\cot x$, $\sec x$, $\operatorname{cosec} x$. Learners	Learners could be introduced to product and quotient rules with formal proofs but these would not be assessed. When learners have mastered differentiation rules they can be shown how to use computer algebra systems (CAS). These cannot be used in assessment but their suitability for difficult/real examples can be discussed. When software is used for differentiation in difficult cases, learners should understand which rules were needed for solution. Learners should be exposed to deriving $\cot x$, $\sec x$, $\operatorname{cosec} x$.

	should be able to use derivatives of $\tan x,\ \cot x,\ \sec x,\ \operatorname{cosec} x$. ➢ Differentiating functions which require more than one application or combination of applications of chain rule, product rule and quotient rule eg i) $y = e^{2x} \tan 3x$ ii) $y = \ln\lvert -3 + \sin 2x \rvert$ iii) $y = \dfrac{\sec 2x}{e^{3x}}$ ➢ Know that $\dfrac{dy}{dx} = \dfrac{1}{\left(\dfrac{dx}{dy}\right)}$	Chain rule: $a = \dfrac{dv}{dt} = \dfrac{dv}{dx} \times \dfrac{dx}{dt} = \dfrac{dv}{dx} \times v = v\dfrac{dv}{dx}$ Apply differentiation to simple rates of change, eg rectilinear motion and optimisation.
Finding the derivative of functions defined implicitly	♦ Use differentiation to find the first derivative of a function defined implicitly including in context eg $x^3 y + xy^3 = 4$ eg Apply differentiation to related rates in problems where the functional relationship is given implicitly. For example, spherical balloon losing air at a given rate. ➢ Use differentiation to find the second derivative of a function defined implicitly	Learners should have a clear understanding of an implicit function and see that some can be manipulated to give an explicit function but this method will allow differentiation to be used with all implicit functions. The use of implicit functions to differentiate exponential functions such as $f(x) = 5^x$ by using logs initially should be explored. Acceleration as an implicit function: $\dfrac{d}{dx}(\dfrac{1}{2}v^2) = \dfrac{1}{2} \times 2v \times \dfrac{dv}{dx} = v\dfrac{dv}{dx}$

Finding the derivative of functions defined parametrically	♦ Use differentiation to find the first derivative of a function defined parametrically eg Apply parametric differentiation to motion in a plane If the position is given by $x = f(t)$, $y = g(t)$ then i) Velocity components are given by $v_x = \frac{dx}{dt}$ $v_y = \frac{dy}{dt}$ ii) Speed = $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ eg Apply differentiation to related rates in problems where the functional relationship is given explicitly. ➢ Solve practical related rates by first establishing a functional relationship between appropriate variables eg A snowball in the shape of a sphere is rolling down a hill with its radius increasing at a uniform rate of $0 \cdot 5$ cms^{-1}. How fast is the volume increasing when the radius is 4 cm?	Parameters should be introduced by using IT to sketch graphs where *x* and *y* are different functions of a variable, eg $x = 4\cos\theta$ $y = 4\sin\theta$ representing a circle. Another example in context could be calculating the rate at which the depth of coffee in a conical filter is changing.

1.3 Applying calculus skills through techniques of integration		
Sub-skill	**Description of Unit standard and added value**	**Learning and teaching contexts**
Integrating expressions using standard results	♦ Use $\int e^x dx, \int \frac{dx}{x}, \int \sec^2 x dx$ eg $\int e^{5x-7} dx, \int \frac{dx}{2x-4}, x \neq 2$ ➢ Recognise and integrate expressions of the form $\int g(f(x)) f'(x) dx$ and $\int \frac{f'(x)}{f(x)} dx$ eg $\int \cos^3 x \sin x dx$ eg $\int x e^{x^2} dx$ eg $\int_0^2 \frac{2x}{x^2+3} dx$ eg $\int \frac{\cos x}{(5+2\sin x)} dx$	

	♦ Use partial fractions to integrate proper rational functions where the denominator may have: i) two separate or repeated linear factors ii) three linear factors with constant numerator eg $\int \frac{4x-9}{(x-2)(x-3)}dx$, $\int \frac{x+3}{(x+5)^2}dx$, $\int \frac{6}{(x-1)(x+2)(x+1)}dx$ ➢ Use partial fractions to integrate proper rational functions where the denominator may have three linear factors with a non-constant numerator eg $\int \frac{x^2-5}{(2-x)(3+x)(1-x)}dx$	Learners should be competent with the process of expressing a rational function in partial fractions (1.1). Some revision of logarithmic functions might be useful before working with integrals here. This can be linked to solving problems involving motion with resistance in section 1.4.
Integrating using a substitution when the substitution is given	♦ Integrate where the substitution is given eg Use the substitution $u = \ln x$ to obtain $\int \frac{1}{x \ln x}dx,$ where $x > 1.$ eg Use the substitution $u = 3x - 2$ to obtain $\int x\sqrt{3x-2}\,dx$ where $x > \frac{2}{3}$	

Integrating by parts	♦ Use integration by parts with one application eg $\int x\sin x\,dx$ ➢ Use integration by parts involving repeated applications eg $\int_0^{\pi} x^2\cos x\,dx$ eg $\int x^2e^{3x}dx$	This may arise again when using the integrating factor to solve first order differential equations.
Applying integration to a range of physical situations	♦ Apply integration to evaluate volumes of revolution about the x-axis ➢ Apply integration to evaluate volumes of revolution about the y-axis ➢ Apply integration to evaluate areas	This can be linked to finding displacement for velocity/time graphs and finding centres of mass in *Linear and Parabolic Motion Unit*.
1.4 Applying calculus skills to solving differential equations		
Sub-skill	**Description of Unit standard and Added Value**	**Learning and teaching contexts**
Finding a general solution of a first order differential equation with variables separable	♦ Solve equations that can be written in the form $\frac{dy}{dx}=\frac{g(x)}{h(y)}$ eg $\frac{dy}{dx}=y(x-1)$ eg $v\frac{dv}{dx}=-\omega^2x$ associated with SHM ➢ Find the particular solution where initial conditions are given	Learners should be reminded of the use of partial fractions, $\int\frac{1}{x}\,dx$ and manipulation of logarithmic terms before starting this work. Many of these equations arise naturally in mathematical modelling of physical situations and will be covered again in this section of study.

	eg $\frac{1}{x}\frac{dy}{dx} = y\sin x$ given that when $x = \frac{\pi}{2}$, $y = 1$ ➢ Use differential equations in context eg Bacterial growth at a rate proportional to the number of bacteria present at time *t:* $\frac{dB}{dt} = kt$ eg Vertical fall with resistive force: $m\frac{dv}{dt} = mg - kv$	Learners should appreciate the link with differentiation and discuss some physical situations such as: electrical circuits, vibrating systems and motion with resistance where models involving differential equations arise. The differential equation will show how the system will change with time (or other variable). Discussion of initial conditions will help lead to the complete solution. Scientific contexts such as chemical reactions, Newton's law of cooling, population growth and decay, bacterial growth and decay provide good examples and can build on the knowledge and use of logarithms.
Solving a simple first order linear differential equation using an integrating factor	♦ Solve equations written in the standard form $\frac{dy}{dx} + P(x)y = f(x)$ eg $\frac{dy}{dx} + \frac{3y}{x} = \frac{e^x}{x^3}$ ➢ Solve equations by first writing linear equations in the standard form $\frac{dy}{dx} + P(x)y = f(x)$ eg $x^2\frac{dy}{dx} + 3xy = \frac{\sin x}{x}$ ➢ Use differential equations in context eg Mixing problems, such as salt water entering a tank of clear water which is then draining at a given rate. $\frac{dM}{dt} = 15 - \frac{1}{100}M$	Learners should be aware of the derivation of the integrating factor method but this will not be assessed. Learners should appreciate the link with differentiation and discuss some physical situations such as: electrical circuits, vibrating systems and motion with resistance where models involving differential equations arise.

	eg Growth and decay problems, an alternative method of solution to separation of variables eg Simple electronic circuits: $L\frac{di}{dt}+Ri=V$ where *L*,*R* and *V* are constant.	The differential equation will show how the system will change with time (or other variable). Discussion of initial conditions will help lead to the complete solution. Scientific contexts such as chemical reactions, Newton's law of cooling, population growth and decay, bacterial growth and decay provide good examples and can build on the knowledge and use of logarithms.
Solving second order homogeneous equations	♦ Find the general solution of a second order homogeneous ordinary differential equation $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$ where the roots of the auxiliary equation are real and distinct eg $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=0$ ➢ where the roots of the auxiliary equation are real and equal eg $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$ ➢ Use differential equations in context eg simple examples of damped simple harmonic motion where the equation of motion is	Learners will use second order differential equation when working with Simple Harmonic Motion in Force, Energy and Periodic Motion Unit. Damped SHM should only form discussion. Learners should understand that real distinct roots of the auxiliary equation lead to heavy damping, equal roots to critical damping and unreal roots to light damping. Assessment would only require a statement in explanation.

	$m\ddot{x} = -m\omega^2 x - mkv$ $\ddot{x} = -\omega^2 x - kv$ $\ddot{x} + k\dot{x} + \omega^2 x = 0$ $\qquad \frac{d^2x}{dt^2} + k\frac{dx}{dt} + \omega^2 x = 0$	